Formal Semantics
Sets, Functions, Language **Sophia Malamud**

1. Relations.

Ordered Pairs and Cartesian Product:

(1) Ordered pair/n-tuple: a set with n-elements where order matters.
$\langle a,b\rangle =_{def} \{\{a\},\{a,b\}\}$

(2) Cartesian Product: $A\times B =_{def} \{\langle x,y\rangle \mid x\in A \text{ and } y\in B\}$

Relations: a relation is a set of pairs (or, more generally, of n-tuples).
E.g., "mother of", "to be sitting to the right of".

Relation in A. E.g. "advisor of" in the set of people.
Relation from A to B. E.g. "advisor of" from the set of professors to the set of students

(3) A *relation* from A to B is a subset of A×B. A *relation* in A is a subset of A×A.

(4) a. Domain of R: {a | there is some b such that $\langle a,b\rangle \in R$}

b. Range of R: {b | there is some a such that $\langle a,b\rangle \in R$}

(5) Complement of a relation R from A to B (a relation $R\subseteq A\times B$): R'
$R' =_{def} (A\times B) - R$

QUESTION 1: Take A={a,b,c}, B={1,2,3} and C={a,b}. R={<a,1>, <b,2>, <b,3>}. What is the complement of the relation R from A to B? What is the complement of the relation R from C to B?

(6) Inverse of a relation: R^{-1}
$R^{-1} =_{def} \{\langle b,a\rangle \mid \langle a,b\rangle \in R\}$

QUESTION 2: Give the denotation of **kiss** (in both variants) in world w_{100}. Do the same for **assign**.

(7) World w_{100}: U = {Ann, Betty, Connor} Ann kisses Ann, Ann kisses Betty, and Betty kisses Connor. Ann assigns Ann to Connor, Betty assigns Ann to Betty, Betty assigns Betty to Ann, and Connor assigns Betty to Betty.

(8) [[**kiss**]] ^{w100} =

(9) [[**assign**]] $^{\mathbf{w100}}$ =

QUESTION 3: What is the relationship between a relation denoted by a transitive verb, and its passive form? For instance, the relationship between **kiss** and **be kissed by**?

2. Functions.

Functions:

(10) A relation R from A to B is a *function* from A to B ($F:A\rightarrow B$) iff:
a. The domain of R is A. (except for partial "functions")
Every member of A appears at least once as first member of a pair.
b. Each element in the domain is paired just with one element in the range
Every member of A appears at most once as first member of a pair.

(11) Argument and value: F(a) = b

(12) a. Functions from A to B are called functions *into* B.
b. Functions from A to B such that the range of the function equals B are called *onto* B.

(13) a. Functions from A to B where every member of B is assigned at most once to a member of A are called *one-to-one*.
b. Otherwise, we'll call them *many-to-one*.

(14) Functions that are at the same time onto and one-to-one are *called one-to-one correspondences*.

The characteristic function of a set:

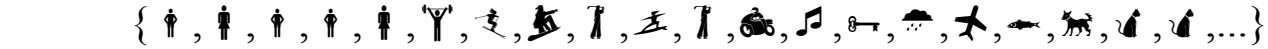
(15) a. Let A be a set. The, $char_A$, the *characteristic function of A*, is the function F such that, for any $x \in A$, $F(x)=1$, and for any $x \notin A$, $F(x)=0$.
b. Let F be a function with range {0,1}. Then, $char_F$, the *set characterized by F*, is $\{x \in D \mid F(x)=1\}$

3. Model-theoretic semantics

Central question: what kinds of model structures are appropriate for interpreting natural language?
Assumption: what works for one language, should apply to all the others.
So, part of our task: a semantic theory applicable to every language,
showing the universal properties of interpretation
and mapping the range of semantic variation

Syntax tells us what strings (of words) are sentences of a language
Semantics tells us about the meanings of different expressions in a language
Interpretation: a way of assigning meanings to expressions

Interpretation, at a minimum
What is needed to interpret a language? Starting with a minimum:

A model:
- universe of things/individuals – Bach's E for 'entity'

- truth values
{0 ,1}

Then, we can have meanings for proper names, and for sentences.
We can also build meanings for other expressions:
- predicates (VPs), nouns, and (some) adjectives *sets of individual entities*
OR *functions from E to {1,0}* (characteristic functions of sets)
- two-place predicates (transitive verbs) are *sets of ordered pairs of entities*
OR *functions from E to other functions, from E to {1,0}*
- three-place predicates (ditransitive verbs) are *sets of ordered triples of entities*

etc.

