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PROCESSING NUMERICAL INFORMATION: A CHOICE TIME ANALYSIS [1]

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The time needed for human adults to identify the numerically larger of two digits was studied. In Exp. I, the digits to be compared were presented in succession at two different exposure durations. In Exp. II, a single visually presented digit was compared with another digit held in memory. In both experiments, the larger the numerical difference, the faster the identification.

The time needed to identify the numerically larger of two digits depends upon their numerical difference (Moyer & Landauer, 1967). In general, the larger the numerical difference, the shorter the time required to make the discrimination. A variety of reasons have compelled us to replicate the Moyer and Landauer finding. First, the result is of considerable theoretical importance for what it may imply about the general storage and use of numerical information. Second, we were surprised that college students, having had 15 or more years of experience with such decisions, would still take longer to decide that "5" is numerically larger than "4" than they do to decide, for example, that "7" is numerically larger than "4." Third, the error rate in the Moyer and Landauer experiment covaried strongly with judgment time. Using their published graphs, we have calculated a Spearman rank-order correlation of .80 between error rate and judgment time. As a number of people have suggested, such a strong relationship between error rate and judgment time raises some obstacles to the unambiguous interpretation of judgment times (Egeth & Smith, 1967; Smith, 1968). Consequently we have tried to devise a situation where the error rates might not be so highly correlated with the expected judgment times. Fourth, we wanted to record judgment times in an experimental situation which controlled the viewing time associated with each member in a pair of digits. This control would permit an estimate of the processing time for a single digit. To meet these requirements we used successive rather than the simultaneous presentation of each pair of digits used by Moyer and Landauer. We expected that successive, controlled exposures might permit the clearer analysis of the various components of overall processing time involved in the ultimate, binary judgment of numerical inequality.

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EXPERIMENT I

Method

Apparatus.—The single digits 1–9 were presented in pairs using a single plane display (IEE Series 10 rear projection display with #1813 lamps installed). The digits, viewed binocularly, were presented in a viewing box at a distance of 60 cm. from *S*'s eyes. Each digit subtended 4°40′ visual angle in the vertical by a maximum of 2°20′ on the horizontal. The luminance of the digits, presented against a constant, dark background, was about 3.4 cd/m².

The entire apparatus was under the control of a high-speed incremental paper tape reader and associated programming system. On each trial the following sequence of events took place. The first digit was illuminated for a predetermined interval. At the end of that interval, the digit was extinguished and there followed immediately the illumination of a second, different digit which appeared in the same spatial location as the first. Simultaneous with the presentation of the second digit, an electronic buffer began to accumulate a count from a crystal clock at the rate of 1,000 pulses per second. When *S* had decided which digit, the first or second, was numerically larger, he threw a toggle switch in

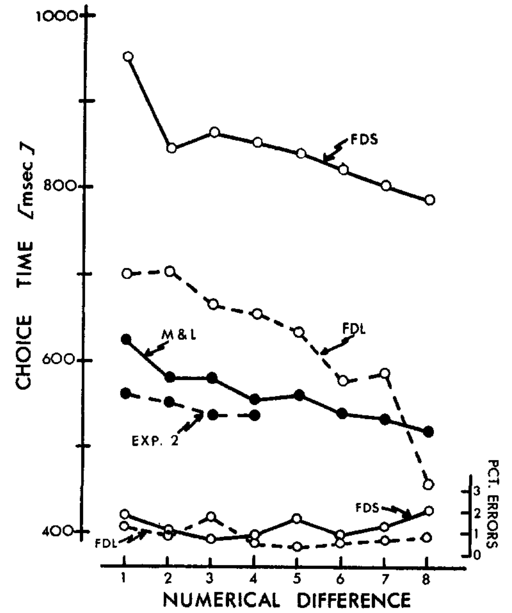

FIG. 1. Mean choice time for identifying the numerically larger of two digits as a function of their numerical difference. (Starting from the top, curves are shown for Exp. I Cond. FDS and FDL, the Moyer and Landauer (1967) experiment (M & L), and Exp. II. The bottom curves show the error rates for the two conditions of Exp. I.)

the assigned direction to signal that judgment. The switch throw had three effects: the second digit was extinguished, the accumulation of a count in the buffer stopped, and the contents of the buffer were read out into a teleprinter. Following a constant intertrial interval of 4.5 sec., the entire sequence was repeated with another pair of digits.

