Math 20A lecture 19

Div and curl

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Announcements

- Homework nine due today.
- Homework ten will be posted soon, due 1 week from today.
- Office hours are 3–4.30pm today
- See the website for all sorts of course-related fun `http://people.brandeis.edu/~tbl/math20a/`
- It is *your responsibility* to log into LATTE and check that the grades are entered correctly. So far, HW 1–6 should be posted.

Previously on math 20a

- We looked at vector fields in two dimensions, and tried to practice visualizing them.
- We looked at several kinds of line integrals. The two most important ones were *vector line integrals* and *flux integrals*.
- Vector line integrals measure how much the vector field is 'helping us' (or if negative, hindering us) as we try to move *along* a path
- Flux integrals measure how much 'vector stuff' is going *through* a boundary.

Example: computing a 2D divergence

Compute the divergence of the vector field

$$\mathbf{F} = \langle \cos x, \sin x + y \rangle$$

Example: computing a 2D divergence

Compute the divergence of the vector field

$$\mathbf{F} = \langle \cos x, \sin x + y \rangle$$

We just use the rule

$$\text{div } \mathbf{F} = \frac{\partial \mathbf{F}_x}{\partial x} + \frac{\partial \mathbf{F}_y}{\partial y}$$

$$= -\sin x + 1$$

Example: computing a 2D divergence

Compute $\nabla \cdot \mathbf{F}$ for the vector field

$$\mathbf{F} = \langle \cos x, y \sin x \rangle$$

Example: computing a 2D divergence

Compute $\nabla \cdot \mathbf{F}$ for the vector field

$$\mathbf{F} = \langle \cos x, y \sin x \rangle$$

We just use the rule

$$\text{div } \mathbf{F} = \frac{\partial \mathbf{F}_x}{\partial x} + \frac{\partial \mathbf{F}_y}{\partial y}$$

$$= -\sin x + \sin x = 0$$

Example: computing a 2D curl

Compute the curl of the vector field

$$\mathbf{F} = \langle \cos x, \sin x + y \rangle$$

Example: computing a 2D curl

Compute the curl of the vector field

$$\mathbf{F} = \langle \cos x, \sin x + y \rangle$$

We just use the rule

$$\text{curl } \mathbf{F} = \frac{\partial \mathbf{F}_y}{\partial x} - \frac{\partial \mathbf{F}_x}{\partial y}$$

$$= \cos x + 0$$

Example: computing a 2D curl

Compute $(\nabla \cdot R\mathbf{F})$ for the vector field

$$\mathbf{F} = \langle x^2 + y, y^2 + x \rangle$$

Example: computing a 2D curl

Compute $(\nabla \cdot R\mathbf{F})$ for the vector field

$$\mathbf{F} = \langle x^2 + y, y^2 + x \rangle$$

We just use the rule

$$\text{curl } \mathbf{F} = \frac{\partial \mathbf{F}_y}{\partial x} - \frac{\partial \mathbf{F}_x}{\partial y}$$

$$= 1 + (-1) = 0$$

Example: computing a 3D divergence

Compute the divergence of the vector field

$$\mathbf{F} = \langle z \cos x, \sin x + y, xyz \rangle$$

Example: computing a 3D divergence

Compute the divergence of the vector field

$$\mathbf{F} = \langle z\cos x, \sin x + y, xyz\rangle$$

We just use the rule

$$\text{div } \mathbf{F} = \nabla \cdot \mathbf{F} = \frac{\partial \mathbf{F}_x}{\partial x} + \frac{\partial \mathbf{F}_y}{\partial y} + \frac{\partial \mathbf{F}_z}{\partial z}$$

$$= -z\sin x + 1 + xy$$

Example: computing a 3D curl

Compute the curl of the vector field

$$\mathbf{F} = \langle z \cos x, \sin x + y, xyz \rangle$$

Example: computing a 3D curl

Compute the curl of the vector field

$$\mathbf{F} = \langle z \cos x, \sin x + y, xyz \rangle$$

We just use the rule

$$\text{curl } \mathbf{F} = \nabla \times \mathbf{F}$$

$$= \langle \frac{\partial \mathbf{F}_z}{\partial y} - \frac{\partial \mathbf{F}_y}{\partial z}, \frac{\partial \mathbf{F}_x}{\partial z} - \frac{\partial \mathbf{F}_z}{\partial x}, \frac{\partial \mathbf{F}_y}{\partial x} - \frac{\partial \mathbf{F}_x}{\partial y} \rangle$$

$$= \langle yz - 0, \cos x - xy, \cos x - 0 \rangle$$