

30. T, since both kernels consist of the zero vector alone.
31. T; Consider any basis $\vec{v}_1, \vec{v}_2, \vec{v}_3$ of V . Then $k\vec{v}_1, \vec{v}_2, \vec{v}_3$ is a basis as well, for any nonzero scalar k .
32. F; The identity matrix is similar only to itself.
33. F; Consider $\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.
34. F; Let $A = I_2$, $B = -I_2$ and $\vec{v} = \vec{e}_1$, for example.
35. F; Let $V = \text{span}(\vec{e}_1)$ and $W = \text{span}(\vec{e}_2)$ in \mathbb{R}^2 , for example.
36. T; If $A\vec{v} = A\vec{w}$, then $A(\vec{v} - \vec{w}) = \vec{0}$, so that $\vec{v} - \vec{w} = \vec{0}$ and $\vec{v} = \vec{w}$.
37. T; Consider the linear transformation with matrix $[\vec{w}_1 \ \dots \ \vec{w}_n][\vec{v}_1 \ \dots \ \vec{v}_n]^{-1}$.
38. F; Suppose A were similar to B . Then $A^4 = I_2$ were similar to $B^4 = -I_2$, by Example 7 of Section 3.4. But this isn't the case: I_2 is similar only to itself.
39. F; Note that \mathbb{R}^2 isn't even a subset of \mathbb{R}^3 . A vector in \mathbb{R}^2 , with two components, does not belong to \mathbb{R}^3 .
40. T; If $B = S^{-1}AS$, then $B + 7I_n = S^{-1}(A + 7I_n)S$.
41. T; Let $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, for example.
42. F; Consider I_n and $2I_n$, for example.
43. T; Matrix $B = S^{-1}AS$ is invertible, being the product of invertible matrices.
44. T; Note that $\text{im}(A)$ is a subspace of $\ker(A)$, so that $\dim(\text{im } A) = \text{rank}(A) \leq \dim(\ker A) = 10 - \text{rank}(A)$.
45. T; Pick three vectors $\vec{v}_1, \vec{v}_2, \vec{v}_3$ that span V . Then $V = \text{im}[\vec{v}_1 \ \vec{v}_2 \ \vec{v}_3]$.
46. T; Check that $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ is similar to $\begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix}$.
47. T; Pick a vector \vec{v} that is neither on the line nor perpendicular to it. Then the matrix of the linear transformation $T(\vec{x}) = R\vec{x}$ with respect to the basis \vec{v} , $R\vec{v}$ is $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, since $R(R\vec{v}) = \vec{v}$.
48. F; If $B = S^{-1}AS$, then $B = (2S)^{-1}A(2S)$ as well.
49. T; Note that $A(B - C) = 0$, so that all the columns of matrix $B - C$ are in the kernel of A . Thus $B - C = 0$ and $B = C$, as claimed.