

Long Run Expectations, Learning and the U.S. Housing Market*

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Abstract

This paper examines key facts about the U.S. housing market. The price to rent ratio is highly volatile and significantly autocorrelated. Returns on housing are positively autocorrelated. The price to rent ratio is negatively correlated with future returns on housing and future rent growth. Finally, housing returns exhibit significant time varying volatility. Given the similarity of these facts to those in U.S. equity markets, I begin by showing that a benchmark general equilibrium asset pricing model is inconsistent with these facts. I modify the model in two ways to improve its fit with the data. First, I allow for pricing frictions so prices adjust slowly to their fundamental value. Second, I assume the agent does not know if housing fundamentals, captured by preference shocks, are trend stationary or difference stationary and has changing beliefs depending on how well each model fits the current data. I find that these modifications allow the model to increase the volatility of the price to rent ratio and to match the autocorrelation of housing returns. The price to rent ratio then negatively forecasts returns and rent growth. Finally the model generates time varying volatility consistent with the data.

JEL Codes: D83, D84, G12, R21, E03

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1 Introduction

Given the recent boom and bust in housing markets there is renewed interest in understanding the determinates of U.S. house prices. In this paper I begin by noting that housing market data present several puzzles that are present in asset market data more broadly, especially equities. First, there is evidence of excess volatility in house prices. The standard deviation of the price to rent ratio is 15% and the standard deviation of housing returns is 6%, while the standard deviation of the underlying housing rents is only 2.3%. Second, housing returns are significantly positively autocorrelated. Finally, the price to rent ratio is negatively correlated with future returns and rent growth and housing returns show evidence of time varying volatility. While housing as an asset is in many ways different than equities, the similarities between these empirical facts and facts in the aggregate U.S. equity market motivates me to examine the equilibrium housing price in a general equilibrium asset pricing framework.^{1,2}

I begin with a standard consumption-based asset pricing model and use a log-linear approximation to the Euler equation as in Campbell (1993) and Restoy and Weil (2011) to solve for the equilibrium house price. The equilibrium house price then depends on future expectations of fundamentals (housing preferences) and consumption. I show that this model is unable to explain the facts outlined in the previous paragraph. Therefore, I adapt the standard model in two ways. First, I allow for prices to adjust slowly to their fundamental value. Second, I assume the agent does not know the true model for housing preference shocks. Specifically, they are unsure if preference shocks are permanent or temporary. They use a Bayesian learning model as in Cogley and Sargent (2005) to learn if the preference process is trend stationary or difference stationary. Their beliefs change over time depending on how well each model fits the data. While the true process is (trend) stationary, the agent does not know this. He puts excessive weight on the difference stationary process, overreacting to temporary changes in market preferences.

Two features of the housing preference processes makes this learning significant. The first is the well know fact that unit root and near unit root processes are very difficult to tell apart in small time series samples. See, for example, Cochrane (1988); Stock (1991). As a result, the agent will almost always put some weight on the difference stationary process even if the

¹For an analysis of similar facts for the S&P 500 index see Laibson et al. (2007) or Tortorice (2014)

²Some of the important differences between housing markets and equity markets include larger transactions costs for buying and selling houses, the use of debt to purchase housing, and the fact that homeowners usually own exactly one house at a time.

true process is trend stationary. Additionally, after a random sequence of shocks which moves housing preferences away from their long run trend the agent will put additional weight on the difference stationary model. Second, analogous to the analysis of the permanent income hypothesis (see for example Deaton (1992)) if the individuals believes the true process is a unit root process then shocks are permanent. As a results, they will react strongly to news about fundamentals and return volatility will increase.

The sticky price assumption and the learning mechanism allow the model to better match the data. Learning amplifies volatility over the rational expectations benchmark. I find that the learning model generates increased volatility in both housing returns and the HP-filtered house price. Sticky prices allow the model to match the autocorrelation of housing returns. Learning about the true nature of the preference process creates a negative correlation between the price to rent ratio and future returns and rent growth. Additionally, the learning model generates time varying volatility consistent with the data. The learning model generates excessive kurtosis of returns. Data from the learning model is consistent with the positive autocorrelation of squared returns and the estimation of GARCH effects in U.S. data.

There is a large literature incorporating housing into macroeconomic models beginning with the contributions of Benhabib et al. (1991) and Greenwood and Hercowitz (1991). These models modify the standard real business cycle (RBC) framework to include housing and home production. Davis and Heathcote (2005) extend this framework to allow for housing specific productivity shocks but find that the model underpredicts the volatility of house prices and predicts a counterfactually negative correlation between house price growth and new home construction. Iacoviello (2005) and Iacoviello and Neri (2010) incorporate housing as a transmission mechanism for monetary policy in a New-Keynesian framework and find that preference shocks are important for explaining house price volatility. While much work has been done incorporating housing into standard macroeconomic models, these models still struggle to explain the volatility of housing prices given observed fundamentals. This observation motivates the current paper as well as other recent work, e.g. Miao et al. (2014) who amplify housing prices in a model where housing is used as collateral by firms engaged in production.

A second strand of literature attempts to explain house prices using supply and demand models with city-specific shocks and search frictions beginning with the work of Wheaton (1990) and Krainer (2001). Recent contributions include: Glaeser and Gyourko (2006); Head et al. (2012). These models use a wide variety of fundamentals to explain the cross section

of house price volatility across cities, e.g. shocks to local amenities and income. The goals of this paper differ, in that I am concerned about explaining the macroeconomic time series of house prices and returns, specifically focusing on return predictability and time varying volatility while this literature is concerned about the cross section and autocorrelation of price growth across metropolitan areas.

My paper also relates to the literature that models housing within an asset pricing framework. Piazzesi et al. (2007) model housing jointly as an asset and a consumption choice in an otherwise standard consumption based asset pricing model. They show that housing increases the risk premium and predicts excess returns in equity markets. Lustig and Nieuwerburgh (2005) reach a similar conclusion in a model where housing is an important source of collateral. Flavin and Nakagawa (2008) explore how the illiquidity of housing influences the stochastic discount factor in a consumption based asset pricing model. Ayuso and Restoy (2006) apply the asset pricing framework of Restoy and Weil (2011) and show that a large part of the fluctuations in Spanish house prices can not be explained with observed fundamentals. The present paper uses the asset pricing framework of Restoy and Weil (2011) to explain house prices, but differs from the above papers by focusing on the volatility and predictability of housing returns and considering a learning based model of expectation formation.

Many recent papers have tried to explain the recent U.S. housing boom and bust in rational expectations models using various institutional features and frictions in the housing market. For example, Chu (2014) uses the fall in down payment requirements as an explanation for the increase in house prices during the boom period. However, the model is unable to explain why interest rates remained low at the same time. Similar mechanisms are explored in Chambers et al. (2009) and Iacoviello and Pavan (2013) with the former examining the impact of down payment requirements on the rise in homeownership rates during the boom and the later arguing a tightening of credit constraints can lead to a large drop in home production though these papers take the path of house prices as given. Corbae and Quintin (2013) and Garriga and Schlagenhaut (2009) explore how leverage led to an increase in foreclosures during the house price bust, but treat the path of prices as exogenous. Titman et al. (2014) and Chatterjee and Eyigungor (2009) argue overbuilding was an important contributor to the bust in housing prices, however they have difficulty explaining the positive correlation between house prices and residential investment. Favilukis et al. (2013) combine foreign capital inflows, relaxed credit constraints and financial market liberalization to explain the fluctuations in the price-to-rent ratio during the boom and bust. The model

has considerable success explaining the volatility of house prices and returns and generates predictability in housing returns as well. However, agents are aware of this predictability and therefore expectations of future returns are low when the price to rent ratio is high. This result is in contrast to expected returns in my model and data on survey expectations discussed below.

One of the first papers to examine data on house price expectations is Case and Shiller (2003). They find that home buyers have unrealistic expectations concerning future house price increases, predicting double digit increases annually over the next ten years. Households also are unlikely to view housing as a risky investment. Case et al. (2012b) replicate these results and argue that long run expectations (10-year) house price expectations are the primary driver of the house price boom and present evidence of extrapolative behavior in agents expectations formation. Piazzesi and Schneider (2009) argue for the presence of momentum traders in the housing market, agents who are always optimistic about price changes, based on data from the Michigan Survey of Consumers. Foote et al. (2012) present evidence that even industry financial analysts were bullish about house prices even at the 2006 house price peak.

Based on this empirical evidence many authors have examined the implication of relaxing rational expectations for house price dynamics. In fact, given the shortcomings of rational expectations models to match the volatility of house prices Glaeser and Gyourko (2006) and Glaeser et al. (2008) argue that deviation from rational expectations and models of learning may be fruitful avenues of research as do Piazzesi and Schneider (2016). Glaeser and Nathanson (2015) study a model where agents neglect to consider the forecasts of other agents when forecasting future prices. They show that this mechanism generates extrapolation in agents price forecasts explaining short run momentum and medium term reversion in price changes. My paper differs in considering the impact of beliefs for time varying volatility of housing returns and also modeling learning which allows agents to abandon models which look unlikely given the data.

There is a large literature on learning and equity prices: see, for example, Pastor and Pietro (2003); Pastor and Veronesi (2009). However, comparatively fewer papers have modeled house prices and expectations using learning models. Burnside et al. (2011) consider a model where, as in the current paper, learning about long run fundamentals is essential. Though in their paper learning comes from social dynamics as opposed to observation of fundamentals. Bolt et al. (2014) generated boom and busts in house prices through an heterogeneous agent model where the agents endogenously switch between different price forecast rules. Adam

et al. (2012) consider a model where agents learn about house price growth in an open economy model. Caines (2015) incorporates the Adam et al. model into a model where expectations affect both the demand and supply side of housing. He finds that a substantial fall in the mortgage rate can replicate key fact of the recent U.S. housing boom. Gelain and Lansing (2013) considers learning in an asset pricing based model of housing where agents learn about rent growth using a misspecified model and extrapolative expectations. All these models increase volatility of the price to rent ratio and the Gelain and Lansing paper also generates predictability in house price returns. However, my paper differs from these in important ways. First, the paper seeks to endogenously explain both the predictability and the time varying volatility in housing returns as well as amplifying volatility.³ Secondly, I present a novel model of learning where agents are unsure about the true process for fundamentals and change their beliefs based on how accurately each model captures the data. Finally, agents make significant mistakes about their long run expectations (as opposed to their short run expectations) consistent with the results in Case et al. (2012b,a).⁴

The rest of the paper proceeds as follows. Section two discusses the data and the key empirical facts. Section three outlines the model and section four explains its calibration. Section five gives the main model results and section six demonstrates the robustness of the results to alternative parameter specifications. Section seven concludes.

