Set theory and meaning.

We already started dealing with sets:

- The meaning of a sentence is ***the set of situations*** in which it is true.
- Entailment: sentence A entails sentence B if ***the set of situations in which B is true*** is contained within ***the set of situations in which A is true***.

(1) a. Fido is a dog. b. Fido is a poodle.
(2) a. John has one good leg. b. John has two good legs.

1. Basic concepts, the meanings of some words.

(3) A SET is a collection of objects.

(4) An object is an ELEMENT OF a set A if that object is a member of the collection A. Notation: "∈" reads as "is an element of" or "belongs to".

(5) A set A is a SUBSET OF a set B if all the elements of A are also in B.
Notation: "⊆" reads as "is a subset of".

QUESTION 1. In the definition of entailment above, does the phrase "is contained within" correspond to "is an element of" or to "is a subset of"?

We can treat meanings of some nouns, verbs, and adjectives as sets (later in the course we will discuss whether this is quite right):

Nouns: denotations of nouns may vary from world to world. A noun denotes a set.

Adjectives: their denotations also vary. An adjective like **red** or **wooden** or **blond** is a set.

red is the set of all the red things in the world or situation,
apple is the set of all the apples in the world or situation

If in w_{100} Betty and Connor smoke and Ann doesn't, we write

$[[\textbf{smoke}]]^{w100} = [[\textbf{smoker}]]^{w100} = \{Betty, Connor\} = \{x: x \text{ smokes in } w_{100}\}$

And if Connor and Ann are blond, while Betty is a brunette,

$[[\textbf{blond}]]^{w100} = \{Ann, Connor\} = \{x: x \text{ is blond in } w_{100}\}$

QUESTION 2. In the example (1) above, what is the relationship between the meanings of the words **dog** and **poodle**? What about the meanings of the words **dog** and **Fido**?

(6) The INTERSECTION of two sets A and B ($A \cap B$) is the set containing all and only the objects that are elements of both A and B.

(7) The UNION of two sets A and B ($A \cup B$) is the set containing all and only the objects that are elements

of A, of B, or of both A and B.

(8) The COMPLEMENT of a set A (A') is the set containing all the individuals in the discourse except for the elements of A.

We can combine meanings of some set-denoting words by intersecting the sets they denote. For instance, with the w_{100} as described above, we can say that

$[[\textbf{blond smoker}]]^{w100} = [[\textbf{blond}]]^{w100} \cap [[\textbf{smoker}]]^{w100} =$
$= \{Ann, Connor\} \cap \{Betty, Connor\} = \{Connor\} = \{x: x \text{ is a blond smoker in } w_{100}\}$

(9) World w_{100}:

U = {Ann, Betty, Connor, apple1, apple2, apple3, cherry, table, chair}

(10) $[[\textbf{ red }]]^{w100}$ = {apple1, apple3, cherry, chair}

$[[\textbf{ wooden }]]^{w100}$ = {table, chair}

$[[\textbf{green}]]^{w100}$ = {apple2, table}

QUESTION 3: Give the denotation of **red apple** (in both variants) in world w100. Do the same for **red wooden thing**.

(11) The POWER SET of a set A (written as $\wp(A)$) is the set whose members are all the subsets of A.

QUESTION 4: Given the sets under (12) and assuming that the universe of the discourse is $\cup\{A, B, C, D, E, F, G\}$, list the members of the following sets:

(12) A= {1, 2, 3, 4}
B = {a, b, c, d, e, f}
C = {1, 2}
D = {1, 3, 4, a, b}
E = { {1}, 2, {a, 1} }
F = {1, c, d}
G = {d, e, 2, 3}

(13) a. C-D =
b. A∩F =
c. A∩B=
d. C'∩F' =
e. E∩C =
f. (C∪D) – (C∪D) =
g. F∪C =
h. G'∩C =
i. A∩E =
j. (E∪B)∪D =

2. Relations.

Ordered Pairs and Cartesian Product:

(14) Ordered pair/n-tuple: a set with n-elements where order matters.
$\langle a,b\rangle =_{def} \{\{a\},\{a,b\}\}$

(15) Cartesian Product: $A\times B =_{def} \{\langle x,y\rangle \mid x\in A \text{ and } y\in B\}$

Relations: a relation is a set of pairs (or, more generally, of n-tuples).
E.g., "mother of", "to be sitting to the right of".

Relation in A. E.g. "advisor of" in the set of people.
Relation from A to B. E.g. "advisor of" from the set of professors to the set of students

(16) A *relation* from A to B is a subset of A×B. A *relation* in A is a subset of A×A.

(17) a. Domain of R: $\{a \mid$ there is some b such that $\langle a,b\rangle \in R\}$

b. Range of R: $\{b \mid$ there is some a such that $\langle a,b\rangle \in R\}$

(18) Complement of a relation R from A to B (a relation $R\subseteq A\times B$): R'
$R' =_{def} (A\times B) - R$

QUESTION 5: Take A={a,b,c}, B={1,2,3} and C={a,b}. R={<a,1>, <b,2>, <b,3>}. What is the complement of the relation R from A to B? What is the complement of the relation R from C to B?

(19) Inverse of a relation: R^{-1}
$R^{-1} =_{def} \{\langle b,a\rangle \mid \langle a,b\rangle \in R\}$

QUESTION 6: Give the denotation of **kiss** (in both variants) in world w_{100}. Do the same for **assign**.

(20) World w_{100}: U = {Ann, Betty, Connor} Ann kisses Ann, Ann kisses Betty, and Betty kisses Connor. Ann assigns Ann to Connor, Betty assigns Ann to Betty, Betty assigns Betty to Ann, and Connor assigns Betty to Betty.

(21) $[\![\textbf{kiss}]\!]^{w100} =$

(22) $[\![\textbf{assign}]\!]^{\textbf{w100}} =$

QUESTION 7: What is the relationship between a relation denoted by a transitive verb, and its passive form? For instance, the relationship between **kiss** and **be kissed by**?