This sequence was repeated 432 times for each *S*. The absolute value of the differences between the first and second digits ranged from 1 to 8. All of these absolute values of the difference between digits occurred with equal frequency during the course of 432 trials in a block-randomized order. For any of the absolute values of the difference between digits, on half of the trials the larger digit appeared first. With one exception, any given numerical difference between the two digits could be constituted by several different pairs of particular digits. The difference 1, for example, could be generated by the presentation of many different pairs: 4 and 3, 5 and 4, 9 and 8, etc. A difference of 8 could be generated only by the digits 1 and 9. The stimulus sequence was constructed so that each of the possible pairs of digits which constituted a given numerical difference occurred with approximately equal frequency. This constraint, taken with the constraint that all numerical differences occurred with equal frequency, meant that particular *pairs* of digits did not occur with equal frequency. For example, 1 and 9, the only pair constituting a difference of 8, occurred many times more often than did 9 and 8, which was only one of the many pairs that constituted a difference of 1. All *S*s were tested with the same sequence of digits.

Subjects.—The *S*s were nine volunteer introductory psychology students and one of the authors (RS). Each of the *S*s was assigned, in alternation, to one of two conditions. In Cond. First Digit Long (FDL) the first digit in each pair was presented for 2.0 sec.[4] In Cond. First Digit Short (FDS) the first digit was presented for only 50 msec. In each group, three of the five *S*s indicated that the second digit was numerically larger by a switch-throw to the right, that the first was larger with a throw to the left. The other two *S*s in each group used the opposite S–R mapping. Since the present data showed no effect of direction of switch-throw, in all analyses the two assignments of response to direction of switch-throw have been combined.

Each *S* was told to make and signal his judgments as quickly as possible while making sure that he made as few errors as possible. Every *S* served in one session which lasted about 35 min. for members of the FDS group and 45–50 min. for members of the FDL group.

Results

Figure 1 shows mean time required for the correct identification of the larger of two digits plotted as a function of the numerical difference between the digits. Data from the Moyer and Landauer (1967) experiment (M & L) are also plotted to facilitate comparisons. Note that for both FDS and FDL conditions, judgment time decreased systematically as the difference between the first and second digit increased. An analysis of variance indicated that the absoulte value of the difference between digits in a pair was a significant source of variance, $F\ (7, 56) = 14.82$, $p < .01$. The analysis also showed First Digit Duration × Numerical Difference between Digits interaction, $F\ (7, 56) = 2.96$, $p < .05$. This interaction

[4] These intervals are defined by the duration for which the reed-relay controlling the display was closed, and do not take into account the delay added by the onset times for the display lamps. We have estimated this delay by determining for how long the reed-relay controlling power to the display lamps had to be closed in order for an observer to correctly recognize what digit was presented. A duration of approximately 30 msec. was needed for recognition.

arose from the difference between the two digit duration groups at the extremes of the numerical difference continuum. Figure 1 shows that over the range of numerical differences from 2–7, there was an approximately constant 150-msec. difference between the FDS and FDL conditions. The analysis of variance also indicated no significant effect ($p = .50$) of the sign of the difference between first and second digits; i.e., as far as judgment times are concerned, it does not matter whether the first digit is larger or smaller than the second as long as the absolute value of the numerical difference is the same. The error rate shown at the bottom of Fig. 1 was less than 2.2% in all cases and was not systematically correlated with judgment times.