2 Data

Data come from Davis et al. (2008).⁵ Data begin in 1960:Q1 and end in 2013:Q1. Data on rents and house prices are obtained from the Decennial Census of Housing from 1960 to 2000. Data on rents are interpolated between census dates using the Bureau of Labor Statistics (BLS) index for the rent of primary residences. Data on house prices are interpolated using the Freddie Mac (CMHPI) series repeat-sales house prices index after 1970 and the median price of new homes sold index before 1970. The Macromarkets LLC national house price index, formally known as the Case-Shiller-Weiss index, is used after 2000 to construct house prices. Prices are deflated using the CPI.

Moments for the data are given in table 1. The full sample expected return on housing,

³In Gelain and Lansing (2013) for example, fundamentals exhibit exogenous time-varying volatility.

⁴Nguyen (2014) uses a similar learning model to explain serially correlated house price forecast errors and house price volatility in a model in which housing is allocated by a central planner who does not know the true process for housing preference shocks.

⁵Data are available at: <http://www.lincolninst.edu/subcenters/land-values/rent-price-ratio.asp>.

given by $E \frac{q_t}{q_{t-1}d_{t-1}}$ where q_t is the house price at time t and d_{t-1} is the rent at time $t-1$, is equal to 6.4% on an annual basis. The standard deviation of the annual return is 6%. Rent growth averages 1% per year with a standard deviation of 2.3%. These data indicate the presence of an excess volatility puzzle with returns being almost three times as volatile as the underlying fundamental rents. The standard deviation of the log price-to-rent ratio is 15% and the standard deviation of the log HP-filtered housing price is 3.7%.⁶

Examining autocorrelations of the data at one to four quarterly lags, we see that the price-to-rent ratio is highly persistent with all autocorrelation coefficients above 0.95. The autocorrelation of returns declines from 0.84 at one lag to 0.51 at four lags. The existence of positive momentum in the housing market has previously been documented by Case and Shiller (1989) among others.

Additionally, there is evidence of return predictability in the housing data. The price-to-rent ratio is negatively correlated with the cumulative return over the next four years $r_{t+1} + \dots + r_{t+16}$ with a coefficient of -0.74. However the current period return is positively correlated with the same cumulative return with a coefficient of 0.19. Finally the PE ratio is negatively correlated with future rent growth, $\ln d_{t+16} - \ln d_t$ with a coefficient of -0.44. This long term mean reversion in housing prices is also noted by Glaeser et al. (2014).⁷

As a further examination of return predictability in the data I estimate the following Campbell-Shiller [Campbell and Shiller (1988)] style regressions:

$$\begin{aligned} r_{t+1} + \dots + r_{t+16} &= \alpha + \beta^{\text{return}} \ln(\text{price} - \text{to} - \text{rent}_t) + \epsilon_t \\ \ln d_{t+16} - \ln d_t &= \alpha + \beta^{\text{rent}} \ln(\text{price} - \text{to} - \text{rent}_t) + \epsilon_t \end{aligned}$$

Results for the regressions are presented in table 2. We find that $\beta^{\text{return}} = -0.8$. This coefficient implies that a 10% increase in the price to rent ratio predicts cumulative returns will be -8% lower over the next four years or about -2% per year. The R-squared of this regression is 0.5 suggesting cumulative returns over the next 4 years are explained fairly well using the price-to-rent ratio. On the other hand, $\beta^{\text{rent}} = -0.12$ and the R-squared of the regression is only 0.2. This suggests that rent growth is less predictable than return growth and importantly high price-to-rent ratios do not seem to forecast periods of high demand for housing. Quite the opposite, if anything, they predict we are entering a period of low demand for housing.

⁶The price-to-rent ratio is calculated as q_t/d_t . There is no need to take a past average of rents as rents (as opposed to equity dividends) are quite smooth in the data.

⁷Results are qualitatively similar for a variety of horizon windows from 3 years onward.

There is also substantial evidence of time varying volatility in the data. I report skewness $E\frac{(x-\mu)^3}{\sigma^3}$ and kurtosis $E\frac{(x-\mu)^4}{\sigma^4}$ of the price to rent ratio. Housing returns are left skewed with a skewness of -2 while the price to rent ratio is right skewed with a skewness of 2.04. Both series also demonstrate high levels of kurtosis of around 7. If the series were normally distributed they would exhibit a kurtosis of 3. The high level of kurtosis is evidence of the existence of fat-tails in the distribution, i.e. increased frequency of extreme values relative to a normal distribution.

As further evidence of time varying volatility I examine the autocorrelation of squared returns and the autocorrelations of squared residuals from an AR(1) return regression, i.e. ε_t^2 from $r_t = \alpha + \rho r_{t-1} + \varepsilon_t$. If large returns and residuals are more likely followed by large returns and residuals, as would be the case with time varying volatility, we would expect to see positive autocorrelation of squared returns and residuals. That is indeed what we see. The autocorrelation of squared returns ranges from 0.7 at one lag to 0.27 and four lags. Similarly, the autocorrelation of squared residuals ranges from 0.1 to 0.39.⁸

For additional evidence of time varying volatility in the aggregate U.S. housing data, I estimate a GARCH(1,1) model on the quarterly return series. The GARCH(1,1) model is:

$$\sigma_t^2 = \kappa + \gamma_1 \sigma_{t-1}^2 + a_1 \varepsilon_{t-1}^2.$$

In this model the variance of ε_t in the AR(1) regression $r_t = \alpha + \rho r_{t-1} + \varepsilon_t$ is varying over time. Positive γ_1 and a_1 are evidence of time varying volatility with data that will exhibit periods of particularly high volatility. For the quarterly return data I estimate $\gamma_1 = 0.73$ and $a_1 = 0.27$. Both estimates are highly statistically significant. Furthermore the Engle test (Engle (1982)) rejects the null of no GARCH effects at the 95% confidence level.

3 Model

3.1 Housing Price

A representative consumer can consume or buy units of housing h_t at a price q_t . The household is subject to stochastic shocks to their preference for housing ξ_t . The household's

⁸Under the null hypothesis of zero autocorrelation, standard errors are calculated as $\frac{1}{\sqrt{T}} = 0.07$, making these estimates statistically significant. See Hamilton (1994) pp. 111.

problem then is to choose

$$\max_{c_t, h_t} E_o \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\gamma}}{1-\gamma} + \xi_t \ln h_t \right]$$

subject to the constraint:

$$c_t + q_t h_t = q_t h_{t-1} + y_t. \quad (1)$$

Here y_t is income at time t , and c_t is consumption at time t . The first order conditions for the consumer's optimal choice are:

$$c_t : c_t^{-\gamma} - \lambda_t = 0 \quad (2)$$

$$h_t : \frac{\xi_t}{h_t} - \lambda_t q_t + \beta E_t [\lambda_{t+1} q_{t+1}] = 0 \quad (3)$$

where λ_t is the Lagrange multiplier on the budget constraint.

Combining these two equations one gets

$$q_t = \frac{\xi_t}{h_t \lambda_t} + \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} q_{t+1} \right]$$

I will assume that housing is in fixed supply so that $h_t = 1$ for all t . From the first order condition we have:

$$q_t = \frac{\xi_t}{\lambda_t} + \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} q_{t+1} \right].$$

Letting $d_t = \frac{\xi_t}{\lambda_t}$ we can write

$$1 = \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \frac{q_{t+1}}{q_t - d_t} \right]$$

This is a highly non-linear condition. But, following Campbell (1993) and Restoy and Weil (2011), it can be linearized in two steps. First assuming that housing returns and consumption are conditionally homoskedastic and jointly log-normally distributed we obtain:

$$\ln(q_t - d_t) = \ln \beta + E_t [-\gamma \Delta \ln c_{t+1} + \ln q_{t+1}] + \frac{1}{2} [\gamma^2 \sigma_c^2 + \sigma_q^2 - 2\gamma \sigma_{c,q}] \quad (4)$$

where $\sigma_c^2 = Var_t \Delta \ln c_{t+1}$, $\sigma_q^2 = Var_t \ln q_{t+1}$ and $\sigma_{c,q} = Cov_t(\Delta \ln c_{t+1}, \ln q_{t+1})$.

Then I linearize $\ln(q_t - d_t)$ about the steady rent to price ratio and obtain:

$$\ln(q_t - d_t) \approx k + (1 - \delta) \ln q_t + \delta \ln d_t \quad (5)$$

where k , δ , and \bar{x} are constants given in the appendix along with the model steady state.

