Math 201a, Topics in Algebra, Spring 2010
Course Orientation and Syllabus

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Class website: http://people.brandeis.edu/~tbl/math201a/
Schedule:  Mon 5:15pm-6:35pm, Thurs 2pm-3:20pm.
Office Hours: will be fixed by a class vote on the first day of class
Textbook:  none (but see below for books which will be useful)

1 Course Grade

Your course grade will be comprised of:

• Homeworks (60% of the grade)
• A final paper (40% of the grade)

2 So, what is the course about?

This will be a course about ‘rigid analytic geometry’; so let me start by saying a few words about what that might be. In undergraduate courses, you will have studied calculus and its big brother analysis. These courses often emphasize that analysis doesn’t work very well over $\mathbb{Q}$, but it does work much better over $\mathbb{R}$; the key reason for this is that the reals are complete (every Cauchy series converges). Analysis becomes even more well behaved if we work over the algebraically closed (and still complete) field $\mathbb{C}$, where many pathologies that exist over $\mathbb{R}$ go away.

But we should be aware that $\mathbb{R}$ and $\mathbb{C}$ are not the only complete fields out there. For each prime $p$, we have another metric on the rationals, the $p$-adic metric, and if we complete the rationals with respect to this metric, we get another complete field, $\mathbb{Q}_p$, which behaves in some respects very like $\mathbb{R}$. (We can also construct an algebraically closed and complete field $\mathbb{C}_p$, which plays the role of $\mathbb{C}$.) It is natural to try to recreate the analysis that worked for $\mathbb{R}$ and $\mathbb{C}$ in this context too: this is the field of $p$-adic analysis, and a brief study of this will be the first order of business in the course. The study will be brief because much ends up being the same (or even easier) than the real case, and so we should be able to speed through the material merrily.

But this will not be the main body of the course. One direction that real analysis can take after the standard undergraduate courses is to connect to topology and geometry. This leads to the study of analysis on manifolds, which exists in many versions (complex, smooth, $C^1$, ...), where a manifold is an object that is ‘stitched together’ out of pieces which locally look like $\mathbb{R}^k$ (or $\mathbb{C}^k$) but may be stitched together in a way which has some interesting topology. For example, the surface of a donut looks like $\mathbb{R}^2$, but the donut is topologically different to $\mathbb{R}^2$. The interaction of geometry, topology and analysis here leads to very rich mathematics.

The main aim of this course will be to develop a theory of ‘$p$-adic manifolds’. This turns out to be significantly more complicated than setting up a theory of real or complex manifolds, because our geometric intuition is less easy to apply. In fact, there are several different approaches with different advantages and disadvantages, and the last word on ‘which is better’ has still not emerged. We will start by covering the ‘classical’ theory, whose study was originated by Tate; this is called the theory of rigid analytic spaces. Just over half-way through the course, we will start to study some of the more recent approaches; the approach of Raynaud and others (and the closely related approach of Fujiwara-Kato) which makes a very strong
connection to formal geometry (a part of algebraic geometry), the approach of Berkovich which makes geometric intuitions much easier to apply; and perhaps also (time permitting) the approach of Huber, which bridges between Berkovich and Raynaud.

3 Content

We will cover the following topics:

- Basic ideas of $p$-adic analysis
- The theory of rigid spaces, including
  - affinoid algebras
  - the Tate topology
  - rigid analytic spaces
  - sheaves and cohomology
- More recent approaches:
  - Raynaud
  - Berkovich
  - Huber

4 Books

There is no textbook for the course. However, the following books may be useful for the first part of the course:

- Non-Archimedean Analysis (A Systematic Approach to Rigid Analytic Geometry), Bosch, S., G"untzer, U., Remmert, R.
- Lectures on Formal and Rigid Geometry, S. Bosch (online notes)
- Rigid analytic geometry and its applications, Fresnel, J., Van Der Put, M.

In the latter part of the course, you might find it helpful to refer to Brian Conrad’s mini-course notes ‘Several approaches to non-archimedean geometry’, and the references therein. (These notes can either be found online, or as part of the book ‘P-adic Geometry: Lectures from the 2007 Arizona Winter School’, published by the AMS.)

5 Official policies

5.1 Documented disabilities

If you are a student who needs academic accommodations because of a documented disability, please contact me and present your letter of accommodation as soon as possible. If you have questions about documenting a disability or requesting academic accommodations, you should contact Beth Rodgers-Kay in Academic Services (x6-3470 or brodgers@brandeis.edu.)
Letters of accommodation should be presented at the start of the semester to ensure provision of accommodations. Accommodations cannot be granted retroactively.

5.2 Academic honesty

You are expected to follow the University's policy on academic integrity, which is distributed annually as section 4 of the Rights and Responsibilities Handbook. Instances of alleged dishonesty will be forwarded to the Department of Student Development and Conduct for possible referral to the Student Judicial System. Potential sanctions include failure in the course and suspension from the University. If you have any questions about how these policies apply to your conduct in this course, please ask. See http://www.brandeis.edu/studentaffairs/sdc/rr/index.html for more information.

5.3 Homework grade queries

Homework grades will be posted on LATTE at the same time as homework is returned to you. You are responsible for checking the posted grade to see if any errors have been introduced as the grades have been entered. You must do this in a timely fashion: any requests to change grades in LATTE must be submitted within one week of the grades being posted.