

Solutions to Homework 13

Section 7.1

Problem 2. By assumption, $A\vec{v} = \lambda\vec{v}$. Multiply both sides by A^{-1} to get $\vec{v} = A^{-1}A\vec{v} = A^{-1}\lambda\vec{v}$. So $A^{-1}\vec{v} = \frac{1}{\lambda}\vec{v}$.

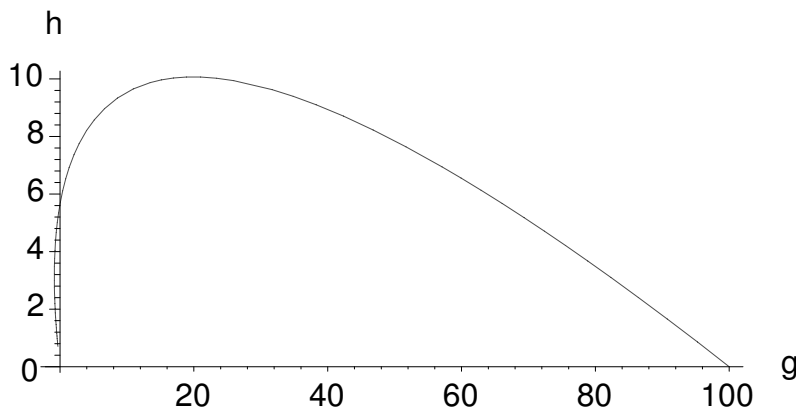
Problem 4. Since $A\vec{v} = \lambda\vec{v}$, we have $(7A)\vec{v} = 7\lambda\vec{v}$ so \vec{v} is an eigenvector of $7A$ with eigenvalue 7λ .

Problem 8. If $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = 5 \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, then $\begin{bmatrix} a \\ c \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$, so $a = 5$ and $c = 0$. So any such matrix will be of the form $\begin{bmatrix} 5 & b \\ 0 & d \end{bmatrix}$.

Problem 18. Any nonzero vector in the plane is sent to itself by the reflection, and hence is an eigenvector with eigenvalue 1. Any vector in the line E^\perp is sent to minus itself, and hence is an eigenvector with eigenvalue -1 . To make a basis of eigenvectors, choose any basis for E (two vectors) and throw in any nonzero vector in E^\perp .

Problem 36. a. $\begin{bmatrix} 0.978 & -0.006 \\ 0.004 & 0.992 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} -0.99 \\ 1.98 \end{bmatrix} = 0.99 \begin{bmatrix} -1 \\ 2 \end{bmatrix}$. Likewise, $\begin{bmatrix} 0.978 & -0.006 \\ 0.004 & 0.992 \end{bmatrix} \begin{bmatrix} 3 \\ -1 \end{bmatrix} = \begin{bmatrix} 2.94 \\ -0.98 \end{bmatrix} = 0.98 \begin{bmatrix} 3 \\ -1 \end{bmatrix}$. So the eigenvalues are 0.99 and 0.98.

b. $\vec{x}_0 = \begin{bmatrix} g_0 \\ h_0 \end{bmatrix} = \begin{bmatrix} 100 \\ 0 \end{bmatrix} = 20 \begin{bmatrix} -1 \\ 2 \end{bmatrix} + 40 \begin{bmatrix} 3 \\ -1 \end{bmatrix}$. So $\vec{x}(t) = 20(0.99)^t \begin{bmatrix} -1 \\ 2 \end{bmatrix} + 40(0.98)^t \begin{bmatrix} 3 \\ -1 \end{bmatrix}$, from which we read off $g(t) = -20(0.99)^t + 120(0.98)^t$ and $h(t) = 40(0.99)^t - 40(0.98)^t$. A plot is shown below; we see that $h(t)$ rises, then falls back to 0. On the other hand $g(t)$ goes a little below 0, then goes up back to 0.



c. We set $g(t) = 0$ and solve for $t \approx 176$ minutes.

Section 7.2

Problem 2. Since the matrix is lower-triangular, there are two eigenvalues, 1 and 2, each having algebraic multiplicity 2.

Problem 4. $\det(\lambda I - A) = \det \begin{bmatrix} \lambda & -4 \\ 1 & \lambda - 4 \end{bmatrix} = \lambda(\lambda - 4) + 4 = (\lambda - 2)^2$, so the eigenvalue 2 has algebraic

multiplicity 2.

Problem 10. The characteristic polynomial is $(\lambda + 1)^2(\lambda - 1)$, so the eigenvalues are -1 (with alg. multiplicity 2) and 1 .

Problem 20. In general, for a 2×2 matrix A , the characteristic polynomial is $\lambda^2 - \text{tr}(A)\lambda + \det(A)$. If the polynomial factors as $(\lambda - \lambda_1)(\lambda - \lambda_2)$, then multiplying this out and comparing terms gets $\det(A) = \lambda_1\lambda_2$ and $\text{tr}(A) = \lambda_1 + \lambda_2$.

Problem 22. Remember that the determinant of matrix is the same as that of its transpose. So the characteristic polynomial for A^T is given by $\det(\lambda I - A^T) = \det((\lambda I - A)^T) = \det(\lambda I - A)$, which is the characteristic polynomial for A . So A and A^T have the same eigenvalues and same algebraic multiplicities.

Problem 26. If you read through problem 25, you will see a couple of eigenvectors for $A = \begin{bmatrix} 0.5 & 0.25 \\ 0.5 & 0.75 \end{bmatrix}$, namely $\vec{v}_1 = \begin{bmatrix} 0.25 \\ 0.5 \end{bmatrix}$ (with eigenvalue 1) and $\vec{v}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ (with eigenvalue 0.25). In the phase diagram, this means that anything in the \vec{v}_1 direction is unchanged, but anything in the \vec{v}_2 direction is shrunk, by factor 0.25. The artist's conception of this is given below.