To make a more detailed analysis possible, we can consider only those trials on which the numerical difference between first and second digits was 1. The only significant term in our analysis of variance on mean judgment times was the interaction between the order of digits (larger first or second) and the digits in each pair, regardless of order, $F(7, 56) = 2.34$, $p < .05$. This relationship can be seen in Fig. 2. Judgment times are plotted against the magnitude of the lesser digit of the pair. The parameter of the curves is the sign of the difference between the digits. When the second digit is numerically larger (solid line), choice times increase with the size of the lesser digit. When the first digit is numerically larger (dashed line), choice times decrease with the magnitude of the smaller digit. Of particular interest in Fig. 2 are the four data points labeled 2–1, 1–2, 8–9, and 9–8. The exceptionally short choice times associated with 1–2 and 9–8 indicate *S*s correctly recognized, while the first digit was presented, the inevitability of the correct response. For example, in the case of 1–2, the first digit was 1 and consequently it was a certainty that the second digit would be numerically larger. In the case of 9–8, the first digit was 9, making it certain that the second digit would be smaller. The lack of significant interaction between duration of the first digit, the sign of the difference between digits, and the pairs involved indicated that the relationships shown in Fig. 2 are essentially unchanged by the duration of the first digit. This means that the effects of expectancy reflected in the short times for Pairs 1–2 and 9–8 are found even with the short first digit duration. This resembles expectancy effects found in somewhat different choice time situations by other investigators (Bernstein & Reese, 1965). The other pairs of data points of particular interest in Fig. 2 are those labeled 2–1 and 8–9. With these pairs too, *S*s could be expected to strongly anticipate, after seeing only the first digit, what the relationship between the two digits would be. In the case 2–1 they would have expected, after the first digit, that the second would be larger; in the case 8–9, they would have expected the second digit to be smaller. Of course these anticipations would have proved wrong. The unusually long judgment times associated with these pairs may

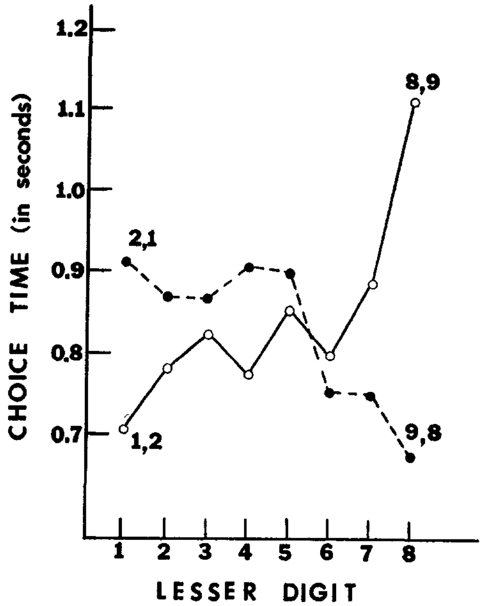

FIG. 2. Mean choice time for trials on which numerical difference between digits was unity as a function of the lesser digit in a pair. (The solid line connects data from conditions in which the lesser digit was presented first; the dashed line connects data from conditions in which the numerically larger digit was presented first.)

reflect the fact that *S*s, after seeing the first digit, prepared a response that would have been the most likely response, and then, upon seeing the second digit, had to abort the already prepared response, make ready and then release the correct one (see Smith, 1968).

Figure 2 shows then, that with numerical difference between digits in a pair held constant, which digit is presented first does influence the choice time. This effect seems not to be controlled by the sign of the difference between digits in a pair (i.e., whether the numerically larger is first or second), but rather seems to depend upon the certainty with which the correct response can be anticipated given only the first digit. When the first digit makes one of the responses "larger" or "smaller" far more certain than the other, the choice time is low; when the correct response is more difficult to anticipate only on the basis of the first digit, choice time lengthens. The relationship between the first digit and the probability of anticipating the correct response, given only that digit, renders the relationships shown in Fig. 1 somewhat ambiguous. The decrease in choice time with increasing numerical difference between digits in a pair could have been produced not by the numerical difference, but by the fact that increasing numerical differences makes it more possible to anticipate the correct response, given only the first digit in a pair. Figure 2 suggests that expectancies based on the first digit alone might mediate a decrease in choice time with increasing numerical difference. A second experiment was performed to measure the effect of numerical difference uncontaminated by associated changes in the predictive value of the first digit in a pair.

