Children's Judgments of Numerical Inequality

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When adults judge which of 2 digits is numerically larger, their response times decrease linearly with the numerical difference. For children in fourth and seventh grades, the function relating judgment time to numerical difference has the same slope as that of adults. For children in kindergarten and first grade the function is considerably steeper. This may reflect a developmental change in the internal representation of the number series.

For adults, the time required to identify the numerically larger of two digits is inversely related to the numerical difference between the digits (Moyer & Landauer 1967). This basic result has been verified and extended several times since its discovery (Buckley & Gillman 1974; Parkman 1971; Sekuler, Rubin, & Armstrong 1971).

The chronometric properties of subjects' judgments of numerical inequality reflect the structural relationships among the digits' internal representations (Shepard, Kilpatric, & Cunningham 1975). Therefore, we should be able to trace the development of the internal representational structure by comparing the pattern of response times for children at various ages. Since the character of the developmental sequence is a matter of practical, pedagogical importance as well as one of theoretical interest, we studied judgment times for numerical inequality in five different age levels, from kindergarten through college.

Method

Subjects.—Four groups, each of six males and six females, were drawn from the kindergarten, first, fourth, and seventh grades of St. Mary's School in Evanston, Illinois. A fifth group of subjects was composed of Northwestern University students. The means and standard deviations of the ages for the four groups of children were 5.89 (0.26), 6.88 (0.56), 10.18 (0.34), and 13.13 (0.36). Mental ages were available only for our kindergarten subjects, mean = 6.18. All groups were tested within a 3-week period toward the end of the 1974–1975 school year. Before each subject was tested we verified his ability to count, unaided, from 1 to 10. Even our youngest subjects had no trouble with this task.

Apparatus and procedure.—The digits 1–9 were presented on a pair of single-plane displays mounted side by side on a black cabinet. Viewed binocularly from a distance of about 40 cm, the maximum extent of each digit subtended 4°40' visual angle on the vertical and 2°20' on the horizontal. Horizontal distance between the center of the displays was approximately 4°. The luminance of the digits, presented against a constant, darker background was about 3.4 cd/m².

On each trial a pair of thumb-wheel switches was used to select the two digits to be illuminated. After making the selection, the experimenter pushed a button to illuminate the digits and start a digital timer which ran at a 100-herz rate. When the subject decided which digit, right or left, was numerically larger, he pushed a toggle switch in a direction corresponding to that digit. The subject's binary choice (left or right) was displayed to the experimenter on indicator lights which only she could see. In addition, the response turned off the digits and stopped the timer, allowing the experimenter to record both response and latency.

Following an intertrial interval of approximately 4 sec, the entire sequence was repeated with another pair of digits. The absolute value of the difference between left and right digits ranged from 1 to 8. During the 64 trials for any subject, each of these numerical

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differences occurred with equal frequency. Moreover, on half the trials the numerically larger digit appeared on the left. Particular pairs of digits did not occur with equal frequency since (1) nearly any numerical difference could be produced by several different pairs, and (2) we did want to present all possible numerical differences with equal frequency. Two different sequences of digit pairs were used, one the reverse of the other.

Though subjects were instructed to make responses as quickly as possible, without errors, some errors did naturally occur. When a subject made an error (chose the wrong digit), that fact was noted and the trial repeated later at a randomly chosen point in the remaining sequence.

Results

All analyses were done on the response times for correct judgments only. Figure 1 shows the main results, each group's response times as a function of the numerical difference between digits in a pair. We did an analysis of variance on the subjects' mean response times for each of the eight possible numerical differences between pairs of digits. Several features of this analysis are noteworthy. First, with increasing age of subjects response times decrease, $F(4,55) = 48.36, p < .001$. Second, over all groups response times decrease as the numerical difference between digits increases, replicating the basic phenomenon reported previously by others (Moyer & Landauer 1967; Sekuler, Rubin, & Armstrong 1971). We used orthogonal polynomials to decompose this effect of numerical difference into linear, quadratic, and residual components. Only the linear portion of the trend was statistically significant, $F(1,55) = 81.13, p < .01$; other components, $p > .25$. Finally, we again used orthogonal polynomials to decompose the interaction between group and numerical difference into linear, quadratic, and residual components. Only the linear component, group $\times$ difference linear, was statistically significant, $F(4,55) = 9.99, p < .001$; other components, $p > .25$. Since both the linear components of the interaction of group and numerical difference and the main effect of group were significant, we need both the slopes and intercepts of the five sets of group data if we are to describe the data adequately. We therefore determined the best fitting (in the least-square sense) straight line for each group's data and have shown these lines in figure 1.

