Review Problems for Midterm 1

The midterm will cover the material we did in Chapters 1 and 2. The book's review problems (some of which I've selected) are all of the form True/False, where you are supposed to justify the true statements by calculation or proof, and give a counterexample to the false statements. I've provided a few more review problems, to cover concepts that are not adequately addressed by the book's list of problems. In addition to doing these review problems, you should go over class notes and homework assignments, and make sure that you know the definitions of the basic concepts we've used so far in the class.

Chapter 1: Pages 37-38/2, 3, 8, 11-16, 21, 23, 26-32, 34, 35
Chapter 2: Pages 94-95/1-8, 10, 11, 12, 13, 15, 18, 19, 20, 24, 26, 28, 29, 31-37, 40, 42.

Additional problems:

1. Give examples of 4×5 matrices, if possible, with ranks 0, 1, 2, 3, 4, and 5.

2. Let A be the $3{\times}4$ matrix $\begin{bmatrix} 1 & 2 & 0 & -1 \\ 3 & 2 & 2 & 1 \\ -1 & 2 & -2 & -3 \end{bmatrix}$. Solve the equations $A\vec{x} = \begin{bmatrix} 1 \\ 4 \\ -2 \end{bmatrix}$ and $A\vec{x} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$.
   Is there any choice of vector $\vec{b}$ that makes the equation $A\vec{x} = \vec{b}$ inconsistent?

3. If the rank of a 5×3 matrix A is 3, what is rref(A)? If the rank of a 4×4 matrix B is 2, what are the possibilities for rref(B)?

4. Show that if A and B are 2×2 matrices representing rotations by angles α and β respectively, then A and B commute (i.e., $AB = BA$). Is this true if instead A and B represent reflections?

5. Write down some reasonably big matrices, and multiply them. If you don't feel like making up your own examples, do a few from problems 1-15 on page 85.

6. Suppose that A is a 2×2 matrix that represents rotation by angle θ, and B is a 2×2 matrix that represents reflection in a line L. Show, geometrically, that ABA^{-1} represents reflection in the line L', where L' is L rotated around by θ.