Substituting (5) into (4) and letting $\sigma = \frac{1}{2} [\gamma^2 \sigma_c^2 + \sigma_q^2 - 2\gamma \sigma_{c,q}]$ we have:

$$\ln q_t \approx \frac{1}{1 - \delta} [\sigma + \ln \beta - k - \delta \ln d_t + E_t(-\gamma \Delta \ln c_{t+1} + \ln q_{t+1})].$$

Iterating forward

$$\ln q_t \approx \frac{k - \ln \beta - \sigma}{\delta} - \frac{\delta}{1 - \delta} E_t \sum_{s=0}^{\infty} \frac{1}{(1 - \delta)^s} \ln d_{t+s} - \frac{\gamma}{1 - \delta} E_t \sum_{s=0}^{\infty} \frac{1}{(1 - \delta)^s} \Delta \ln c_{t+s+1}$$

Since $d_t = \frac{\xi_t}{\lambda_t}$ we obtain $\ln d_t = \ln \xi_t - \ln \lambda_t = \ln \xi_t + \gamma \ln c_t$. Therefore we can solve for the following closed form solution for the log house price:

$$\ln q_t \approx \frac{k - \ln \beta - \sigma}{\delta} - \frac{\delta}{1 - \delta} E_t \sum_{s=0}^{\infty} \left[\frac{1}{(1 - \delta)^s} (\ln \xi_{t+s} + \gamma \ln c_{t+s}) \right] - \frac{\gamma}{1 - \delta} E_t \sum_{s=0}^{\infty} \frac{1}{(1 - \delta)^s} \Delta \ln c_{t+s+1}$$

The appendix gives analytically expressions for the sums and the conditional variances as a function of the underlying processes for consumption growth and housing preferences listed below. ⁹

Iacoviello (2010) argues that institutional rigidities lead to sluggish adjustment in the housing market. For example, listing agents often used “comps” or comparable sales when setting prices. These are listings which are similar to the property for sale and have sold recently. Likewise, banks often will not grant a mortgage on a property that does not appraise for the sale price. I model these institutional features in a reduced form way, assuming rigidity in the price such that the market price q^* is given by:

$$\ln q_t^* = \lambda \ln q_t + (1 - \lambda) \ln q_{t-1}^* \quad (6)$$

where λ is a measure of the degree of institutional price rigidity in the market. In this

⁹In general these sums are finite given the calibration as $(1 - \delta) > 1$ and the presence of the *log* of the variable in the numerator. To see this let $x_{t+s} = (1 + g_x)^s x_t$. Then $\ln x_{t+s} = s \ln(1 + g_x) + \ln x_t$ and $\sum_{s=0}^{\infty} \frac{1}{(1 - \delta)^s} \ln x_{t+s} = \frac{1 - \delta}{\delta^2} \ln x_t + \frac{g_x}{\delta^2}$ for $(1 - \delta) > 1$.

formulation, the market price is assumed to slowly adjust to the fundamental price. The smaller is λ the more sluggishly prices adjust in the housing market.¹⁰

To close the model it is necessary to specify the exogenous processes for income, consumption and housing preference. First note that given the budget constraint ($c_t + q_t h_t = q_t h_{t-1} + y_t$) and the assumption that $h_t = 1$ for all t we have that $c_t = y_t$ for all t . Therefore one can specify either the income or consumption process. Here I specify the consumption process. In order to focus on the role of future expectations of housing fundamentals for driving house prices, I assume that consumption growth is i.i.d. though it can be correlated with the housing preference process as described in the calibration section. To focus on learning about future housing fundamentals I allow the consumer to know the exact process for consumption.

In contrast, there is uncertainty about the nature of the housing preference process. The housing preference process will be a trend stationary process. However, the consumer does not know the true form of the preference process ξ_t . Specifically, he is uncertain if the preference process is trend stationary or not. He believes that:

$$\xi_t = \rho_0^s + \beta t + \rho_1^s \xi_{t-1} + \dots + \rho_p^s \xi_{t-p} + \varepsilon_t^s \quad (7)$$

with probability p_t^s and that

$$\Delta \xi_t = \alpha + \rho_1^{ns} \Delta \xi_{t-1} + \dots + \rho_p^{ns} \Delta \xi_{t-p} + \varepsilon_t^{ns} \quad (8)$$

with probability $p_t^{ns} = 1 - p_t^s$. These beliefs will vary over time given how well each model fits the data as described in the following subsection.

In this paper, consistent with much of the literature on macroeconomic models of house prices, shocks to housing preferences are a key driver of fluctuations. For example, Iacoviello and Neri (2010), Adam et al. (2012), and Liu et al. (2013) among others all rely on preference shocks to generate volatility in house prices.¹¹ While the literature is vague about what the housing preference process represents, it is intended to be a reduced form representation of the various shocks which can affect house prices. It could represent interest rates, credit availability as well as a strict preference shock to the demand for housing. It can also

¹⁰For a complete theory of nominal rigidities with strategic complementarities leading to sluggish pricing in housing markets see Guren (2014).

¹¹Other examples of papers with exogenous fluctuations in housing preferences include: Chambers et al. (2009); Miao et al. (2014); Guerrieri and Iacoviello (2013); Sterk (2015); Lambertini et al. (2013); Sá and Wieladek (2015)

represent macroeconomic conditions and housing supply conditions which affect house prices. Modeling these factors in a reduced form way allows the model to better focus on the role learning and expectations can have in driving house prices.

However, it is worth noting that the emphasis on housing preference shocks receives strong support the literature. In a general equilibrium, open economy model Sá and Wieladek (2015) find that a housing preference shock is an important determinate of house prices. Iacoviello and Neri (2010) also find a large fraction of the variation in house prices can be attributed to preference shocks, even controlling for a wide variety of fundamentals like income, interest rates, availability of subprime mortgages and mortgage fees. They then show that their estimated preference shocks are correlated with many of the factors we believe drive housing markets. These factors include, fees to purchase a home, inflation, fraction of subprime mortgages, and demographic variables. Time varying importance of housing for utility is also supported by Iacoviello (2010) who emphasizes that many press articles explain changes in house prices with changes in the nature of what housing consumers are looking for, e.g. looking for larger homes or viewing homes as an investment vehicle. In addition to this empirical evidence, Iacoviello and Neri (2010) also provide a theoretical explanation for time varying housing preference. The first is a model where the housing stock delivers time varying housing services, the second is a model where the resources, in terms of other goods, required to purchase a house is varying over time.

An important question here is the order of integration of the preference process: is it trend or difference stationary? While most papers assume a stationary preference process, this is not always supported by the literature. For example, Adam et al. (2012) use a unit root process for housing preferences. They then show that after estimating the preference process from rental data, across six different countries, they are unable to reject the possibility of a unit root in any of the countries. It is also natural to assume some type of trend in preference for housing, either stochastic or deterministic, to match the growth of housing fundamentals (rents) in the data.

Finally, given the broad interpretation of the preference process, it is worthwhile to consider uncertainty about the preference process. Of course if we consider the world to consist of one agent whose preferences are the only factor that can drive prices it is hard to see how an individual would be uncertain concerning the driver of prices. However, thinking beyond a strict interpretation of the model, the drivers of housing preference may be unknown to any one agent and in a heterogeneous agent context it would be hard to know how aggregate housing preference would evolve. One can consider then the uncertainty about

the preference process as standing in for the lack of heterogeneity in the model.

3.2 Beliefs

I use the methods of Cogley and Sargent (2005) to calculate the parameters of each model of housing preference and the probability weights on the trend stationary and difference stationary model. Their model uses Bayesian methods to recursively update the parameters on each model and then uses the likelihood of each model to calculate a probability weight on each model. For a given model (i.e. trend or difference stationary) indexed by $i = \{s, ns\}$, and a housing preference history Ξ^{t-1} , we assume that agents prior beliefs about the model parameters are distributed normally according to:

$$p(\Theta_{i,t-1}|\sigma_i^2, \Xi^{t-1}) = N(\Theta_{i,t-1}, \sigma_i^2 P_{t-1}^{-1})$$

and their prior beliefs concerning the model residual variance are given by:

$$p(\sigma_{i,t-1}^2|\Xi^{t-1}) = IG(s_{t-1}, v_{t-1})$$

Here N represents the normal distribution function and IG represents the inverse-gamma distribution function. P_{t-1} is the precision matrix that captures the confidence the agent has in his belief for $\Theta_{i,t-1}$, σ_i^2 is the estimate of the variance of the model residuals, s_{t-1} is an analogue to the sum of squared residuals, and v_{t-1} is a measure of the degrees of freedom to calculate the residual variance such that the point estimate of $\sigma_{i,t-1}^2$ is given by s_{t-1}/v_{t-1} . After observing aggregate housing preference ξ_t the agent's posterior beliefs are given by:

$$\begin{aligned} p(\Theta_{i,t}|\sigma_i^2, \Xi^t) &= N(\Theta_{i,t}, \sigma_i^2 P_t^{-1}) \\ p(\sigma_i^2|\Xi^t) &= IG(s_t, v_t) \end{aligned}$$

Cogley and Sargent (2005) gives the following recursion to update the parameters of the beliefs:

$$\begin{aligned} P_t &= P_{t-1} + x_t x_t' \\ \theta_t &= P_t^{-1}(P_{t-1} \theta_{t-1} + x_t y_t) \\ s_t &= s_{t-1} + y_t^2 + \theta_{t-1}' P_{t-1} \theta_{t-1} - \theta_t' P_t \theta_t \\ v_t &= v_{t-1} + 1 \end{aligned}$$

Here x_t is the vector of right hand side variables for the model at time t and y_t is the left hand side variable for the model at time t . This recursion gives the parameters of each model. Now it is necessary to calculate the probability weight on each model.

Given a set of model parameters: $\{\Theta_i, \sigma_i\}$ we can calculate the conditional likelihood of the model as:

$$L(\Theta_i, \sigma_i^2, \Xi^t) = \prod_{s=1}^t p(y_s | x_s, \Theta_i, \sigma_i^2)$$

where y_s and x_s are the left and right hand side variables of the model at time s and Ξ^t is the housing preference history up to time t . Based on this likelihood, one can write the marginalized likelihood of the model by integrating over all possible parameters:

$$m_{it} = \iint L(\Theta_i, \sigma_i^2, \Xi^t) p(\Theta_i, \sigma_i^2) d\Theta_i d\sigma_i^2$$

Then we have the probability of the model given the observed data $p(M_i | \Xi^t) \propto m_{i,t} p(M_i) \equiv w_{i,t}$. Here we have defined the weight on model i , $w_{i,t}$ and $p(M_i)$ is the prior probability on model i .

Cogley and Sargent (2005) show that Bayes's rule implies

$$m_{it} = \frac{L(\Theta_i, \sigma_i^2, \Xi^t) p(\Theta_i, \sigma_i^2)}{p(\Theta_i, \sigma_i^2 | \Xi_t)}$$

and therefore:

$$\frac{w_{i,t+1}}{w_{i,t}} = \frac{m_{i,t+1}}{m_{i,t}} = p(y_{i,t+1} | x_{i,t}, \Theta_i, \sigma_i^2) \frac{p(\Theta_i, \sigma_i^2 | \Xi_t)}{p(\Theta_i, \sigma_i^2 | \Xi_{t+1})}.$$

We assume that regression residuals are normally distributed allowing us to use the normal p.d.f to calculate $p(y_{i,t+1} | x_{i,t}, \Theta_i, \sigma_i^2)$. Cogley and Sargent (2005) show that $p(\Theta_i, \sigma_i^2 | \Xi_t)$ is given by the normal-inverse gamma distribution and provide the analytical expressions for this probability distribution. Any choice of Θ_i, σ_i^2 will give the same ratio of weights; I use the posterior mean in my calculations.