A Newman-Keuls test ($\alpha = 0.05$) on the mean response times for each group indicated that (1) the kindergarten group was significantly slower than all other groups, (2) the first grade was significantly slower than all groups except the kindergarten, (3) fourth grade, seventh grade, and adult subjects did not differ significantly from one another. Before these differences among the group mean response times can be evaluated, the possibility of a speed-accuracy trade-off must be considered. In other words, we must determine whether the faster groups achieved their greater speeds by adopting a less stringent performance criterion and at the expense of more errors. In general, the error rate decreased from kindergarten through adult subjects: 11.75, 10.67, 7.58, 7.92, and 5.08 mean errors per subject (out of 64 trials). Although the trends were not strictly monotonic, in general, for each group, the mean number of errors per subject tended to decline with increasing numerical difference between digits in a stimulus pair. Over all groups, the average subject made 1.77, 1.47, 1.12, 0.97, 0.87, 0.73, 0.75, and 0.75 errors (out of eight trials) for numerical differences of 1, 2, 3, 4, 5, 6, 7, and 8, respectively. One fact about these error rates is relevant to our discussion of response times:
the error rate differences are opposite what is expected from speed-accuracy trade-off. In our data, the slower groups tend to make more errors, meaning, if anything, that the obtained speed differences may slightly underestimate the differences that would obtain if all groups could be made to perform at the same level of accuracy. This adjustment could have been achieved, for example, by encouraging the older subjects to work less carefully, thereby making more errors and presumably achieving even shorter response times.

In general, the effects observed in the response time data of our adult subjects are about the same magnitudes reported by others: average response times change by about 100 msec over the entire range of numerical differences.

Finally, we wondered whether, for our youngest subjects, there might be some relationship between performance on our task and mental age. But the Pearson product-moment correlation between mental age and number of errors for the kindergarten subjects was nonsignificant, \( r(11) = -0.13, p > 0.25 \).

**Discussion**

Here we shall first consider the developmental character of our results. In the trend analyses we reported earlier, the only significant terms were the linear component of numerical difference and the linear component of the interaction between numerical difference and group. So, at least to a first approximation, we can consider the effect of numerical difference a linear one and, intercept differences aside, treat differences among groups on that effect primarily as matters of slope (hence, the best fitting straight lines shown in fig. 1). The fact that the shape of the numerical difference effect does not change from group to group encourages the assumption that the same basic processes are responsible for the numerical difference effect, whatever the age of the subject. In other words, we can consider group differences as matters of quantity rather than quality. Before trying to explicate this quantitative developmental change we must consider how the numerical difference effect which we and others have observed is most likely generated.

Previous work on judgments of numerical inequality demonstrated that the processes which support such judgments must preserve at least the ordinal character of the digits being judged. More specifically, Moyer and Landauer (1967) proposed that the digits evoked internal analog responses and that the psychophysical judgment depended upon some comparison between the pair of such responses evoked on any trial. According to this line of argument, judgment times reflect the *subjective distances* on the judged dimension between the analog representations of the digit referents.

Our own results can be interpreted within this framework if we add an elaboration from its most recent formulation (Moyer & Bayer 1976). As is common in psychophysical models, we shall assume that the analog representation of any digit is somewhat noisy, producing a distribution of analog responses upon repeated presentations of the same digit. These discriminable dispersions about each of the means provide a metric for the dimension to be judged. Large discriminable dispersions produce greater overlap between distributions about adjacent digit representations, thereby reducing the effective subjective distance between them. This would increase the difficulty of the judgment and lengthen judgment time. Accordingly, then, the steeper slopes for younger subjects (in fig. 1) would imply either that (1) the average distance between analog representations of the digits is reduced in younger subjects (i.e., the representation of the number series is compressed), or (2) the discriminable dispersions about each mean representation are larger in younger subjects, or (3) there is some combination of both series compression and increased discriminable dispersions. We cannot see how these possibilities can be distinguished experimentally, so it is fortunate that they are basically variants on a single proposition: smaller effective subjective distances among number representations in younger subjects.

This analog representation model, although quite counterintuitive, has two distinct advantages: good fit to available data on numerical inequality judgments, and parsimonious explanation of data from a variety of other judgmental tasks involving nonnumerical judgments (Moyer 1973). Various details of this analog comparison model and its application to other psychophysical tasks are treated thoughtfully by Moyer and Bayer (1976) and need not be elaborated here.
References


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