Math 20A lecture 17

Line integrals

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Announcements

- Homework eight due today.
- Homework nine will be posted soon, due 1 week from today.
- Office hours are 3–4.30pm today
- See the website for all sorts of course-related fun `http://people.brandeis.edu/~tbl/math20a/`
- It is *your responsibility* to log into LATTE and check that the grades are entered correctly. So far, HW 1–6 should be posted.

Previously on math 20a

- We looked at vector fields in two dimensions, and tried to practice visualizing them.

Example: computing a line integral

Compute the line integral

$$\int_C \mathbf{F} \cdot d\mathbf{r}$$

where C is the curve given parametrically as $\mathbf{r}(t) = \langle t, t^2 \rangle$ (t between 0 and 1), and $\mathbf{F}$ is the vector field $\mathbf{F}(x, y) = \langle x + y, y - x \rangle$.

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- We work out:

$$\mathbf{r}'(t) = \langle 1, 2t \rangle$$
$$\mathbf{F}(\mathbf{r}(t)) = \langle t + t^2, t^2 - t \rangle$$

- We then use the formula...

Example: computing a line integral

$$\mathbf{r}'(t) = \langle 1, 2t \rangle$$

$$\mathbf{F}(\mathbf{r}(t)) = \langle t + t^2, t^2 - t \rangle$$

The formula gives:

$$\begin{aligned}
\int_C \mathbf{F} \cdot d\mathbf{r} &= \int_0^1 \mathbf{F}(\mathbf{r}(t)) \cdot \mathbf{r}'(t)\, dt \\
&= \int_0^1 \langle t + t^2, t^2 - t \rangle \cdot \langle 1, 2t \rangle\, dt \\
&= \int_0^1 (t + t^2) + (t^2 - t)2t\, dt = \int_0^1 t + t^2 + 2t^3 - 2t^2\, dt = \ldots
\end{aligned}$$

Example: computing a scalar line integral

Compute the line integral

$$\int_C xy^4 ds$$

where C is the curve given parametrically as $\mathbf{r}(t) = \langle \sin t, \cos t \rangle$, for t between 0 and π.

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$$|\mathbf{r}'(t)| = \sqrt{(\cos t)^2 + (\sin t)^2} = 1$$

$$f(\mathbf{r}(t)) = \sin t (\cos t)^4$$

Example: computing a scalar line integral

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$$|\mathbf{r}'(t)| = \sqrt{(\cos t)^2 + (\sin t)^2} = 1$$

$$f(\mathbf{r}(t)) = \sin t (\cos t)^4$$

We then use the formula...

$$\int_C xy^4 ds = \int_0^\pi f(\mathbf{r}(t))|\mathbf{r}'(t)|dt = \int_0^\pi \sin t (\cos t)^4 \, 1 \, dt$$

$$= \int_1^{-1} -u^4 \, du = \left[-u^5/5\right]_1^{-1} = 2/5$$