This recursion implies the following recursion for model weights.

$$\frac{w_{s,t+1}}{w_{ns,t+1}} = \frac{m_{s,t+1}/m_{s,t}}{m_{ns,t+1}/m_{ns,t}} \frac{w_{s,t}}{w_{ns,t}}$$

Finally, to calculate the model probabilities, the consumer normalizes the weights to one,

and therefore the weight on the stationary model is given by:

$$p_{s,t} = \frac{1}{1 + w_{ns,t}/w_{s,t}}$$

Since housing preference shocks are an exogenous process, the model will eventually put all the weight on the true process. To allow for perpetual learning, I adapt the concept of constant gain learning from the least squares learning literature to the current setup.¹² I introduce a gain parameter (g) that over-weights current observations. The gain probability can be interpreted as the probability of a structural break in the economy, such that the history of the housing preference process no longer has any bearing on the current process generating housing preferences, hence the previous model weight ratio is set to one.

$$p_{s,t} = (1 - g) \frac{1}{1 + w_{ns,t}/w_{s,t}} + g \frac{1}{1 + \frac{m_{s,t+1}/m_{s,t}}{m_{ns,t+1}/m_{ns,t}}}$$

Using the estimated probabilities, he can then calculate the price by:

$$\ln q_t^L = p_{s,t} \ln q_t^S + (1 - p_{s,t}) \ln q_t^{NS} \quad (9)$$

where $\ln p_t^S$ and $\ln p_t^{NS}$ represent the log prices conditional on the stationary and non-stationary models being true.¹³

4 Calibration and Simulation

The model is tightly parameterized. Most calibrated parameters are standard and I set them to standard values. Additionally, robustness to the parameter choices is shown in section 6. Time is quarterly and I set the discount rate in the model $\beta = 0.9975$. This implies a 3% annual real interest rate slightly higher than average rates on 10-year Treasury inflation protected securities (TIPS).¹⁴ Low discount rates are consistent with the evidence in Giglio et al. (2014) who find very low discount rates when comparing the prices on housing with

¹²See Evans and Honkapohja (2001)

¹³Importantly I make a standard assumption from the learning literature, that of anticipated utility (Kreps (1998)), i.e. the agent makes decisions assuming his future beliefs will be the same as his current beliefs. This includes his beliefs about both the likelihood of each process and the implied covariances. However, beliefs can and do change in the future.

¹⁴The real interest rate in the model is given by $(1 + g^c)^\gamma / \beta$ where g^c is the growth rate of consumption which is calibrated to be 0.5% per quarter.

temporary ownership contracts versus permanent ownership contracts in the U.K. and Singapore. In the sticky price version of the model I set $\lambda = 0.25$ to better match the positive autocorrelation of returns.¹⁵ I set the lag length of the trend stationary and difference stationary model of housing preferences to 4.¹⁶ I set $\gamma = 1$, a standard assumption, consistent with log utility.

I also need to calculate the prior beliefs of the agent, however these do not matter much for the results because I simulate the model 500 times for 2,000 periods each time and keep only the last $212 = (2013-1960)*4$ observations to match the length of my data. The initial prior on the stationary model $p_{s,0} = 0.5$. To calibrate the initial beliefs for the preference processes (Θ_o) I estimate ordinary least squares regressions on the log housing rents series from Davis et al. (2008) deflated with the CPI. I assume that ε_t^s and ε_t^{ns} are distributed $N(0, \sigma_i^2)$ where σ_i^2 is estimated from the residuals in the previous estimation. I set the initial precision matrix $P_o = 10^{-2} * I$. This is a very diffuse prior setting the standard error of the coefficients to 10 times the standard deviation of the regression residuals. I set s_o to the variance of the regression residuals and set the initial degrees of freedom (v_o) equal to 1. Finally, I assume that consumption growth is i.i.d, and estimate the process using real per-capita consumption of non-durables and services from the national income and product accounts. To estimate the covariance matrix C between the preference process shocks and the consumption shocks, I take the residuals from the preference process estimation and the consumption process estimation and calculate the variance-covariance matrix between these residuals. To simulate the stochastic shocks I use a Cholesky decomposition of this variance-covariance matrix with the preference shock ordered first multiplied by a 2X1 vector of standard normal random variables. That is to say $[\varepsilon_t^i \ \varepsilon_t^c] = chol(C)' * [\varepsilon_t^1 \ \varepsilon_t^2]$ where ε_t^1 and $\varepsilon_t^2 \sim N(0, 1)$ for $i = \{s, ns\}$.

I set the gain parameter (g) equal to 0.005 which implies a structural break in the economy once every 50 years. Robustness to different values of the gain and a justification based on analysis of implied forecast errors are considered in section 6.¹⁷

To evaluate the model I assume the true preference process is the stationary process and

¹⁵This calibration has one-quarter of houses updating their prices each quarter consistent with assumptions made in good markets, see Galí (2009)

¹⁶Four lag forecasting VARs are common in the literature. See for example Christiano et al. (1999); Stock and Watson (2003, 2005).

¹⁷While the setup in this paper with model learning makes direct comparisons difficult my choice of gain appears to be on the low end of the choices in the literature. Least squares learning and Kalman gains in the literature range from 0.002 to 0.05. See for example: Branch and Evans (2006); Eusepi and Preston (2011); Kuang and Mitra (2015); Milani (2014)

simulate 500 trials of length 2,000 keeping the last 212 observations to match the length of my data set. I then report median statistics across the trials. Initial housing preferences are normalized to the steady state value of the trend stationary model when t equals zero and consumption is initialized to be twice this value in line with U.S. CPI data which suggests housing represents 30% of the U.S. consumption basket, though I find this initialization does not affect the results.

I am motivated to make the true process the stationary process by a variety of concerns. The first is that the survey evidence outlined in the introduction suggests that individuals overreacted to the run up in house prices and extrapolated current price changes far into the future. This evidence supports a true process for fundamentals being one with temporary deviations from trend and agents overrating to these temporary deviations by assuming they are permanent. Additionally, in the U.S. housing market temporary increases in prices may be persistent because of a slow response of supply. However, eventually supply can respond to bring prices down. A model where agents believe temporary shocks are permanent is consistent with neglecting the long run supply response of housing.¹⁸ Finally, Shiller (2005) and Reinhart and Rogoff (2009) argue that individuals often attribute new-era stories of fundamental change to justify high valuations or current booms as being permanent instead of temporary. This view of the world is consistent with my modeling. Of course, it would be possible to have the underlying process be a true switching process and agents form rational beliefs about what state they are in. However, I believe the spirit of that model is different than my goal in this paper. In that model agents are as likely to underreact as overreact. But here I try to capture the general notion that in speculative bubbles agents are overreacting in their long run expectations and neglecting the tendency of fundamentals to return to long run trends.

5 Results

In this section I compare results from the learning model with a rational expectations benchmark. The benchmark model is one in which the preference shock follows a stationary process and the household knows this. Results are reported for the flexible price model where $\lambda = 1$ so that prices immediately adjust to the fundamental value and then for $\lambda = 0.25$ so prices adjust more slowly.

¹⁸Fuster et al. (2012) argue that neglecting long run mean reversion is a key psychological bias that is useful for understanding equity market puzzles.

5.1 Model Mechanism

First, to provide intuition for the main mechanism of the model I examine a single simulated housing preference series and the implied path of rents and beliefs. Then, in the subsequent section I will provide median statistics over many simulations to evaluate the model. Examining figure 1 we see plotted the probability the learning model puts on the (trend) stationary model being true for a single simulated housing preference series from the (trend) stationary model. We see that on average the model puts more weight on the trend stationary model than the difference stationary model. However, this weight is not constant. Around time 175 we see the beliefs drift away from the trend stationary model where the agent goes from putting 75% weight on the stationary model to only putting 55% weight on the trend stationary model.

Recall that beliefs are endogenous here and depend on the realized housing preference and implied rent series. Figure 2 plots the rent series that corresponds to the simulated housing preference series. At time 175, we can see the growth rate of rents increases resulting in a housing fundamental series that is persistently above trend. Because the series is not reverting to trend, the agent begins to put more and more weight on the possibility that the housing preference series is difference stationary, revising his beliefs. Finally, around time 200 growth slows down, and the agent revises his beliefs, going back to putting about 70% of his weight on the trend stationary model.

Figure 3 plots the price-to-rent ratio under the learning model (dashed line) versus a rational expectations benchmark where the agent knows the true process is trend stationary. We see a large spike up in the price-to-rent ratio, increasing almost 20% relative to a slight fall under the rational expectations benchmark. There is a temporary housing boom while the agents believe that there has likely been a permanent increase in housing fundamentals which is then reversed with an abrupt fall in the price-to-rent ratio once the agent reverses his belief around time 200.

This mechanism is responsible for the main results of this paper. Agents overreact to news when the world looks as if it may be non-stationary. The overreaction is corrected once the fundamental begins to mean revert. This mechanism results in predictability of returns. Additionally, when the agent believes the world may be non-stationary he reacts strongly to news results in a higher volatility of returns. These reactions generate time varying volatility in returns.

5.2 Main Moments

Now I report median statistics for the model across the 500 simulations described in section 4. Results from the model compared with the main moments in the data are presented in table 3. Examining the flexible price case first, note that both models imply a 3% annual average return on housing and a 3% growth rate of rent. In the data, these number are 6.4% and 1% respectively.

The standard deviation of the log price-to-rent ratio $\sigma(\frac{P_t}{R_t}) = 15\%$ in the data. The rational expectations benchmark generates 1/5 this volatility, predicting a standard deviation of 3%. The learning model predicts 4%. The learning model also better matches the standard deviation of the HP-filtered price (P_t^{HP}) which was 3.7% in the data. The learning model predicts 3% versus only 0.8% for the rational expectations benchmark. I obtain a similar result for the standard deviation of returns, $\sigma(r_t)$, with the learning model predicting 8% versus the 6% in the data. The rational expectations benchmark model predicts only 3%.

