Math 20A lecture 13

Crank up the volume

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Announcements

- Homework six due today.
- Homework seven will be posted soon, due 1 week from today.
- Office hours are 3–4.30pm today
- See the website for all sorts of course-related fun `http://people.brandeis.edu/~tbl/math20a/`
- It is *your responsibility* to log into LATTE and check that the grades are entered correctly. So far, HW 1–4 should be posted.

Previously on math 20a

- We talked about finding maxima. We saw several different ways functions could have (or fail to have) maxima, and figured out how one might go about computing their coordinates exactly in each case.
- We also looked at some ‘IFR’ rules for finding maxima/minima in simple cases when we can’t see the function at all.
- We looked at how to find critical points subject to a constraint, using the method of Lagrange multipliers.

Previously on math 20a

Method of Lagrange multipliers: To find a critical point of the function $f(x,y,z)$ subject to the constraint that $g(x,y,z)=0$, introduce the function

$$L(x,y,z,\lambda) = f(x,y,z) + \lambda g(x,y,z)$$

The critical points of f subject to the constraint are just the ‘ordinary’ critical points of L. We can find them by setting

$$\frac{\partial L}{\partial x} = \frac{\partial L}{\partial y} = \frac{\partial L}{\partial z} = \frac{\partial L}{\partial \lambda} = 0$$

And now for something completely different

Example: Finding the volume under a graph

What is the volume under the graph of $f(x, y) = xy^2$ above the region $0 \leq x \leq 2$, $-1 \leq y \leq 1$?

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- We use the technique of iterated integration. If we make a slice $x =$*(const)*, the cross-sectional area is:

$$\int_{-1}^{1} f(x, y) dy = \int_{-1}^{1} xy^2 dy$$

- Combining all the cross sectional areas together, we get that the total area is

$$\int_0^2 \int_{-1}^{1} f(x, y) dy\, dx = \int_0^2 \int_{-1}^{1} xy^2 dy\, dx$$

Example: Finding the volume ctd

$$\int_0^2 \int_{-1}^1 f(x,y)dy\,dx = \int_0^2 \int_{-1}^1 xy^2 dy\,dx$$

- *Always do the inside integral first!*

$$\int_0^2 \int_{-1}^1 xy^2 dy\,dx = \int_0^2 \left[x\frac{y^3}{3}\right]_{y=-1}^1 dx$$

$$= \int_0^2 (x\frac{1^3}{3}) - (x\frac{(-1)^3}{3})dx = \int_0^2 (2x/3)dx$$

$$= \left[\frac{x^2}{3}\right]_{x=0}^2 = \frac{2^2}{3} = 4/3$$

If we'd sliced the other way...

If we make a slice $y =$*(const)*, the cross-sectional area is: $\int_0^2 f(x,y)dx = \int_0^2 xy^2 dx$ Combining all the cross sectional areas together, we get that the total area is

$$\int_{-1}^{1}\int_{0}^{2} f(x,y)dx\,dy = \int_{-1}^{1}\int_{0}^{2} xy^2 dx\,dy$$

Always do the inside integral first!

$$\int_{-1}^{1}\int_{0}^{2} xy^2 dx\,dy = \int_{-1}^{1}\left[\frac{x^2y^2}{2}\right]_{x=0}^{2} dy = \int_{-1}^{1} 2y^2 dy$$

$$= \left[\frac{2y^3}{3}\right]_{x=-1}^{1} = \frac{2(1)^3}{3} - \frac{2(-1)^3}{3} = 4/3$$

Sometimes, order matters

Even though you get the same answer in either order, it often matters a lot to us which order we do things in—an integral which is easy in one direction can be impossible or much harder in the other direction.

e.g.: What is the integral of the function

$$f(x, y) = x \cos(xy)$$

over the region $0 \leq x \leq \pi$, $0 \leq y \leq 1$?