**Math 110b Spring 2004** **Homework 3, due Friday, May 7**

1. Let $E_1$ and $E_2$ be vector bundles with connections $\nabla^1, \nabla^2$.

   (a) Show that there is a natural connection $\nabla^1 \oplus \nabla^2$ on the Whitney sum $E_1 \oplus E_2$.

   (b) Describe the curvature of this connection.

2. Roe, question 2.29 (p. 37).

3. (a) Roe, question 2.31 (p. 37).

   (b) Work this out for the special case of $SO(3)/SO(2) = S^2$. Remember that since the group G acts transitively on $G/H$, you only have to find the curvature at one point.

4. (a) Roe, question 2.36, (i)–(iv) (p. 39).

   (b) Show that if the oriented bundle $V$ splits as a Whitney sum $V = W \oplus \mathbb{R}$, then (for an appropriately chosen connection, cf. problem 2), $Pf(K) = 0$. Conclude that if $V$ has a nowhere vanishing section, then its Euler class is 0.

   (c) Using the result of problem 3, compute the Euler class of the oriented bundle $SO(3) \times_{SO(2)} \mathbb{R}^2 \to S^2$. What bundle is this?

5. Show that a principal $G$-bundle $\pi : P \to X$ is trivial (isomorphic to $\pi_1 : X \times G \to X$) if and only if it has a section. (Contrast with vector bundles, which have lots of sections.)

6. Find all the places in your notes where I said 'this is an exercise' and do the exercise.