Recall, that both the price-to-rent ratio and returns are highly positively autocorrelated. Both models are consistent with the first fact, however since the housing price is modeled as an asset price neither model can explain the positive autocorrelation of returns.

Finally, I examine the ability of the model to explain the predictability of housing returns. The price-to-rent ratio is negatively correlated with future housing returns, $\rho(\frac{P_t}{R_t}, r_{t+1} + \dots + r_{t+16}) = -0.74$. The rational expectations benchmark predicts a small positive correlation.¹⁹ The learning model however predicts a correlation of -0.49. Similarly, the price-to-rent ratio is negatively correlated with future rent growth $\rho(\frac{P_t}{R_t}, \lnrent_{t+16} - \lnrent_t)$. The learning model predicts a correlation coefficient of -0.36 vs. -0.44 in the data. The rational expectations benchmark obtains the wrong sign for the correlation, predicting a substantial positive correlation. Finally, neither model can match the fact that the current return is positively correlated with future returns.

Campbell-Shiller style regressions tell a similar story to the raw correlations. The rational expectations model predicts no predictability of returns. The coefficient on returns $\beta^{return} = 0.06$ and the $R^2 = 0.04$. In contrast, the learning model generates a coefficient equal to -1 versus -0.8 in the data with an appreciable higher R^2 of 0.24. Similarly, the rational expectations model poorly explains rent growth predictability. The coefficient on rent growth in the Campbell-Shiller regression $\beta^{rentgrowth} = -0.12$ however the rational expectations model predicts a coefficient equal to 0.46. Because fundamentals are mean reverting, a

¹⁹Small sample bias leads to a slight negative correlation instead of a value of zero.

high price-to-rent ratio predicts future rent growth. However, in the learning model a high price-to-rent ratio is consistent with overreaction to growth in fundamentals. As a result, a high price-to-rent ratio forecasts lower rental growth in the future not higher rental growth. This mechanism allows the learning model to match the data much more closely. It predicts a coefficient of $\beta^{rentgrowth} = -0.42$ and an $R^2 = 0.13$.

Next, I examine the ability of the sticky price model to match the data, highlighting the differences relative to the flexible price model. Sticky prices lower the volatility of the HP filtered price for the learning model, 2% vs. 3% before, and returns 4% vs. 8% as before. In both cases the learning model still increases volatility over the rational expectations benchmark.

Sticky prices have little effect on the autocorrelation predictions for the price-to-rent ratio. However, the model is now able to generate positive autocorrelation of returns. The rational expectations model predicts autocorrelations ranging from 0.74 at one lag to 0.3 at four lags vs. 0.84 to 0.51 in the data. The learning model also predicts positive autocorrelation ranging from 0.44 to 0.15.

Finally, the rational expectations model predicts a small negative correlation between the price-to-rent ratio and future returns and a positive correlation between the price-to-rent ratio and rent growth. In the data these correlations are strongly negative. The learning model generates negative correlations consistent with the data. Indeed it predicts a correlation between the price-to-rent ratio and future returns equal to -0.51 versus -0.74 in the data and between the price-to-rent ratio and future rent growth equal to -0.43 versus -0.44 in the data. Both models now are consistent with the positive correlation between the current return and future returns. The rational expectations model predicts 0.25 and the learning model predicts 0.14 vs. 0.32 in the data.

Campbell-Shiller regressions for the sticky price model show a similar result. The rational expectations model again predicts no predictability of returns. The coefficient on returns $\beta^{return} = -0.06$ and the $R^2 = 0.04$. In contrast, the learning model generates a coefficient equal to -1 versus -0.8 in the data with an R^2 of 0.27. Similarly, the rational expectations model again does not explain rent growth predictability. The coefficient on rent growth $\beta^{rentgrowth} = -0.12$ however the rational expectations model predicts a coefficient equal to 0.37. The learning model matches the data much more closely. It predicts a coefficient of $\beta^{rentgrowth} = -0.5$ and an $R^2 = 0.19$.

5.3 Time Varying Volatility

Neither model can explain the skewness in the return data, however the learning model generates positive skewness in the price to rent ration. The learning model also generates higher kurtosis than the rational expectations benchmark. For the price-to-rent ratio, the learning model generates kurtosis equal to 4.4 vs 2.2 for the rational expectations benchmark. For returns, the learning model generates return kurtosis equal to 4.1 vs. 2.9 for the rational expectations model. In the data kurtosis is about 7. In the sticky price case the learning model generates more kurtosis than the rational expectations model and comes close to matching the data on kurtosis of returns with a value of 5.1.

The flexible price results indicate that the rational expectations model does not generate any autocorrelation in squared returns $\rho(r_t^2, r_{t-1}^2)$ or residuals $\rho(\varepsilon_t^2, \varepsilon_{t-1}^2)$. In contrast, model learning allows for endogenous time varying volatility, though in the flexible price case the magnitudes of the correlations predicted by the model are smaller than in the data. The learning model predicts and autocorrelation of squared returns equal to 0.07 vs 0.5 in the data; the learning model predicts and autocorrealtion of 0.09 on average for AR(1) return residuals versus 0.2 in the data. In the sticky price case, the rational expectations benchmark has autocorrelation in squared returns but not in the squared residuals. However, the learning model predicts positive autocorrelation for both squared returns and residuals. Additionally, for the squared residuals the magnitude predicted by the learning model is approximately correct, about 0.18 for the model versus 0.2 for the data.

Results from estimating GARCH models on the simulated data, give a similar result.²⁰ There is no evidence of GARCH effects in the rational expectations benchmark. However, we consistently find significant GARCH effects in the learning model data and of a similar magnitude to the data when we allow for sticky prices. For the sticky price model the GARCH parameter equals 0.68 versus 0.73 in the data, while the ARCH parameter equals 0.21 versus 0.27 in the data. For the flexible price model, the median GARCH and ARCH paramaters are zero. However, even with flexible prices the learning model shows more evidence of GARCH effects. The Engle test for GARCH effects reject 42% of the time for the learning model but only 7% of the time for the rational expectations model.

²⁰To estimate the GARCH parameters, I first run an Engle test for the null hypothesis of no conditional heteroscedasticity on the simulated data. I estimate the GARCH parameters only if the test rejects, otherwise I assign zeros for the GARCH parameters. This procedure is required because absent GARCH effects I am unable to identify the GARCH parameters using the maximum likelihood procedure.

5.4 Expected Returns

One of the important features of the model is that it generates predictability in housing returns without generating time varying expected returns.²¹ This is in contrast to models with time varying risk which generate predictability in returns which are expected by investors. Applying these models to the housing market one would find that when the price-to-rent ratio is high investors would expect lower returns in the future. While these models are able to explain a negative correlation between the price-to-rent ratio and subsequent housing returns, they are at odds with an increasing large literature on survey expectations. For equity markets, survey results indicates that investors' expectations regarding future returns seems to be increasing in past stock market performance. As a results, high price to earnings ratios are correlated, if anything, with higher expectations about future returns not lower. See for example: Fisher and Statman (2002); Shiller (2000); Greenwood and Shleifer (2013); Vissing-Jorgensen (2004). Similarly, in the housing market, Case and Shiller (2003); Piazzesi and Schneider (2009); Shiller (2007); Case et al. (2012b) all find that expectations about future returns were increasing during the housing boom of the 2000s not declining. While, I am unable to generate increasing expectations of future returns when the price-to-rent ratio rises, I am able to explain predictability in housing returns without time varying expected returns. In this manner, my results are more inline with the survey evidence than models which require low expected returns when the price-to-rent ratio is high.

6 Robustness

6.1 Paramter Variation

The model has a small number of free parameters and therefore is straightforward to calibrate. However, I did set the AR lag length, the gain level g , the risk aversion coefficient γ and the sticky price parameter λ . Table 5 gives the results from varying each of these parameters one at a time, while keeping the others at their original calibrated value. Except when varying the sticky price parameter λ , I use the sticky price model as the benchmark and set $\lambda = 0.25$. Table 5, therefore, demonstrates the robustness of the results to the various parameter choices.

²¹One period ahead expected returns are given by: $E_t[r_{t+1}] = E_t \frac{q_{t+1}}{q_t - d_t} = \exp[E_t(\ln q_{t+1}) + \frac{1}{2}\sigma_q^2]/(q_t - d_t)$. This quantity is approximately constant in the model and equal to $(1 + g^c)^\gamma/\beta$ where g^c is the growth rate of consumption.

There is little effect on the results of varying the AR lag length. I consider an AR length 2 and 6. We see that the learning model exhibits slightly more volatility when the AR lag length is 2 versus 4. For example the standard deviation of the price-to-rent ratio is now 0.05 versus 0.03. The predictability correlations are all of the same sign and same magnitude under the alternative AR lag length calibrations as they are under the baseline calibration. The evidence of time varying volatility remains; the autocorrelation of residuals from the AR(1) return regression are all positive and of similar magnitude.

Similarly increasing the gain from 0.005 to 0.01 slightly amplifies volatility of the price-to-rent ratio. Additionally, the predictability of rent growth increases slightly. The correlation of the price-to-rent ratio with future returns is -0.52 versus -0.51 for the baseline calibration. Similarly the correlation of the price-to-rent ratio with future rent growth is equal to -0.45 versus -0.43 for the benchmark calibration. I also consider lowering the gain to 0.001. Here volatility falls, for example the standard deviation of returns is now 0.03 versus 0.04 in the benchmark calibration. And the model predicts less kurtosis of the price-to-rent ratio. However the model still explains the kurtosis of returns, the predictability of returns and rent growth and the autocorrelation of squared returns.

Increasing γ from 1 to 3 has very little effect on the results. We see a slight fall in the level of volatility. At a level of $\gamma = 3$ the model exhibits a small degradation in its ability to explain kurtosis of returns, predictability of future rent growth, and the autocorrelation of squared return residuals. However, it is still clear that the learning model improves over the benchmark rational expectations model in this case as well. Finally, increasing λ , the sticky price parameter, from 0.25 to 0.5 and reducing it from 0.25 to 0.1 has little effect on the results. We only see that with more flexible prices the model has slightly more difficulty explaining the predictability of future returns using current returns and the autocorrelation of square return residuals.