Interpretation function – Bach's D:
a function from expressions in the language
to things they stand for in the model

Our way of saying D(Bill) or D(b) = 🧍 (a certain entity in the model, the guy named Bill) is
(16) ⟦ Bill ⟧ = 🧍
We will often use shorthand, and use expressions in PC to stand for meanings of expressions in English (we can, because going from PC to the model is so straightforward), e.g.
(17) ⟦ Bill ⟧ = b

Compositionality:
For complex expressions, we will need a **recipe** for determining what D (or ⟦ . ⟧) will assign, *based on* what D assign to parts of the expression, and the way the expression is put together

Variable assignment – Bach's G:
a function from variables in the language
to things they stand for in the model

- What are variables? Placeholder expressions:
  - Things that depend on context in a particular way:
    (18) He runs. Mary saw them. She loves him.
  - Also a way to connect one part of expression with another:
    (19) Everyone loves their mother.
    (20) Every person is such that s/he runs. (Everyone runs)
    (21) Some apple is such that it is red. (Some apple is red)
- So, variable assignment (in natural language) is sometimes a matter of extra-linguistic context (the person I'm pointing to, the people I just mentioned); and at other times just a device to help us figure out a meaning of an expression.
  - Just knowing how to understand English doesn't help with interpreting words like 'he'
  - So, the interpretation function D doesn't help with variables (need assignments)

4. Possible world semantics
That's all we need to interpret PC – but not for natural language!
Here is one set of expressions we are missing: modal auxiliaries and adverbs
(22) a. John *should* walk in the park.
b. *Necessarily*, 2+2=4.
c. Mary *can* wash the dishes.

Possible worlds: a set W, representing *other ways things could be*
(Montague – also a set of times J)

Possible worlds:
(23) a. It may be raining. <=> There is some possible world in which it's raining
b. Mary must be home right now. <=> In every possible world, Mary is home right now

Possible worlds can also help us with some of Frege's problems:

Think of the reference ***inside various possible worlds*** of these expressions:
(24) a. Sophia

b. the professor of Ling 130 at Brandeis in 2008
c. Sherlock Holmes
d. The celestial body most distant from Earth
e. the perfect woman
f. the 20th student in this class
g. the biggest prime number

QUESTION 4. Can we express the difference between the sense and the reference of these expressions using possible worlds?

- Think how Frege's thinking about sense and reference of sentences may be recast using possible world semantics. What can we say about the truth of (25)?

(25) Odysseus was set ashore while sound asleep.

THIS WILL BE OUR CENTRAL ASSUMPTION: That the sense of a sentence is something called a ***proposition*****,** which is ***a set of possible worlds*** in which the sentence is true. OR, ALTERNATIVELY, a characteristic function of such a set: ***a function from each possible world to the truth-value*** of the sentence in that world

- Bach already mentioned that counterfactuals are "world-creating contexts" - we will most certainly use possible worlds to talk about these sentences.

(26) If it were April now, lily-of-the-valley would be in bloom.

- How can we use possible worlds to talk about speech and attitude reports?

(27) a. Holmes knows that the murderer is Bill
b. Sophia hopes that everyone will do their homework
c. John believes that the Earth is flat

We can have different flavours of modality by only looking at a ***subset*** of all possible worlds, called our ***Modal Base***:

(28) For any situations/possible worlds w and actual world w0 :
a. Epistemic modality:
$w \in Epi_x(w0)$ iff w conforms to what x knows in w0.
b. Deontic modality:
$w \in Deo(w0)$ iff all the obligations/requirements (to reach a given goal) are fulfilled in w, and w is maximally similar to w0 otherwise.
c. Doxastic R:
$w \in Dox_x(w0)$ iff w conforms to what x believes in w0 to be the case.
d. Bouletic:
$w \in Bou_x(w0)$ iff w conforms to what x desires in w0 for it to be the case.

Consider these sentences in different modalities:
(29) a. You must believe in spring.
b. Can John walk?