Experiment II

In order to eliminate the differential predictive value of various digits when each is presented first in a pair, only one digit was presented on each trial in Exp. II. The *S* was instructed to compare the numerical magnitude of that single digit with the magnitude of a single reference digit (5). Moreover, for each *S*, the reference digit was the larger on half of the trials and smaller on the other half.

Method

Subjects.—The *S*s were 20 undergraduates, 10 in each of two groups.

Apparatus and method.—The apparatus was like that of Exp. I with the following exceptions. The viewing distance was increased to 125 cm. and the comparison digit remained on for 1.92 sec. instead of terminating with *S*'s response. The interval between the onset of stimuli on successive trails was constant at 4.75 sec. The response switch was changed to one with a shorter handle. In addition, the switch was remounted so that it was centered in *S*'s midsagittal plane with the handle protruding toward the *S* and the possible switch-throw directions being left and right. As the data suggest, remounting the switch from the original position at *S*'s right-hand side produced a significant asymmetry in the difficulty with which the switch could be thrown in the two directions.

Each *S* was tested in one session consisting of 30 practice trials, a short rest, and an unbroken string of 192 trials. Stimuli were block randomized so that each of the possible digits from 1–9, excepting 5, appeared twice in each block of 16 trials. Ten different sets of 192 stimuli were prepared, each being used with one *S* from each of the two groups. The *S*s in Group 1 were told to throw the switch to the right when the digit presented was numerically larger than the digit 5 and to the left if the digit was numerically smaller. The *S*s in Group 2 were given the opposite S-R mapping. In order to control the error rate and response times, *S*s were informed that their base pay for the session would be competitively scaled from $1.50 to $4.50 depending upon their speed while $.25 could be deducted for each error.

Results

To avoid the effects of warm-up on response times, only data from the last 160 of the total 192 experimental trials are considered. Errors occurred on 1.3% of the trials and did so in a manner that was not systematically related to either test digit or group. The mean response times on correct trials are shown in Fig. 3 as a function of the comparison digit. Data for the two groups are plotted separately. The significant effects, according to an analysis of variance, are digits, F (7, 126) = 7.63, $p < .01$, and the interaction between groups and digits, F (7, 126) = 8.33, $p < .01$. The main effect of groups was not statistically significant, F (1, 18) = 2.81, $p > .10$. The Groups × Digits interaction can be interpreted as the result of the differential difficulty with which *S*s could throw the switch in the two response directions. It will be recalled that the switch, for this experiment, was changed and, more importantly, its position altered. Apparently it was easier for *S*s to throw the switch to the right than to the left. Although the main effect of groups was not statistically significant, the appearance of a consistent separation between the groups in Fig. 3 is somewhat disquieting. Such a group difference would reflect the differential ease with which *S*s can operate under the two S–R mappings used in this experiment. Each *S* was asked which S–R mapping seemed the more "natural" to him. All indicated that the natural mapping was the switch thrown to the right being indicative of a larger digit, the switch thrown to the left being indicative of a smaller digit. Figure 3 shows, however, that this natural mapping, used by Group 1, surprisingly produced uniformly slower times.

The middle curve segments in Fig. 3 represent the mean response times for both groups. They show that as the comparison digit approaches the numerical value of the standard (5) from either side, the response time lengthens. This is the same relationship that was found in Exp. I. To facilitate comparison between this experiment and Exp. I, we have calculated the mean times for each absolute value of the difference between standard and comparison digits. As an example, we have averaged the times for comparison digits 1 and 9 since both represent numerical differences of 4. These means have been plotted in Fig. 1, and the line connecting them is labeled Exp. II. Although this experiment differs in several respects from Exp. I, as well as from the Moyer and Landauer (1967) study, the curves shown in Fig. 1 all show the same general trends, a decrease in response time with increasing numerical difference between standard and comparison digits. The absolute heights of the various curves are somewhat more difficult to compare since the instructions (or lack of them) concerning the speed-accuracy trade-off differ.

