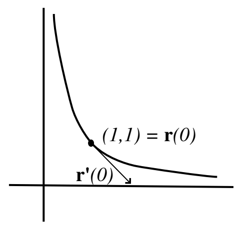
Solutions to Homework 5

14.2: (5), 6, 12, (15), 16, 18, 22, 24, (31), 32, 34, (41) 44.

Section 14.2

Problem 6. $\mathbf{r}'(t) = e^t\mathbf{i} - e^{-t}\mathbf{j}$.



Problem 12. $\mathbf{r}'(t) = \frac{1}{\sqrt{1-t^2}}\mathbf{i} - \frac{t}{\sqrt{1-t^2}}\mathbf{j}$.

Problem 16. Use the product rule: $\mathbf{r}'(t) = (t\mathbf{a})'\times(\mathbf{b}+t\mathbf{c})+t\mathbf{a}\times(\mathbf{b}+t\mathbf{c})' = \mathbf{a}\times(\mathbf{b}+t\mathbf{c})+t\mathbf{a}\times\mathbf{c} = \mathbf{a}\times\mathbf{b}+2t\mathbf{a}\times\mathbf{c}$. Alternatively, expand first: $\mathbf{r}(t) = t\mathbf{a}\times(\mathbf{b}+t\mathbf{c}) = t\mathbf{a}\times\mathbf{b}+t^2\mathbf{a}\times\mathbf{c}$, so $\mathbf{r}'(t) = \mathbf{a}\times\mathbf{b}+2t\mathbf{a}\times\mathbf{c}$.

Problem 18. $\mathbf{r}'(t) = \frac{2}{\sqrt{t}}\mathbf{i}2t\mathbf{j}+\mathbf{k}$, so $\mathbf{r}'(1) = 2\mathbf{i}+2\mathbf{j}+\mathbf{k}$ which has length $\sqrt{4+4+1} = 3$. So $\mathbf{T}(1) = \mathbf{r}'(1)/|\mathbf{r}'(1)| = 2/3\mathbf{i}+2/3\mathbf{j}+1/3\mathbf{k}$.

Problem 22. $\mathbf{r}'(t) = \langle 2e^{2t}, -2e^{-2t}, e^{2t}+2te^{2t}\rangle$ and $\mathbf{r}''(t) = \langle 4e^{2t}, 4e^{-2t}, 4e^{2t}+4te^{2t}\rangle$. Plugging in gives $\mathbf{r}'(0) = \langle 2,-2,1\rangle$, so $\mathbf{T}(0) = \frac{1}{3}\langle 2,-2,1\rangle$, and $\mathbf{r}''(0) = \langle 4,4,4\rangle$. After some simplification, we get $\mathbf{r}'\cdot\mathbf{r}'' = (8t^2+12t+12)e^{4t} - 8e^{-4t}$.

Problem 24. $\mathbf{r}(t) = \langle t^2-1, t^2+1, t+1\rangle$, so $\mathbf{r}'(t) = \langle 2t, 2t, 1\rangle$. The point $(-1,1,1)$ corresponds to the parameter value $t=0$, so the tangent vector is $\mathbf{r}'(0) = \langle 0,0,1\rangle$. Thus the tangent line is $\mathbf{l}(t) = \langle -1,1,1\rangle + t\langle 0,0,1\rangle$.

Problem 32. We need to first find s and t so that $t = 3-s$, $1-t = s-2$, and $s^2 = 3+t^2$. Note that the first and second equations are equivalent. Substitute the first in the third, to get $s^2 = 3+9-6s+s^2$, or $s=2$ (and therefore $t=1$.) Differentiating $\mathbf{r}_1$ and $\mathbf{r}_2$, we get that the tangent vectors are $\mathbf{r}_1'(1) = \langle 1,-1,2\rangle$ and $\mathbf{r}_2'(2) = \langle -1,1,4\rangle$. So the angle is determined by $\cos(\theta) = \frac{6}{\sqrt{6}\sqrt{18}} = \frac{1}{\sqrt{3}}$. So $\theta = \cos^{-1}(1/\sqrt{3}) \approx 55°$.

Problem 34. $\int_0^1 \left(\frac{4}{1+t^2}\mathbf{j} + \frac{2t}{1+t^2}\mathbf{k}\right)\, dt = \left[4\tan^{-1} t\mathbf{j} + \ln(1+t^2)\mathbf{k}\right]_0^1 = 4\frac{\pi}{4}\mathbf{j} + \ln(2)\mathbf{k} - 0\mathbf{j} - 0\mathbf{k} = \pi\mathbf{j} + \ln(2)\mathbf{k}$.

Problem 44. Apply the 1-variable chain rule in each component: Write $\mathbf{u}(s) = \langle u_1(s), u_2(s), u_3(s)\rangle$. Then

$$\begin{aligned}\frac{d}{dt}[\mathbf{u}(f(t))] &= \langle \frac{d}{dt}[u_1(f(t))], \frac{d}{dt}[u_2(f(t))], \frac{d}{dt}[u_3(f(t))]\rangle \\ &= \langle f'(t)u_1'(f(t)), f'(t)u_2'(f(t)), f'(t)u_3'(f(t))\rangle = f'(t)\mathbf{u}'(f(t))\end{aligned}$$

Section 14.3

Problem 4. $\mathbf{r}'(t) = 2t\mathbf{i} + 2\mathbf{j} + \frac{1}{t}\mathbf{k}$, so $|\mathbf{r}'(t)| = \sqrt{4t^2+4+\frac{1}{t^2}} = \frac{1+2t^2}{t}$. (In the last step I used that t is positive.) So

$$L = \int_1^e \frac{1+2t^2}{t}dt = \int_1^e \left(\frac{1}{t} + 2t\right) dt = \left[\ln t + t^2\right]_1^e = e^2.$$

Problem 6. $\mathbf{r}'(t) = 12\mathbf{i} + 12\sqrt{t}\mathbf{j} + 6t\mathbf{k}$, so $|\mathbf{r}'(t)| = \sqrt{144+144t+36t^2} = 6(t+2)$. So $L = \int_0^1 6(t+2)\, dt = \left[3t^2+12t\right]_0^1 = 15$.