Math 20A lecture 10

When I was a lad

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Announcements

- Homework five will be posted soon, due 1 week from today
- Office hours are 3–4.30pm today
- See the website for all sorts of course-related fun `http://people.brandeis.edu/~tbl/math20a/`
- It is *your responsibility* to log into LATTE and check that the grades are entered correctly. So far, HW 1–4 should be posted.
- Please give me feedback!
- I'm afraid your 'total score' in LATTE is not meaningful

Previously on math 20a

- We talked about partial derivatives, the gradient, and directional derivatives.
- We spent a lot of time trying to visualize what they all *mean*.

Example: contour plots the old fashioned way

Sketch a contour plot for the function

$$f(x, y) = x^2y - 2xy - \frac{y^2x}{2} + y^2$$

Example: contour plots the old fashioned way

$$f(x, y) = x^2y - 2xy - \frac{y^2x}{2} + y^2$$

- The partials are

$$f_x = 2xy - 2y - (1/2)y^2$$
$$f_y = x^2 - 2x - yx + 2y = (x - 2)(x - y).$$

- $f_y = 0$ tells us either $x = 2$ or $x = y$.
- If $x = 2$, $f_x = 0$ becomes $4y - 2y - (1/2)y^2 = 0$, or $4y = y^2$; so $y = 4$ or 0.
- If $x = y$, $f_x = 0$ becomes $2y^2 - 2y - (1/2)y^2 = 0$, or $3y^2 = 4y$; so $y = 0$ or $4/3$.

Example: contour plots the old fashioned way

- The second partials are

$$f_{xx} = 2y$$
$$f_{yy} = 2 - x$$
$$f_{xy} = 2x - 2.$$

	$(0,0)$	$(2,4)$	$(2,0)$	$(4/3,4/3)$
f_{xx}	0	8	0	8/3
f_{yy}	2	0	0	2/3
f_{xy}	-2	-2	2	-2/3
type	sdl	sdl	sdl	min

Example: finding maxima exactly

I am a master horse whisperer, and I happen to know that two horses, Arm-n-a-leg and Buy-buy-baby, have a better than evens chance of winning their respective races. In particular, I know they both have a 0.7 chance of winning.

The bookies' are offering me even odds, so for every $1000 I bet, I get $2000 back if one of my horses wins, nothing back otherwise.

How much should I bet? (My current wealth W is $10000, and my utility function is $\log(W/1000)$.)

Example: finding maxima exactly

We work out the expected utility function. (If you're not into stats or econ, ignore this slide!)

Suppose I put x thousand dollars on A and y thousand dollars on B.

- with prob 0.49, both win, and $U = \log(10 + x + y)$
- with prob 0.21, just A wins, and $U = \log(10 + x - y)$
- with prob 0.21, just B wins, and $U = \log(10 - x + y)$
- with prob 0.09, neither wins, and $U = \log(10 - x - y)$

Overall $U = 0.49 \log(10 + x + y) + 0.21 \log(10 + x - y) + 0.21 \log(10 - x + y) + 0.09 \log(10 - x - y)$

Example: finding maxima exactly

Graphing the function, we see there's a single critical point which is the maximum we seek. We can find it by setting the partials to 0.

$$f_x = \frac{0.49}{10+x+y} - \frac{0.21}{10-x+y} + \frac{0.21}{10+x-y} - \frac{0.09}{10-x-y}$$
$$f_y = \frac{0.49}{10+x+y} + \frac{0.21}{10-x+y} - \frac{0.21}{10+x-y} - \frac{0.09}{10-x-y}$$

When $f_x = f_y = 0$,

$$0 = \frac{0.49}{10+x+y} - \frac{0.21}{10-x+y} + \frac{0.21}{10+x-y} - \frac{0.09}{10-x-y}$$
$$0 = \frac{0.49}{10+x+y} + \frac{0.21}{10-x+y} - \frac{0.21}{10+x-y} - \frac{0.09}{10-x-y}$$

Example: finding maxima exactly

$$0 = \frac{0.49}{10+x+y} - \frac{0.21}{10-x+y} + \frac{0.21}{10+x-y} - \frac{0.09}{10-x-y}$$

$$0 = \frac{0.49}{10+x+y} + \frac{0.21}{10-x+y} - \frac{0.21}{10+x-y} - \frac{0.09}{10-x-y}$$

Subtracting, we get

$$0 = \frac{0.21}{10-x+y} - \frac{0.21}{10+x-y}$$

So $0.21(10+x-y) = 0.21(10-x+y)$, so $x = y$.

Example: finding maxima exactly

$$0 = \frac{0.49}{10+2x} - \frac{0.21}{10} + \frac{0.21}{10} - \frac{0.09}{10-2x}$$

So

$$0 = \frac{0.49}{10+2x} - \frac{0.09}{10-2x}$$

So $0 = (10-2x)0.49 - (10+2x)0.09$, so
$0 = 4.9 - 0.98x - 0.9 - 0.18x$, so $1.16x = 4$, so $x = 3.448$.
y is also 3.448.