6.2 Gain Justification and Forecast Errors

This subsection evaluates the magnitude of the expectational errors that agents make. First note that the model is able to generate results using a very low gain value. For example, the gain value of 0.005 implies a structural break in the economy once every 50 years which seems eminently plausible. Additionally, it is worth noting that the model provides a direct check on the reasonableness of beliefs. Specifically, if the likelihood of the difference stationary model is low the agent puts less weight on this model.

To further investigate the reasonableness of beliefs table 6 reports the median autocorrelation and standard deviation of one-step ahead forecast errors across the model simulations for the model rent and return series. It compares the results from the rational expectations benchmark and the learning model. First note that while the learning model implies a larger autocorrelation in forecast errors for the rent series the number is still very small: -0.005 versus -0.002 for the rational expectations benchmark.²² The results are similar for the sticky price model. Similarly, the standard deviation of forecast errors is the same across these two models: 0.72% implying that the uncertainty does not generate large one-step ahead forecast errors in the learning model.

Turning our attention to returns. The autocorrelation of forecast errors in the learning model is -0.007 slightly larger than the value of -0.001 for the rational expectations model. Given, that both the learning model and the rational expectations benchmark are models of constant expected returns the standard deviation of return forecast errors just equals the unconditional standard deviation of returns. Again, results for the sticky price model are essentially same, though the standard deviations fall as the volatility of returns falls.

These autocorrelation values are very small and would be impossible to distinguish from zero in reasonably sized samples. In contrast, one-step ahead forecast errors for professional forecasters are often quite large with autocorrelation of 0.3 or higher (see Eusepi and Preston (2011)) across many macroeconomic time series. Similarly, Caines (2015) finds an autocorrelation of house price return forecast errors equal to 0.4. Suggesting that this model, if anything, underestimates the degree of expectational errors. Additionally, the assumption of a constant expected return allows agents here to be better forecasters than surveys on U.S. equity markets tend to indicate, e.g. Greenwood and Shleifer (2013), where agents expectations of returns rise when future returns are forecastably low.

7 Conclusion

Motivated by the large recent swing in U.S. house prices and the dramatic impact the housing crash had on real economic activity this paper has sought to explain key moments in the U.S. macroeconomic time series on house prices and rents and specifically the role expectations may have played in generating these empirical facts. Given that the housing markets boom and bust was similar to booms and busts that have occurred in equity markets in the U.S. and beyond we have focused on data moments that have received considerable attention in

²²Small sample bias prevents this correlation from being exactly zero.

the analysis of equity markets.

The paper has documented that the price-to-rent ratio and housing returns are substantially more volatile than the underlying rent fundamentals. Both the price-to-rent ratio and housing returns exhibit momentum effects with strong positive autocorrelation in both the price-to-rent ratio and housing returns. Returns on housing are predictable with current returns forecasting higher returns in the future, while the price-to-rent ratio negatively forecasts both future returns and future rent growth. Finally, housing returns exhibit time varying volatility as evidenced both my autocorrelation in squared returns and significant GARCH effects.

I show that a standard rational expectations benchmark is unable to match these facts. I then modify the standard model in two ways. I first allow for sticky prices so that house prices slowly adjust to their fundamental value. Then, I incorporate learning about the true nature of the housing preference process, specifically is the process in trend stationary (so shocks are temporary) or difference stationary (so shocks are permanent).

I find that these modifications substantially improve the fit of the model. They amplify the volatility of prices and returns and explain the positive autocorrelation of returns. They also allow the model to explain the ability of the price-to-rent ratio to predict future returns and rent growth and help the model generate time varying volatility similar to what is observed in the data.

This paper suggests that modeling expectations, particularly outside a strict rational expectations framework is key to understanding the determinates of aggregate U.S. house prices, especially in periods of booms and busts. The paper suggests that non-rational expectations should be incorporated into a wide variety of housing models and could significantly improve the models fit with the data.

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A House Price Derivation

A.1 Steady State

Given the model first order condition

$$1 = \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \frac{q_{t+1}}{q_t - d_t} \right]$$

we can write, letting $\bar{X} = d/q$,

$$\begin{aligned} 1 &= \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \frac{d_{t+1}}{d_t} \frac{q_{t+1}/d_{t+1}}{q_t/d_t - 1} \right] \\ 1 &= \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \frac{d_{t+1}}{d_t} \frac{\bar{X}^{-1}}{\bar{X}^{-1} - 1} \right] \\ \frac{\bar{X}^{-1} - 1}{\bar{X}^{-1}} &= \beta E_t [(1 + g_c)^{-\gamma} (1 + g_d)] \\ 1 - \bar{X} &= \beta \frac{(1 + g_d)}{(1 + g_c)^\gamma} \\ \bar{X} &= 1 - \beta \frac{(1 + g_d)}{(1 + g_c)^\gamma} \end{aligned}$$

given the calibration of the model $\beta \frac{(1+g_d)}{(1+g_c)^\gamma} < 1$.

A.2 Calculation of Log Price

From the first order condition we have

$$q_t = \frac{\xi_t}{\lambda_t} + \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} q_{t+1} \right]$$

Letting $d_t = \frac{\xi_t}{\lambda_t}$ we have

$$1 = \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \frac{q_{t+1}}{q_t - d_t} \right]$$

If we assume that housing returns and consumption are conditionally homoskedastic and

jointly log-normally distributed we can take logs of both sides

$$\begin{aligned} 0 &= \ln \beta + E_t [\Delta \ln \lambda_{t+1} + \ln q_{t+1} - \ln(q_t - d_t)] + \frac{1}{2} Var_t [\Delta \ln \lambda_{t+1} + \ln q_{t+1}] \\ \ln(q_t - d_t) &= \ln \beta + E_t [\Delta \ln \lambda_{t+1} + \ln q_{t+1}] + \frac{1}{2} [\sigma_\lambda^2 + \sigma_q^2 + 2\sigma_{\lambda,q}] \end{aligned}$$

Where $\sigma_\lambda^2 = Var_t \Delta \ln \lambda_{t+1}$, $\sigma_q^2 = Var_t \ln q_{t+1}$ and $\sigma_{\lambda,q} = Cov_t(\Delta \ln \lambda_{t+1}, \ln q_{t+1})$ Now since $\Delta \ln \lambda_{t+1} = -\gamma \Delta \ln c_{t+1}$ we have

$$\ln(q_t - d_t) = \ln \beta + E_t [-\gamma \Delta \ln c_{t+1} + \ln q_{t+1}] + \frac{1}{2} [\gamma^2 \sigma_c^2 + \sigma_q^2 - 2\gamma \sigma_{c,q}]$$

We linearize $\ln(q_t - d_t)$ about the mean rent to price ratio

$$\begin{aligned} \ln(q_t - d_t) &= \ln[q_t(1 - \frac{d_t}{q_t})] \\ \ln(q_t - d_t) &= \ln q_t + \ln(1 - \exp[\ln(\frac{d_t}{q_t})]) \\ \ln(q_t - d_t) &\approx \ln q_t + \ln(1 - \exp(\bar{x})) - \frac{\exp(\bar{x})}{1 - \exp(\bar{x})} \left[\ln(\frac{d_t}{q_t}) - \bar{x} \right] \\ \ln(q_t - d_t) &\approx k + (1 - \delta) \ln q_t + \delta \ln d_t \end{aligned}$$

where $\bar{x} = E \ln(\frac{d_t}{q_t})$, $k = \ln(1 - \exp(\bar{x})) + \frac{\exp(\bar{x})}{1 - \exp(\bar{x})} \bar{x}$, and $\delta = \frac{-\exp(\bar{x})}{1 - \exp(\bar{x})}$. Letting $\sigma = \frac{1}{2} [\gamma^2 \sigma_c^2 + \sigma_q^2 - 2\gamma \sigma_{c,q}]$ we have

$$\begin{aligned} \ln q_t &\approx \frac{1}{1 - \delta} [\sigma + \ln \beta - k - \delta \ln d_t + E_t(-\gamma \Delta \ln c_{t+1} + \ln q_{t+1})] \\ \ln q_t &\approx \frac{k - \ln \beta - \sigma}{\delta} - \frac{\delta}{1 - \delta} E_t \sum_{s=0}^{\infty} \frac{1}{(1 - \delta)^s} \ln d_{t+s} - \frac{\gamma}{1 - \delta} E_t \sum_{s=0}^{\infty} \frac{1}{(1 - \delta)^s} \Delta \ln c_{t+s+1} \end{aligned}$$

Now recall $d_t = \frac{\xi_t}{\lambda_t}$ so $\ln d_t = \ln \xi_t - \ln \lambda_t = \ln \xi_t + \gamma \ln c_t$. Therefore

$$\ln q_t \approx \frac{k - \ln \beta - \sigma}{\delta} - \frac{\delta}{1 - \delta} E_t \sum_{s=0}^{\infty} \left[\frac{1}{(1 - \delta)^s} (\ln \xi_{t+s} + \gamma \ln c_{t+s}) \right] - \frac{\gamma}{1 - \delta} E_t \sum_{s=0}^{\infty} \frac{1}{(1 - \delta)^s} \Delta \ln c_{t+s+1}$$

A.3 Calculation of sums and conditional variances.

For the stationary preference process we have $\vec{\xi}_{t+1} = \Phi^s \vec{\xi}_t + \vec{\varepsilon}_{t+1}^s$ where $\vec{\xi}_t = [1 \ t \ \xi_{t-1} \ \dots \ \xi_{t-p}]'$ and $\vec{\varepsilon}_t^s = [0 \ 0 \ \varepsilon_t^s \ 0_{p-1}]'$ and for the non-stationary process we have $\Delta \vec{\xi}_{t+1} = \Phi^{ns} \Delta \vec{\xi}_t + \vec{\varepsilon}_{t+1}^{ns}$

where $\Delta \vec{\xi}_t = [1 \quad \xi_{t-1} \quad \Delta \xi_{t-1} \quad \dots \quad \Delta \xi_{t-p}]'$ and $\vec{\varepsilon}_t^{ns} = [0 \quad \varepsilon_t^{ns} \quad \varepsilon_t^{ns} \quad \vec{0}_{p-1}]'$

For the preference part of the sum assuming the trend stationary process we can calculate, letting $\tilde{\Phi} = \Phi^S / (1 - \delta)$

$$\Sigma_{\xi,S,t} = \frac{-\delta}{1-\delta} E_t \sum_{s=0}^{\infty} \frac{1}{(1-\delta)^s} \ln \xi_{t+s} = \frac{-\delta}{1-\delta} e'_{3,p+2} [I - \tilde{\Phi}]^{-1} \vec{\xi}_t$$

where $e'_{i,j}$ is a row vector of length j with zeros in all places except row i .

