Math 21a, Fall 2004. Review sheet for exam 1, Thursday Oct. 14

The exam will consist of a mix of computations (for example, to find the solutions of some system of equations) and more conceptual questions. So for instance you should certainly be able to multiply a pair of matrices, but you should also be able to use the algebraic properties of matrix multiplication such as the distributive or associative laws. You should know the definitions of the basic concepts and be able to use them to do simple proofs.

In studying, you may find the "summary" at the end of each section to be helpful. Here is a list of the basic ideas that we've covered:

1. Vector addition; more generally linear combinations of the vectors $\vec{v_1}, \ldots, \vec{v_k}$. Geometric interpretation of addition (parallelogram rule) and scalar multiplication.
2. The span of $\vec{v_1}, \ldots, \vec{v_k}$.
3. Dot products; the norm (length) of the vector $\vec{v} = [v_1, \ldots, v_n]$. Unit vectors.
4. The Cauchy-Schwarz and triangle inequalities.
5. The angle between vectors $\vec{v}$ and $\vec{w}$ in $\mathbb{R}^n$; perpendicularity of vectors.
6. The transpose A^T of a matrix A.
7. If A and B are matrices such that AB makes sense, the relation between $(AB)^T$, A^T and B^T? Can you prove your assertion?
8. What is a symmetric square matrix?
9. What are the elementary row operations and what is an elementary matrix? What is the effect of multiplying by an elementary matrix? What is the inverse of an elementary matrix?
10. What does if mean to say that a matrix A is row equivalent to a matrix B? If they are row equivalent, what is the relation between the null space of A and that of B?
11. What properties does a matrix in row echelon form have? A matrix in reduced row echelon form? What is a pivot?
12. Consistency of the linear system $A\vec{x} = \vec{b}$. How do you decide if this system is consistent or not?
13. Suppose that A is $m \times n$ where $m < n$. How many solutions could there be to a system $A\vec{x} = \vec{b}$. What if $m = n$? $m > n$?
14. If A is in row echelon form and you consider the equation $A\vec{x} = \vec{0}$, which variables x_i are free? How do these various numbers interact: m, n, the number of pivots, the number of non-pivots, the number of free variables, the number of 0 rows?
15. Invertibility or non-invertibility of the $n \times n$ matrix A is invertible. What does it mean, how do you decide this for a given A, how does it relate to solving $A\vec{x} = \vec{b}$?
16. If A and B are invertible $n \times n$ matrices, what is $(AB)^{-1}$? Can you prove your assertion?
17. If A is $n \times n$, how do you find A^{-1} if it exists.
18. W is a linear subspace of $\mathbb{R}^n$.
19. $\{\vec{w_1}, \ldots, \vec{w_k}\}$ is a basis of the linear subspace W.

20. What is the standard basis of $\mathbb{R}^n$?
21. If $A\vec{p} = \vec{b}$, what is the relation between solutions to the homogeneous system $A\vec{x} = \vec{0}$ and solutions to $A\vec{x} = \vec{b}$?
22. Show that a nonempty subset W of $\mathbb{R}^n$ is a linear subspace iff for any $\vec{v}$, $\vec{w} \in W$ and $r \in \mathbb{R}$, the vector $\vec{v} + r\vec{w}$ is in W.
23. What are the row space, column space and nullspace of a matrix A?
24. Let A be $n \times n$. What are the relations between the following:
    (a) A is invertible.
    (b) $A\vec{x} = 0$ has a nontrivial solution.
    (c) $A\vec{x} = \vec{b}$ has a solution for any $b \in \mathbb{R}^n$.
    (d) $A\vec{x} = \vec{b}$ has a unique solution for any $b \in \mathbb{R}^n$.
    (e) A is not a product of elementary matrices.
    (f) The column space of A is $\mathbb{R}^n$.
    (g) The columns of A are linearly dependent.

I would suggest that you do some problems from the book as part of your studying. Here are some suggestions, although probably you don't want to do them all. First, each section has True/False questions; these are very useful as review questions. In addition, I suggest the following problems. When you do the problems in the earlier sections, use the knowledge you've gained in the later sections.
§1.1: 7, 10 (is there a linear combination of these vectors that adds up to $[1, 3, -2, 3]$?), 21, 25, 29, 30, 40.
§1.2: 5, 10, 12, 14, 17, 23, 25-30 (do them in your head), 36, 44.
§1.3: As many of 1-16 as you feel like doing, 17 (state a generalization), 19, As many of 25-34 as you feel like doing, 36, 38, 39, 41, 44.
§1.4: 4, 5, 9, 10, 12, 23 (while you're at it, in these last 4, give a basis for the solution space for the homogeneous equation), 26, 27, 35, 41, 49, 57.
§1.5: 8, 11 & 12 (in these, if the span is $\mathbb{R}^4$, find the inverse), 15, 17, 19, 25, 27, 33, 34.
§1.6: 1-9, 17, 20, 21 (while you're at it, in these three examples, find a basis for the column space of the associated matrix), 23, 25, 29, 30, 33, 35, 37, 42, 43, 47.