This is a kind of dictionary comparing a general sort of function $f : X \to Y$ with a special kind of function, that is a linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m$. As you can see from the table, linear functions have many special features that don't mean much for an arbitrary function.

Functions $\leftrightarrow$ Linear Transformations	
Function $f : X \to Y$.	Linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m$.
f is determined by its values $f(x)$ for every possible $x \in X$.	T determined by just n values: the vectors $T(\vec{e}_1),\ T(\vec{e}_2), \ldots, T(\vec{e}_n)$. $T(\vec{x}) = A\vec{x}$ where A is the matrix whose columns are $T(\vec{e}_1),\ T(\vec{e}_2), \ldots, T(\vec{e}_n)$.
f is one-to-one **Equivalent:** $f(x) = f(y) \Rightarrow x = y$.	T is one-to-one **Equivalent:** $T(\vec{u}) = f(\vec{v}) \Rightarrow \vec{u} = \vec{v}$. **Equivalent:** $T(\vec{x}) = \vec{0} \Rightarrow \vec{x} = \vec{0}$. **Equivalent:** $\ker(T) = \{\vec{0}\}$. **Equivalent:** Null space of A is $\{\vec{0}\}$. **Equivalent:** All columns of rref(A) have pivots. **Equivalent:** rank(A) $= n$.
f is onto **Equivalent:** For every $y \in Y$ there is an $x \in X$ with $f(x) = y$. **Equivalent:** Range(f) $= Y$.	T is onto **Equivalent:** For every $\vec{y} \in \mathbb{R}^m$ there is an $\vec{x} \in \mathbb{R}^n$ with $T(\vec{x}) = \vec{y}$. **Equivalent:** For every $\vec{b} \in \mathbb{R}^m$, the system $A\vec{x} = \vec{b}$ is consistent. **Equivalent:** The column space of A is all of $\mathbb{R}^m$. **Equivalent:** All rows of rref(A) have pivots (ie, no 0-rows). **Equivalent:** rank(A) $= m$.
$f : X \to Y$, $g : Y \to Z$, composition $g \circ f : X \to Z$	$T : \mathbb{R}^n \to \mathbb{R}^m$, $S : \mathbb{R}^m \to \mathbb{R}^k$ both linear $\Rightarrow$ composition $S \circ T : \mathbb{R}^n \to \mathbb{R}^k$ linear. Matrix for $S \circ T$ is BA where $B =$ matrix for S and $A =$ matrix for T.
$f : X \to Y$, $g : Y \to X$, are inverses. **Equivalent:** Composition $g(f(x)) = x$ and $f(g(y)) = y$.	$T : \mathbb{R}^n \to \mathbb{R}^m$, $S : \mathbb{R}^m \to \mathbb{R}^n$ are inverses. **Equivalent:** Composition $S(T(\vec{x})) = \vec{x}$ and $T(S(\vec{y})) = \vec{y}$. Then $m = n$, and the matrices A, B are inverses in the sense of multiplication: $AB = I = BA$.