We can also calculate the unforecastable part of this sum as:

$$\Sigma_{\xi,S,t+1} - E_t \Sigma_{\xi,S,t} = -\frac{\delta}{1-\delta} [I - \tilde{\Phi}]_{3,3}^{-1} \varepsilon_{t+1}^s \equiv q_{1,s} \varepsilon_{t+1}^s$$

For the non-stationary model the analogous results are, letting $\tilde{\Phi} = \Phi^{NS} / (1 - \delta)$:

$$\begin{aligned} \Sigma_{\xi,NS,t} &= \frac{-\delta}{1-\delta} E_t \sum_{s=0}^{\infty} \frac{1}{(1-\delta)^s} \ln \xi_{t+s} = \frac{-\delta}{1-\delta} (e'_{2,p+1}) [I - \tilde{\Phi}]^{-1} \Delta \vec{\xi}_t \\ \Sigma_{\xi,NS,t+1} - E_{t+1} \Sigma_{\xi,NS,t+1} &= -\frac{\delta}{1-\delta} \left([I - \tilde{\Phi}]_{2,2}^{-1} + [I - \tilde{\Phi}]_{2,3}^{-1} \right) \varepsilon_{t+1}^{ns} \equiv q_{1,ns} \varepsilon_{t+1}^{ns} \end{aligned}$$

For the consumption process we have $\Delta \vec{c}_{t+1} = \Theta \Delta \vec{c}_t + \vec{\varepsilon}_t^c$ where $\Delta \vec{c}_t = [1 \quad c_{t-1} \quad \Delta c_{t-1} \quad \dots \quad \Delta c_{t-p}]'$ and $\vec{\varepsilon}_t^c = [0 \quad \varepsilon_t^c \quad \varepsilon_t^c \quad 0]'$

For the first consumption sum we get, letting $\tilde{\Theta} = \Theta / (1 - \delta)$

$$\Sigma_{c,t} = \frac{-\delta\gamma}{1-\delta} E_t \sum_{s=0}^{\infty} \frac{1}{(1-\delta)^s} \ln c_{t+s} = \frac{-\delta\gamma}{1-\delta} e'_{2,p+2} [I - \tilde{\Theta}]^{-1} \Delta \vec{c}_t$$

For the second consumption sum we obtain

$$\Sigma_{\Delta c,t} = \frac{-\gamma}{1-\delta} E_t \sum_{s=0}^{\infty} \frac{1}{(1-\delta)^s} \Delta \ln c_{t+s+1} = \frac{-\gamma}{1-\delta} e'_{3,p+2} [I - \tilde{\Theta}]^{-1} \Theta \Delta \vec{c}_t$$

The unforecastable part of the sum of these two sums is then given by

$$\begin{aligned} [\Sigma_{c,t} + \Sigma_{\Delta c,t}] - E_{t+1} [\Sigma_{c,t} + \Sigma_{\Delta c,t}] &= \left[\frac{-\delta\gamma}{1-\delta} \left([I - \tilde{\Theta}]_{2,2}^{-1} + [I - \tilde{\Theta}]_{2,3}^{-1} \right) - \frac{\gamma}{1-\delta} \left([I - \tilde{\Theta}]_{3,3}^{-1} \theta \right) \right] \varepsilon_{t+1}^c \\ &\equiv c_1 \varepsilon_{t+1}^c \end{aligned}$$

Therefore we can write the unforecastable part of $\ln q_{t+1}^s$ as $a_s [\varepsilon_{t+1}^s \quad \varepsilon_{t+1}^c]'$ where $a_s =$

$[q_{1,s} \ c_1]$, the unforecastable part of lnq_{t+1}^{ns} as $a_{ns}[\varepsilon_{t+1}^{ns} \ \varepsilon_{t+1}^c]'$ where $a_{ns} = [q_{1,ns} \ c_1]$ and the unforecastable part of the consumption sums as $b[\varepsilon_{t+1}^s \ \varepsilon_{t+1}^c]'$ and $b[\varepsilon_{t+1}^{ns} \ \varepsilon_{t+1}^c]'$ where $b = [0 \ c_1]$. We can now calculate the variance and covariance terms as

$$\begin{aligned}\sigma_{q_s}^2 &= a_s C_s a_s' \\ \sigma_{q_{ns}}^2 &= a_{ns} C_s a_{ns}' \\ \sigma_{c,q_s} &= a_s C_s b' \\ \sigma_{c,q_{ns}} &= a_{ns} C_s b'\end{aligned}$$

where C_s is the variance-covariance matrix of $[\varepsilon_{t+1}^s \ \varepsilon_{t+1}^c]'$ and C_{ns} is the variance-covariance matrix of $[\varepsilon_{t+1}^{ns} \ \varepsilon_{t+1}^c]'$.

Figure 1: Probability of the Stationary Model

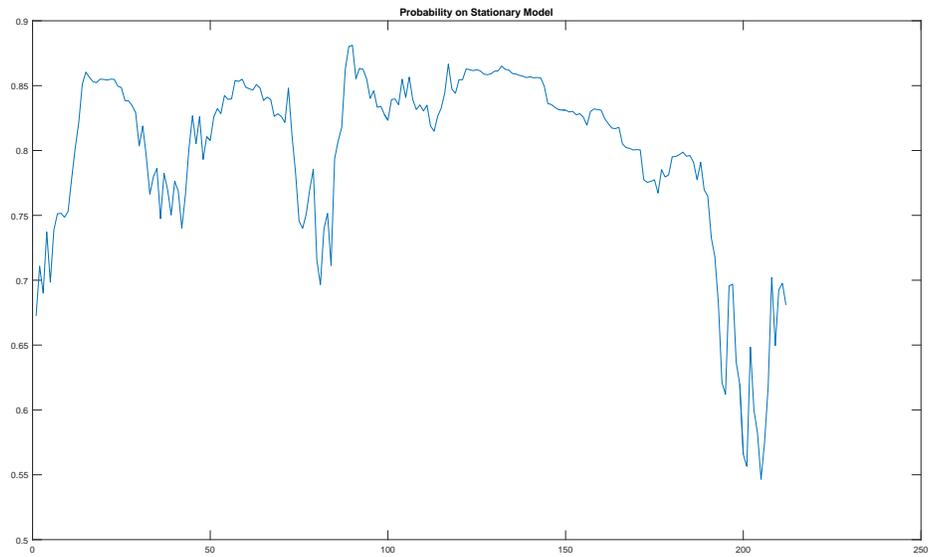


Figure 2: Rent

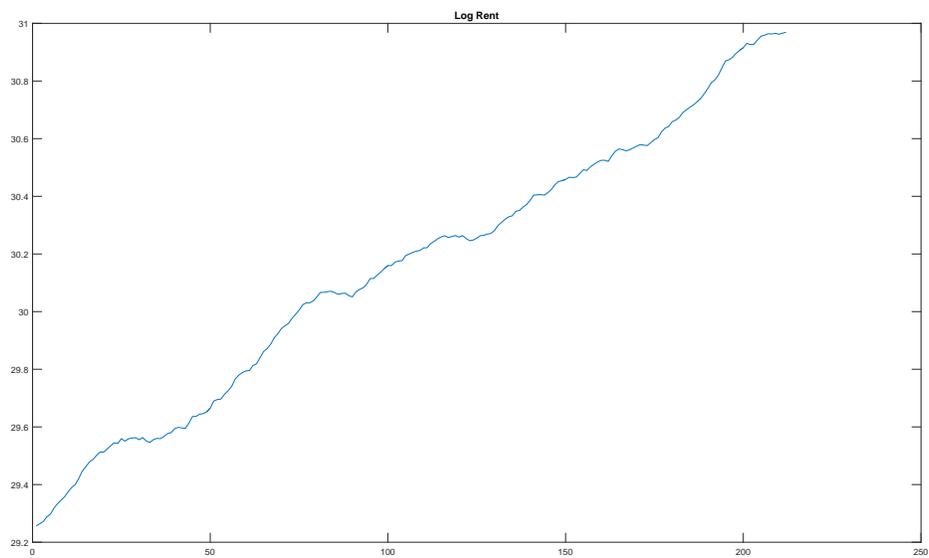


Figure 3: Price-to-rent Ratio: Learning Model (dashed), RE model (solid)