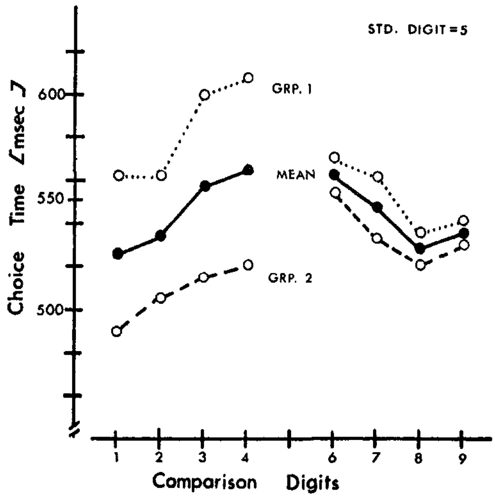

FIG. 3. Mean choice time in Exp. II for Group 1, Group 2, and over all means as a function of the comparison digit. (Group 1 threw a switch to the right to indicate that the comparison digit was larger than 5; Group 2 threw a switch to the left for that condition.)

Discussion

The present experiments have replicated in all major details the earlier findings of Moyer and Landauer (1967). With successive presentations of the digits in each pair, time required for judgment of numerical inequality declines with increasing differences between the digits. Even when only a single digit is presented to be compared against the memorial representation of a standard digit, we find that the larger the numerical difference between the two digits, the shorter the response time. As Moyer and Landauer suggest, this finding argues against what is probably a priori the most likely system for generating judgments of numerical inequality. Assume that each visually presented digit activates a process which searches out the psychological representation of the numerical magnitude associated with the digit. If the psychological representations are themselves arranged according to magnitude, an *S* having already located the representation of one digit should require an amount of time to find the representation of a second digit that is a nondecreasing function of the distance between storage locations of both representations. This system for storing numerical magnitudes should generate data that are essentially the opposite of our own; i.e., it implies that time required for judgment of numerical inequality should increase with increasing differences between the two digits which are to be compared. We think that the alternative system suggested by Moyer and Landauer (1967) should be accepted at least as a working hypothesis. This hypothesis is that some analog representation is generated for each digit which is to be compared. The two analog representations are then compared to produce a judgment of numerical inequality. This places the psychological response to digits and pairs of digits in the same general theoretical framework as that used for other psychophysical judgments.

The average difference of 150 msec. between the two conditions, FDS and FDL, in Exp. I is of some theoretical importance. It will be recalled that in Cond. FDS the first digit was presented for only 50 msec. and was followed immediately by the second digit. In Cond. FDL the first digit remained on for a full 2 sec. The shorter response times in this latter condition probably reflect, among other things, the fact that the 2-sec. exposure of the first stimulus permitted the stimulus to be entered into whatever form of short-term store (visual or other) serves in this task. We may take the difference between Cond. FDS and FDL as a rough estimate of the time required to process the first digit. The validity of this estimate needs to be examined in a study involving parametric manipulation of the exposure duration of the first digit in the sequence.

Recently, Lovelace and Snodgrass (1970) repeated the experiment of Moyer and Landauer (1967) using letters of the alphabet instead of digits as stimuli. Lovelace and Snodgrass found that when the alphabetic distance between two letters increases, *S*s require less time to identify which of the two comes first in the alphabet. This is, of course, consistent with both the present findings and those of Moyer and Landauer (1967). Unlike the experiments with numerical stimuli, however, the Lovelace and Snodgrass experiment found a strong "order" effect: when the left-hand letter occurred earlier in the alphabet than did its right-hand mate, *S*s took significantly less time to identify the alphabetic order of the two than they did when the left-hand stimulus occurred later in the alphabet. As indicated above, we found no analogous effect. For the present, we assume that this difference reflects the differential practice *S*s have with backward number series and backward alphabets. Because of space-launch countdowns and Sesame Street and other common situations, we have considerable practice with the number series in backwards order. We do not have nearly so much practice with the alphabet in its backward order and indeed a normal *S* finds that it is quite hard to recite the alphabet backward. We assume that such differential practice, rather than an inherent alphabetic-numeric difference, caused the alphabetic stimuli to show much stronger order effects than did their numerical stimuli counterparts.

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