
0. Preliminaries

- **Material implication**: this is the familiar connective from Propositional Logic: “→

  The truth table for material implication:

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>p→q</th>
</tr>
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<tbody>
<tr>
<td>1</td>
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- **Indicative sentences**: “run-of-the-mill” assertions – NOT interrogative
  NOT imperative
  NOT subjunctive

  **Conditionals**: sentences of the form “if …. then…”

  So, **indicative conditionals** are, e.g.

  1. a. If my hen has laid eggs today, then the Cologne cathedral will collapse tomorrow
     b. If it is raining, I’m the monkey’s uncle
     c. If John came to class last Monday, he got the handout

  **Non-indicative conditionals** are, e.g.

  2. a. If I were you, I wouldn’t do it. **subjunctive** (this one is a **counterfactual**)
     b. If my hen laid eggs, will the Cologne cathedral collapse tomorrow? **interrogative**
     c. If you value your life, please don’t go to the moor after dark **imperative**

  We will concentrate on examples like the ones in (1).

- **Logical form**: various intermediate steps between “**John loves Mary**” (or the corresponding tree), and the corresponding stuff in the world or stuff in the mental model: e.g., translations of natural language sentences into PL, PredL, λ-calculus,…

1. Meaning of indicative conditionals: material implication?

  - **Grice’s analysis**:

    (1a) means

    3. “my hen has laid eggs today” → “the Cologne cathedral will collapse tomorrow”

      - Why does anyone hearing (1a) think there’s some kind of connection (causal, for instance) between egg-laying and cathedral collapsing?
It’s non-literal meaning: roughly, hearers expect that speakers will say **enough to satisfy the hearer**, and **no more than is relevant**.

- Suppose speaker knows p=my hen has laid eggs today and also q=the cathedral will collapse. Then a **stronger statement** would have provided better info: p & q
- Since speaker didn’t say p&q, it’s not true (or speaker doesn’t know it’s true).
- Now, p → q is equivalent to ~p v q.

What possible reason could there be for saying ~p v q?
- Suppose speaker knows ~p: then saying “My hen hasn’t laid eggs today” would be more relevant (p → q = ~p v q contains extra, irrelevant info)
- Suppose speaker knows q: same thing, saying “Cathedral will collapse” is more helpful.

And yet, for all I know, I told you the truth, and I’m not trying to be misleading.

- So, there must be some **non-truth-conditional** (extra) connection (like cause-effect) between p and q.

### Pros of Grice’s analysis:

Explanation of why “→” is all over the place in logic and math: that’s just the literal meaning of natural-language “if-then” statements

Going from natural language “if-then” to logical material implication is not such a leap – all you have to do is drop those expectations about speakers/hearers in everyday conversation.

### Additional arguments: Gibbard

Theorem:
Suppose we have some new connective, “#”
Suppose it satisfies the following three conditions:

I. p # (q # r) and (p & q) # r are logically equivalent
II. p # q is false in all the worlds in which p is true and q is false
   (at this point, p # q could also be false in some of the other worlds)
III. Suppose q is true in all the worlds in which p is true. Then, p#q is true in all the worlds.

Then, “#” is just “→”.

Actually, natural language “if…then” seems to satisfy this theorem, so it seems that it’s gotta be “→”

### 2. Decline of material implication in semantics

#### Logical form of sentences with quantifiers

Promising:

4. a. “All students run”  a’. ∀x. student(x) → run(x)  **material implication!**

Not so good:

4. b. “Some students run”  NOT b’. ∃x. student(x) → run(x)
- this is true if there is something, which is not a student, REGARDLESS of whether any students run!

OK, so we’ll just do b”. ∃x. student(x) & run(x)

Real trouble:
4. c. “Most students run” no good translation with →, or anything else in PredL!
   d. “Many students run” Need something like Generalized Quantifiers!
   e. “Few students run”

• **Something like GQ:**
  5a [∀x: x is a student] x runs
  5c [Most x: x is a student] x runs
  5b [∃x: x is a student] x runs

Literally: restrict your attention (your variable assignments) to the set of students. For all/most/some individuals in this set, if you assign that individual as the value of x, “x runs” will be true.

• **Logical form of sentences with Q-adverbs**
6. a. Sometimes, if a man buys a horse, he pays cash for it
   b. Always, if a man buys a horse, he pays cash for it
   c. Most of the time, if a man buys a horse, he pays cash for it

First, what are Q-adverb doing? Suppose, they’re quantifying over events e:

6. a’. ∃e e is an event of a man buying a horse → e is an event of the man paying cash for it
   b’. ∀e e is an event of a man buying a horse → e is an event of the man paying cash for it
   c’. Most e e is an event of a man buying a horse → e is an event of the man paying cash for it

Problems with (6a’-c’):
- What’s that “Most e”? It’s not in PredL!
- (6a’) is hopeless as meaning for (6a), for the same reasons that (4b’) is not what (4b) means
- Consider (6c’): it’s true in a world in which 1000 events occurred, of which 200 were events of a man buying a horse and in all 200 the man paid with a check. **Make sure you can explain why!**

• **Conclusion:** “→” is no good for if-then statements with Q-adverbs

**Something better:**
7a [∃e: e is an event of a man buying a horse] e is an event of the man paying cash for it
7b [∀e: e is an event of a man buying a horse] e is an event of the man paying cash for it
7c [Most e: e is an event of a man buying a horse] e is an event of the man paying cash for it

**Generalization so far:** what used to be p (antecedent of conditional) turns out to be the restriction on the domain of quantifiers.
• **Indicative conditionals and probability:** Grice’s paradox

Stats: 100 games,
   Yog has white in 90, black in remaining 10
   there are no draws,
   Yog won 80 times out of 90 when he had white,
   Yog lost all 10 times when he had black.
Situation: we don’t know anything about game # 99.
True statements about game #99:
8.  a. If Yog had white, then there is a probability of 8/9 that he won
    b. If Yog lost, then there is a probability of ½ that he had black

Possible logical forms for (8) with “→”:
9. a. “Y had white” → 8/9-probably “Y won”
    b. “Y lost” → ½-probably “Y had black”

This is wrong: 9a is true if Yog had black in game #99, 9b is true if Yog lost.