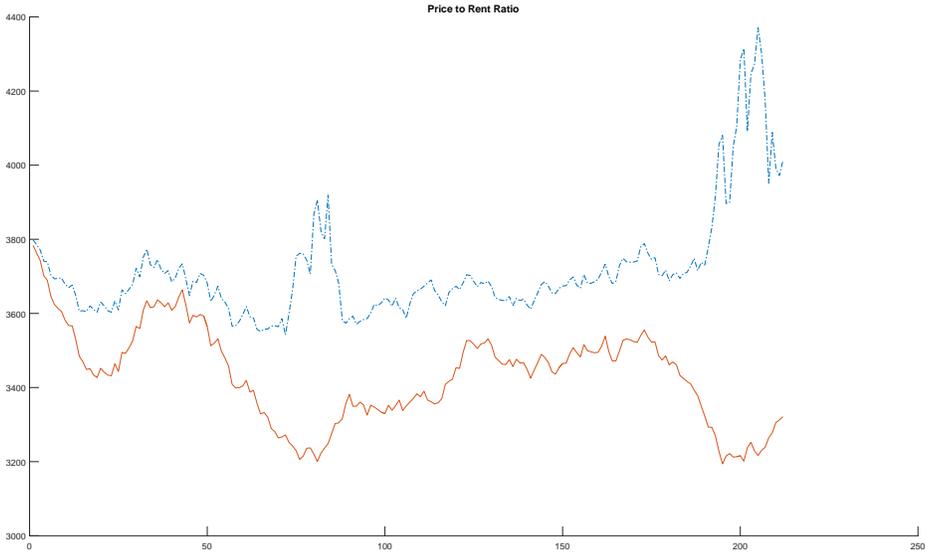


Table 1: Data Moments

<u>Means and Standard Deviations</u>		<u>Skewness and Kurtosis</u>	
$E(r_t)$	0.064	skew(P_t/R_t)	2.04
$E(\Delta \ln(\text{rent}_t))$	0.009	skew(r_t)	-1.7
$\sigma(P_t/R_t)$	0.15	kurtosis(P_t/R_t)	7.1
$\sigma(P_t^{\text{HP}})$	0.037	kurtosis(r_t)	7.4
$\sigma(r_t)$	0.06	<u>Predictability</u>	
$\sigma(\Delta \ln(\text{rent}_t))$	0.023	$\rho(P/R_t, r_{t+1} + \dots r_{t+16})$	-0.74
		$\rho(r_t, r_{t+1} + \dots r_{t+16})$	0.32
		$\rho(P/R_t, \ln(\text{rent}_{t+16}) - \ln(\text{rent}_t))$	-0.44
<u>Autocorrelations</u>		<u>Squared Autocorrelations</u>	
$\rho(P_t/R_t)$	0.99	r_t^2	0.7
	0.98		0.56
	0.95		0.45
	0.92		0.27
$\rho(r_t)$	0.84	$(\varepsilon_t^r)^2$	0.14
	0.73		0.39
	0.66		0.1
	0.51		0.32
		<u>GARCH Model</u>	
		γ_1 (GARCH)	0.73
			(0.06)
		a_1 (ARCH)	0.27
			(0.07)
		p-value Engle test	0.04

Note: This table provides the key moments on house prices, rents and price to rent ratio.

Table 2: Campbell-Shiller Regressions

Future Return Regression ($r_{t+1} + \dots + r_{t+16}$)

β	R^2
-0.8	0.5
(0.06)	

Future Rent Growth Regression ($\ln \xi_t + \dots + \ln \xi_{t+16}$)

β	R^2
-0.12	0.2
(0.02)	

Note: This table presents results from running Campbell-Shiller regressions on the housing data. The price-to-rent ratio is used to predict returns and rent growth over the next four year. Standard errors are in parentheses.

Table 3: Model Results -- Main Moments

		<u>Flex Price</u>		<u>Sticky Price</u>	
	<u>Data</u>	<u>RE</u>	<u>Learn</u>	<u>RE</u>	<u>Learn</u>
<u>Means, Stdev</u>					
$E(r_t)$	0.064	0.03	0.03	0.03	0.03
$E(\Delta \ln(\text{rent}_t))$	0.009	0.03	0.03	0.03	0.03
$\sigma(P_t/R_t)$	0.15	0.03	0.04	0.03	0.03
$\sigma(P^{\text{HP}}_t)$	0.037	0.008	0.03	0.005	0.02
$\sigma(r_t)$	0.06	0.03	0.08	0.01	0.04
$\sigma(\Delta \ln(\text{rent}_t))$	0.023	0.03	0.03	0.03	0.03
<u>Autocorrelations</u>					
$\rho(P_t/R_t)$	0.99	0.99	0.9	0.97	0.96
	0.98	0.97	0.82	0.94	0.92
	0.95	0.94	0.76	0.91	0.88
	0.92	0.91	0.7	0.87	0.84
$\rho(r_t)$	0.84	0	-0.01	0.74	0.44
	0.73	-0.01	-0.01	0.54	0.31
	0.66	0	-0.01	0.4	0.22
	0.51	0	0	0.3	0.15
<u>Predictability</u>					
$\rho(P/R_t, r_{t+1} + \dots r_{t+16})$	-0.74	0.07	-0.49	-0.08	-0.51
$\rho(r_t, r_{t+1} + \dots r_{t+16})$	0.32	-0.03	-0.04	0.25	0.14
$\rho(P/R_t, \ln(\text{rent}_{t+16}) - \ln(\text{rent}_t))$	-0.44	0.29	-0.36	0.25	-0.43
<u>Campbell Shiller Regression</u>					
β^{return}	-0.8	0.06	-1	-0.06	-1
R^2 (return regression)	0.5	0.04	0.24	0.04	0.27
$\beta^{\text{rent growth}}$	-0.12	0.46	-0.42	0.37	-0.5
R^2 (rent growth regression)	0.2	0.09	0.13	0.07	0.19

Note: This table gives the model predictions for the main data moments.

Table 4: Model Results -- Time Varying Volatility

	<u>Data</u>	<u>Flex Price</u>		<u>Sticky Price</u>	
		<u>RE</u>	<u>Learn</u>	<u>RE</u>	<u>Learn</u>
<u>Skewness and Kurtosis</u>					
skew(P_t/R_t)	2.04	0.02	0.89	0.05	0.68
skew(r_t)	-1.7	0.03	0.13	-0.03	0.15
kurtosis(P_t/R_t)	7.1	2.2	4.4	2.4	3.5
kurtosis(r_t)	7.4	3	4.1	2.9	5.1
<u>Squared Autocorrelations</u>					
r_t^2	0.7	-0.01	0.08	0.72	0.35
	0.56	0	0.07	0.53	0.27
	0.45	-0.01	0.08	0.39	0.19
	0.27	-0.01	0.06	0.28	0.15
$(\varepsilon_t^r)^2$	0.14	-0.01	0.09	-0.01	0.21
	0.39	-0.01	0.1	-0.01	0.19
	0.1	-0.01	0.1	-0.01	0.17
	0.32	-0.01	0.08	-0.01	0.17
<u>GARCH Model</u>					
γ_1 (GARCH)	0.73	0	0	0	0.68
	(0.06)				
a_1 (ARCH)	0.27	0	0	0	0.21
	(0.07)				
p-value Engle test	0.04				

Note: This table gives the model predictions for the key data moments.

Table 5: Robustness

<u>Moment</u>	<u>Data</u>	<u>AR</u>	<u>4</u>	<u>2</u>	<u>6</u>	<u>ν</u>	<u>1</u>	<u>2</u>	<u>3</u>
$\sigma(P_t/R_t)$	0.15		0.03	0.05	0.03		0.03	0.024	0.02
$\sigma(P_t^{HP})$	0.037		0.02	0.02	0.02		0.02	0.02	0.02
$\sigma(r_t)$	0.06		0.04	0.04	0.04		0.04	0.04	0.03
kurtosis(P_t/R_t)	7.1		3.5	2.6	3.5		3.5	3.4	3
kurtosis(r_t)	7.4		5.1	3.9	5.7		5.1	4.3	3.5
$\rho(P/R_t, r_{t+1} + \dots r_{t+16})$	-0.74		-0.51	-0.5	-0.5		-0.51	-0.48	-0.29
$\rho(r_t, r_{t+1} + \dots r_{t+16})$	0.32		0.14	0.19	0.13		0.14	0.17	0.21
$\rho(P/R_t, \ln(\text{rent}_{t+16}) - \ln(\text{rent}_t))$	-0.44		-0.43	-0.44	-0.37		-0.43	-0.29	-0.01
Autocorrelation: $(\varepsilon_t^r)^2$	0.14		0.21	0.14	0.21		0.21	0.18	0.14
	0.39		0.19	0.14	0.19		0.19	0.16	0.12
	0.1		0.17	0.12	0.17		0.17	0.15	0.11
	0.32		0.17	0.11	0.15		0.17	0.14	0.1
<u>Moment</u>	<u>Data</u>	<u>g</u>	<u>0.001</u>	<u>0.005</u>	<u>0.01</u>	<u>λ</u>	<u>0.25</u>	<u>0.1</u>	<u>0.5</u>
$\sigma(P_t/R_t)$	0.15		0.02	0.03	0.04		0.03	0.03	0.03
$\sigma(P_t^{HP})$	0.037		0.01	0.02	0.02		0.02	0.01	0.02
$\sigma(r_t)$	0.06		0.03	0.04	0.04		0.04	0.03	0.06
kurtosis(P_t/R_t)	7.1		2.9	3.5	3.4		3.5	2.9	4
kurtosis(r_t)	7.4		5.2	5.1	4.9		5.1	5.8	4.6
$\rho(P/R_t, r_{t+1} + \dots r_{t+16})$	-0.74		-0.4	-0.51	-0.52		-0.51	-0.57	-0.5
$\rho(r_t, r_{t+1} + \dots r_{t+16})$	0.32		0.15	0.14	0.14		0.14	0.26	0.05
$\rho(P/R_t, \ln(\text{rent}_{t+16}) - \ln(\text{rent}_t))$	-0.44		-0.22	-0.43	-0.45		-0.43	-0.27	-0.42
Autocorrelation: $(\varepsilon_t^r)^2$	0.14		0.21	0.21	0.19		0.21	0.23	0.14
	0.39		0.19	0.19	0.17		0.19	0.19	0.14
	0.1		0.17	0.17	0.18		0.17	0.17	0.13
	0.32		0.17	0.17	0.16		0.17	0.18	0.13

Note: This table reports the robustness of the results from table 3 when varying some of the parameters.

Table 6: Analysis of Forecast Errors

	<u>Flex Price</u>		<u>Sticky Price</u>	
	<u>RE</u>	<u>Learn</u>	<u>RE</u>	<u>Learn</u>
<u>Rent</u>				
$\rho(\varepsilon_t, \varepsilon_{t-1})$	-0.0022	-0.0049	-0.0043	-0.0052
$\sigma(\varepsilon_t)$	0.0072	0.0072	0.0072	0.0072
<u>Returns</u>				
$\rho(\varepsilon_t, \varepsilon_{t-1})$	-0.0009	-0.0071	-0.004	-0.007
$\sigma(\varepsilon_t)$	0.0269	0.0842	0.007	0.0348
$\sigma(\varepsilon_t)/\sigma(r_t)$	1	1	0.66	0.87

Note: This table calculates autocorrelation and standard deviations for forecast errors under the learning model and the rational expectations benchmark model. The last row normalizes the standard deviation by the standard deviation of returns