Solutions to Homework 2

Section 1.1

Problem 6. $\left|\begin{array}{rcr} x+2y+3z & = & 8 \\ x+3y+3z & = & 10 \\ x+2y+4z & = & 9 \end{array}\right| \begin{array}{c} \\ -I \\ -I \end{array} \Rightarrow \left|\begin{array}{rcr} x+2y+3z & = & 8 \\ y & = & 2 \\ z & = & 1 \end{array}\right| \begin{array}{c} -2(II) \\ \\ \\ \end{array} \Rightarrow \left|\begin{array}{rcr} x & = & 1 \\ y & = & 2 \\ z & = & 1 \end{array}\right|$

Problem 20. The total demand for the product of industry A is 1000 (the consumer demand) plus $0.1b$ (the demand from industry B). The output a must meet this demand, so we have $a = 1000 + 0.1b$. Similarly, we have $b = 780 + 0.2a$. Solving this system gives $a = 1100$ and $b = 1000$.

Problem 36. Let b = Boris' money, m = Marina's money, and c = cost of a chocolate bar. The first statement gives the equation $\frac{1}{2}b + m = 2c$ and the second gives$b + \frac{1}{2}m = c$. Solving these gives $b = 0$ (and $m = 2c$, although we don't really care).

Section 1.2

Problem 2. $\left[\begin{array}{cccc} 3 & 4 & -1\vdots & 8 \\ 6 & 8 & -2\vdots & 3 \end{array}\right] \div 3 \Rightarrow \left[\begin{array}{cccc} 1 & \frac{4}{3} & -\frac{1}{3}\vdots & \frac{8}{3} \\ 6 & 8 & -2\vdots & 3 \end{array}\right] -6(I) \Rightarrow \left[\begin{array}{cccc} 1 & \frac{4}{3} & -\frac{1}{3}\vdots & \frac{8}{3} \\ 0 & 0 & 0\vdots & -13 \end{array}\right]$ This system is inconsistent, ie has no solutions.

Problem 12. The system reduces to $\left|\begin{array}{ccccccc} x_1 & & & & +3.5x_5 & +x_6 & =0 \\ & x_2 & & & +x_5 & & =0 \\ & & x_3 & & & -\frac{5}{3}x_6 & =0 \\ & & & x_4 & +3x_5 & +x_6 & =0 \end{array}\right| \Rightarrow \begin{array}{rl} x_1 = & -3.5x_5 - x_6 \\ x_2 = & -x_5 \\ x_3 = & \frac{5}{3}x_6 \\ x_4 = & -3x_5 - x_6 \end{array}$

Now let $x_5 = r$ and $x_6 = t$ and write $x_1, \ldots x_4$ in terms of r, t.

Problem 18. (a) No, since the third column contains two leading ones. (b) Yes. (c) No, since the third row contains a leading one, but the second row does not.

Problem 24. Yes, each elementary row operation is reversible, that is, it can be "undone.". The operation of row swapping can be undone by swapping the same rows again. The operation of dividing a row by a scalar can be reversed by multiplying the same row by the same scalar. Finally, the effect of adding c times row i to row j can be reversed by adding $-c$ times row i to row j.

Problem 34. We want all vectors $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ such that $\begin{bmatrix} x \\ y \\ z \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix} = 0$. Multiplying out the dot product gives the equation $x + 3y - z = 0$, whose matrix is already in rref. There is one leading 1 (coefficient of x) and two non-leading entries. So we can choose y and z arbitrarily, and the solution is $y = r$, $z = s$ and $x = s - 3r$. The set of vectors perpendicular to $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ forms a plane.