## Solutions to Homework 3

## Section 1.3

**Problem 2.** The rank is 3 since each row contains a leading one.

**Problem 4.** This matrix has rank 2 since its rref is $\begin{bmatrix}1 & 1 & 1\\0 & 0 & 0\\0 & 0 & 0\end{bmatrix}$.

**Problem 6.** There is no solution, since $\vec{v}_2$ is parallel to $\vec{v}_1$, and so any linear combination of $\vec{v}_1$ and $\vec{v}_2$ will be parallel to $\vec{v}_1$. But $\vec{v}_3$ isn't parallel to $\vec{v}_1$.

**Problem 10.** $\begin{bmatrix}1\\2\\3\end{bmatrix} \cdot \begin{bmatrix}1\\-2\\1\end{bmatrix} = 1 - 4 + 3 = 0.$

**Problem 18.** $\begin{bmatrix}1 & 2\\3 & 4\\5 & 6\end{bmatrix} \begin{bmatrix}1\\2\end{bmatrix} = 1\begin{bmatrix}1\\3\\5\end{bmatrix} + 2\begin{bmatrix}2\\4\\6\end{bmatrix} = \begin{bmatrix}5\\1\\17\end{bmatrix}.$

**Problem 22.** By fact 1.3.4, the rref is $\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$.

**Problem 28.** a. True; you can check that the new matrix satisfies the requirements for a matrix to be in rref listed on page 16.

b. False; removing the first column in $\begin{bmatrix}1 & -3 & 0\\0 & 0 & 0\end{bmatrix}$ gives a counterexample.

**Problem 48.** Saying that $\vec{x}_1$ is a solution to $A\vec{x} = \vec{b}$ means that $A\vec{x}_1 = \vec{b}$. So
a. $A(\vec{x}_1 + \vec{x}_h) = A\vec{x}_1 + A\vec{x}_h = \vec{b} + \vec{0} = \vec{b}$, and so $\vec{x}_1 + \vec{x}_h$ is also a solution.
b. If $\vec{x}_1, \vec{x}_2$ are both solutions, then $A(\vec{x}_1 - \vec{x}_2) = A\vec{x}_1 - A\vec{x}_2 = \vec{b} - \vec{b} = \vec{0}$.
c. Parts (a) and (b) show that the solutions of $A\vec{x} = \vec{b}$ are exactly the vectors of the form $\vec{x}_1 + \vec{x}_h$, where $\vec{x}_h$ is a solution of $A\vec{x} = \vec{0}$. To see this, note that if $\vec{x}_2$ is a solution to $A\vec{x} = \vec{b}$, then $A(\vec{x}_2 - \vec{x}_1) = \vec{0}$. On the other hand, we can write $\vec{x}_2 = \vec{x}_1 + (\vec{x}_2 - \vec{x}_1)$, so we can just set $\vec{x}_h = \vec{x}_2 - \vec{x}_1$.

## Section 2.1

**Problem 2.** Linear; matrix is $\begin{bmatrix}0 & 2 & 0\\0 & 0 & 3\\1 & 0 & 0\end{bmatrix}$.

**Problem 4.** $A = \begin{bmatrix}9 & 3 & -3\\2 & -9 & 1\\4 & -9 & -2\\5 & 1 & 5\end{bmatrix}$.

**Problem 6.** Note that $x_1\begin{bmatrix}1\\2\\3\end{bmatrix} + x_2\begin{bmatrix}4\\5\\6\end{bmatrix} = \begin{bmatrix}1 & 4\\2 & 5\\3 & 6\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}$, so that T is linear, with matrix $\begin{bmatrix}1 & 4\\2 & 5\\3 & 6\end{bmatrix}$.