10. a. 8/9-probably “Y had white → Y won”
    b. ½-probably “Y lost → Y had black”

This is also wrong: since there are no draws, not losing is winning;
also not having white is having black
So, the contraposition of “Y had white → Y won” is
   “Y didn’t win → Y didn’t have white” = “Y lost → Y had black”
and vice versa.
But in 10, we have different probabilities for these logically equivalent statements! – they can’t be both correct!

• **Reasoning and restrictions on probabilities:**
Think about the reasoning behind the truth of (8):
11. a. [8/9-probably: g is a game and Y had white in g] Y won in g
    b. [1/2-probably: g is a game and Y lost in g] Y had black in g

Again, if-clause becomes a restriction!

3. Right, let’s get back to simple indicative conditionals now

1. a. If my hen has laid eggs today, then the Cologne cathedral will collapse tomorrow
   b. If it is raining, I’m the monkey’s uncle
   c. If John came to class last Monday, he got the handout

What could the if-clauses be restricting in (1)?
**Kratzer:** hidden/silent modals
12. [Must: my hen has laid eggs today] the Cologne cathedral will collapse tomorrow
Meanings of modals: possible worlds
12' \[ \forall w: \text{my hen has laid eggs today in } w \] the Cologne cathedral will collapse tomorrow in \( w \)

Accessibility relations
R: set of ordered pairs of worlds. (reflexive, symmetrical)

Correction on the way: sentences are no longer “true” or “false”.
They are now sets of worlds \( \iff \) functions from worlds to truth-values
13. “John loves Mary” = the set of worlds in which John and Mary are in the “love” relation.
We write
13' \( \lambda w. \text{love}(j,m,w) \) OR 13" \( \lambda w. \text{love}(j,m) \) in \( w \)

In the future, I’ll go back and forth between “functions from worlds to truth-values” and “truth-values”, as convenient. I’ll plug in the actual world \( w_0 \) when I wish to do “truth-values”.

12" \( \lambda v. \left[ \forall w: R(v,w) \& \text{my hen has laid eggs today in } w \right] \) the Cologne cathedral will collapse tomorrow in \( w \)
Plugging in the “actual world” \( w_0 \), we get
12" \( \left[ \forall w: R(w_0,w) \& \text{my hen has laid eggs today in } w \right] \) the Cologne cathedral will collapse tomorrow in \( w \)

What worlds are accessible? “Flavors” of modality: epistemic, deontic, bouletic, etc.

14. This guy might be Fred  “might” is a possibility modal = \( \exists w \)
14' \( [\exists w: R(w_0,w)] \) this-guy = Fred

15. a. This guy cannot be Fred;  “can” also possibility modal
    b. he must be Martin  “must” is a necessity modal = \( \forall w \)
15' a. \( \sim [\exists w: R(w_0,w)] \) this-guy = Fred
    b. [\( \forall w: R(w_0,w) \)] this-guy = Martin

How can different people say 14, 15 in the same situation, and both be truthful?
- different R!

• Different Rs in indicative conditionals

Situation: Pete and Stone playing poker on a Mississippi riverboat (don’t ask me why!).
Zack helps Pete: he signals to Pete what Stone’s hand is
Jack sees both hands and knows that Stone’s hand beats Pete’s no matter what
16. a. Zack: “If Pete called, he won”
    b. Jack: “If Pete called, he lost”

• Back to Gibbard’s proof
If that theorem applies to indicative conditionals, how can they fail to be material implications?
there is no two-place connective “#”!
now, “if p then q” statements are not really connecting two propositions;
p and q don’t even form a constituent! If anything, p forms a constituent with a (silent or
pronounced) operator; alternatively, we have a “tri-partite” (non-binary) structure.

So, the theorem doesn’t apply.

So, why do “if … then …” sentences seem to satisfy the theorem (at least, the main condition,
I)? E.g., (17a <=> 17b)

17. a. If you are back before 8, then if the roast is ready, we will have dinner.
   b. If you are back before 8 and the roast is ready, we will have dinner.

Because two successive if-clauses can restrict the same modal or quantifier:
17. a'=b' [Must: you are back before 8 & the roast is ready] we will have dinner.

• Back to Grice

18. a. If this is a guy in a red shirt, it’s Fred
   b. If a transversal line crosses two parallel lines, then alternate interior, alternate exterior, and
      corresponding angles are congruent.

Grice had material implication as a special case of “if … then …” for use in logic:
   Logicians are not obeying Cooperative Principle of conversation, so they are not getting the
   extra stuff like causality.

Kratzer: material implication is indeed a special case of “if … then …”. Our “silent” modal is a
necessity modal (must), usually epistemic. Logicians pretend that they have all the evidence in
the world, so epistemic necessity modal gives the same result as material implication:

Non-logical point of view
18. a' Given what I know, [Must: this-guy=guy in a red shirt] this-guy=Fred

Logical point of view
18. a" Given the way things are, [Must: this-guy=guy in a red shirt] this-guy=Fred
    b" Given the way things are, [ Must: a transversal line crosses two parallel lines] alternate
       interior, alternate exterior, and corresponding angles are congruent.