Math 20A lecture 8

The chain rule

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Announcements

- Homework four due Friday, accepted until 10.50am
- Office hours are 2–3.30pm today
- See the website for all sorts of course-related fun
  `http://people.brandeis.edu/~tbl/math20a/`

Previously on math 20a

- We did some stuff. Not much of it will be relevant today!

Previously in math

- We learned about the chain rule. This states that if you have a composition of functions $c(x) = f(g(x))$, then

$$c'(x) = f'(g(x))g'(x)$$

- If we write y for the output of g and z for the output of f (which is also the output of c), we can also write this as

$$\frac{dz}{dx} = \frac{dz}{dy}\frac{dy}{dx}$$

Chain rule examples

- $y = (x^2 + \sqrt{x} + \sin x)^3 + e^{x^2 + \sqrt{x} + \sin x} + \sqrt{x^2 + \sqrt{x} + \sin x}$
- $y = \sin \cos x$
- $y = e^{3x}$
- $y = f(x^2 + 3)$; what is $\frac{dy}{dx}$ if we know $x = 3$, and

$$f'(3) = 7,\ f'(12) = 4\ f'(6) = 9$$

The higher chain rule

What about:

- $y = (x^2 + \sqrt{x} + \sin x)^3 (\cos x + \ln x) + e^{x^2 + \sqrt{x} + \sin x} + \sqrt{(x^2 + \sqrt{x} + \sin x)(\cos x + \ln x)}$
- $\Gamma(t^2, e^t)$
- t^t

Physics! (sort of)

A particular electrical component can be used to generate magnetic fields. The (rms) strength of the magnetic field produced is affected both by the temperature of the apparatus and the frequency of the current driving the apparatus.

Suppose that, starting from a fixed starting point, we have measured that the magnetic field gets stronger by 3.1 nanotesla for each $^{\circ}C$ the temperature falls, and gets stronger by 2.3 nanotesla for each extra miliHertz of frequency.

We start changing the temperature at an instantaneous rate of $1^{\circ}C/min$ and the frequency by $1mHz/min$. What is the rate of change of the strength of the magnetic field?

A cautionary tale

Chain rule example

- Suppose that $w = xe^{yz}, x = t^2, y = 1 - t, z = 1 + 2t$.
  What is $\frac{dw}{dt}$?

Chain rule example

- Suppose that $w = xe^{yz}, x = t^2, y = 1 - t, z = 1 + 2t$. What is $\frac{dw}{dt}$?
- We just work out

$$\frac{dx}{dt} = 2t, \qquad \frac{dy}{dt} = -1 \qquad \frac{dz}{dt} = 2$$
$$\frac{\partial w}{\partial x} = e^{yz}, \qquad \frac{\partial w}{\partial y} = xze^{yz} \qquad \frac{\partial w}{\partial z} = xye^{yz}$$

then

$$\frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{dy}{dt} + \frac{\partial w}{\partial z}\frac{dz}{dt}$$
$$= 2te^{(1-t)(1+2t)} + (-1)t^2(1+2t)e^{(1-t)(1+2t)} + 2t^2(1-t)e^{(1-t)(1+}$$