

Solutions to Homework 6

Section 2.4

Problem 38. We need to find all matrices $X = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ such that $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix}$.

Multiplying this out gives $\begin{bmatrix} a & 2a+3b \\ c & 2c+3d \end{bmatrix} = \begin{bmatrix} a+2c & b+2d \\ 3c & 3d \end{bmatrix}$. From $c = 3c$, we get $c = 0$, and the only remaining equation is that $2a+3b = b+2d$. This is satisfied if $d = a+b$, so X must be of the form $\begin{bmatrix} a & b \\ 0 & a+b \end{bmatrix}$ for arbitrary a and b .

Problem 46. Written in terms of matrices, we are looking for a matrix A with

$$A \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 7 & 1 \\ 5 & 2 \\ 3 & 3 \end{bmatrix} \quad \text{which gives} \quad A = \begin{bmatrix} 7 & 1 \\ 5 & 2 \\ 3 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix}^{-1} = \begin{bmatrix} 33 & -13 \\ 21 & -8 \\ 9 & -3 \end{bmatrix}$$

Section 3.1

Problem 4. We're solving $A\vec{x} = \vec{0}$; the matrix is already in rref, so we read off the solution $\vec{x} = \begin{bmatrix} -2r-3s \\ r \\ s \end{bmatrix} = r \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + s \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}$. So the vectors $\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}$ span the kernel. (In fact, they're manifestly linearly independent, so they are a basis for the kernel.)

Problem 12. Solving the system of equations gives $\ker(A) = \text{span} \left(\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ -1 \\ 1 \\ 0 \end{bmatrix} \right)$.

Problem 16. The second and third columns are evidently multiples of the first, so the first column spans the image. Since there is a nonzero vector in the image, this one column is a minimal spanning set.

Problem 24. The image of the projection is the plane P itself, since $\text{Proj}(\vec{v}) = \vec{v}$ for $\vec{v} \in P$. The kernel is the line perpendicular to P , which I called P^\perp in class. This line is spanned by any nonzero vector that is perpendicular to P , for example $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$.

Problem 32. Take the transformation $T : \mathbb{R}^1 \rightarrow \mathbb{R}^3$ with $T\vec{e}_1 = \begin{bmatrix} 7 \\ 6 \\ 5 \end{bmatrix}$. Equivalently, think of T as being

given by the 3×1 matrix $\begin{bmatrix} 7 \\ 6 \\ 5 \end{bmatrix}$.

Problem 40. AB is the zero matrix. To see this, note that for any vector \vec{v} , $B\vec{v}$, being a vector in the image of B , must be in the kernel of A . Thus, $(AB)\vec{v} = A(B\vec{v}) = \vec{0}$.