Volatility of Stock Return Variance and Capital Gains Tax

Xia Meng, Junbo L Wang, Zhipeng Yan, Yan Zhao¹

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Abstract

In this paper, we develop a two-period portfolio selection model with differential capital gains tax rates. Our focus is the relationship between the volatility of stock return variance and an investor’s intertemporal decision of realizing capital gains/losses. We predict that a higher volatility of stock return variance will lead to a larger short-term capital loss realization, a larger ratio of short-term capital gain realization to the total net capital gain, and a larger trading volume. Those predictions are consistent with the empirical evidence. In addition, we also demonstrate that higher volatility of stock return variance leads to higher trading volume.

¹ Xia Meng is a Ph.D student, International Business School, Brandeis University. Email: xiameng@brandeis.edu. Junbo L Wang is a Ph.D student, Marshall School of Business, University of Southern California. Email: Junbo.Wang.2013@marshall.usc.edu. Zhipeng Yan is an assistant professor, School of Management, New Jersey Institute of Technology. Email: zyan@adm.njit.edu. Yan Zhao is an assistant professor, Department of Economics, City College, CUNY. Email: yzhao2@ccny.cuny.edu. We would like to thank Larry Ozanne at Congressional Budget Office for providing the data of capital gains realized by US individual taxpayers, as well as detailed descriptions. We would like to thank Blake LeBaron, Carol Osler, and Jens Hilscher at Brandeis University for their valuable advice and suggestions.
1 Introduction

The existing literature on the capital gains/losses is mostly devoted to the long-term capital gains. At the same time, the studies on the short-term capital gains are almost non-existent in the literature largely because “most realized gains are long-term ones.” (Auerbach and Poterba (1988)). However, the relative importance of the short-term capital gains compared to total net capital gains is more than doubled after the late 1980s in the United States. Although the long-term gain realizations are still dominant in the total capital gain realizations, it would be amiss to ignore short-term gains any more. In this paper, we aim to fill the gap in the existing literature by examining factors affecting short-term capital gains/losses realizations. In particular, we develop a simple theoretical model explaining the phenomenon of increasing short-term capital gains relative to total net capital gains both from the perspective of tax effect and from a novel aspect of the stock market - the volatility of stock variance.

A standard mean-variance framework provides the intuition for the argument that the volatility of stock return variance influences the intertemporal decisions of investors to realize their capital gains or losses. Consider two hypothetical economies with constant and time-varying stock return volatility, respectively. In the first economy, all stocks have constant expected returns and variances. Thus for any stock, the volatility of stock return variance is zero. In the second economy, the expected stock returns are constant but stock variances change over time. If investors choose their portfolios according to the mean-variance framework, investors in economy 1 will not change their portfolio weights once they are set, and will not conduct any short-term trading, while investors in
economy 2 must change their portfolio weights over time in responding to the changes in stock variances, which may lead to short term trading activities. Without considering tax effects, a typical investor may realize either gains or losses short term. However, with the tax benefits of short term capital losses and the tax disadvantage of short term capital gains in mind, an investor in economy 2 may be willing to realize more short-term losses rather than short-term gains. If the long-term capital gains and loss are roughly the same in the two economies, economy 2 will have a higher ratio of short term capital gains to total net capital gains.

In addition to the changes in the stock market conditions, the differential capital gains tax rates may also play an important role in the investors’ intertemporal decisions. In the United States, the capital gains tax rate for individuals is lower on “long-term capital gains,” which are gains on assets held longer than one year. As of 2011, the tax rate on long-term gains is 15 percent, while short-term gains are subject to ordinary federal income tax, which ranges up to 35 percent. With the tax rate on long term capital gains and losses being significantly lower than that on short term capital gains and losses, the tax law provides a timing option to realize losses short term and realize gains long term, if at all. Constantinides (1984) illustrates that this timing option is more valuable the higher the stock variance, for essentially the same reason that a call option is more valuable the higher the stock variance.

In this study, we extend the work of Constantinides (1984) by including the volatility of stock return variance as an explanatory factor for the changing pattern of investors’ intertemporal decision. We first build a two-period two-economy portfolio selection model with tax. We show that in a world without any tax, an investor who
maximizes her quadratic utility adjusts the weight in risky assets more significantly between two time periods in the economy with higher volatility of stock return variance. The bigger adjustment leads to a larger amount of short term gains and short-term losses. However, when the tax effect is taken into account, due to the tax disadvantage of realizing gains short term and tax benefits from realizing losses short-term, one should expect that the short term gains can increase, decrease or remain unchanged but the short term capital losses should increase. As a result, the ratio of short-term capital gains realization to long-term capital gains net of short-term losses increases with the volatility of return variance. We also illustrate that high volatility of return variance can lead to high trading volume.

We then check the key assumptions of our model and test the model’s implications in the real world. We collect NYSE/AMEX/NASDAQ daily value-weighted market return from CRSP and annual total capital gains and losses realized by U.S. individual taxpayers from the Department of Treasury for the period of 1970-2008. The whole sample period is divided into two sub-periods: 1970-1986 and 1987-2008. We find that the volatility of stock market return variance increases significantly after 1987. In addition, the short-term gains (normalized by the value of stock market index at the beginning of the year) do not increase, while the long-term gains net of the short-term losses decrease.

Many previous studies have examined the impact of capital gains taxes on stock returns and trading volume. Few studies investigate the relation between capital gains

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taxes and the second moment of stock returns\textsuperscript{3}. Even fewer studies, if any, investigate the possibility that the causality can run from the change in stock market risk to capital gains/losses realization. In this paper, we are focused on the volatility of stock return variance and illustrate theoretically that it, together with differential capital gain taxes, can affect investors’ intertemporal decisions.

It was generally believed that investors should defer their long-term capital gains realization in order to minimize net present value of their tax payments (“taxes deferred are taxes saved”). As a consequence, investors may be less likely to change their portfolios - the so-called ‘lock-in effect’ phenomenon. Constantinides (1983) derives the optimal trading strategies for investors when the timing option exists. In another important paper on tax options, Constantinides (1984) points out that when tax rate on long-term gains and losses is relatively low (compared with short-term tax rate), taxable investors should realize long-term gains in high variance stocks and repurchase stock in order to realized potential future short-term losses. In a more recent study, Dai et al. (2010) investigate the effect of changes in capital gain taxes on stock return volatility. They find, after passage of the 1978 and 1997 capital gain tax rate reductions, larger increases in the return volatility for more appreciated stocks than for less appreciated stocks possibly due to the reduced risk sharing (by the government) and the reduced future capital gain taxes. They also find larger increases in the return volatility for non-dividend-paying stocks than for dividend-paying stocks. Our work differs from these studies, and explores the impact on the capital gain/loss realization from both the volatility of stock return variance and the differential tax rates on the long-term and

\textsuperscript{3} Dai et al. (2010), which examines changes of stock return volatility around capital gains tax reduction events, is a rare exception that focuses on the second moment of stock returns.
short-term capital gains.

Our paper is also related to a large literature that connects trading volume to stock return volatilities. Empirically, researchers find that the correlation between the trading volume and the return volatilities is significant, e.g. Clark(1973), Epps and Epps (1976), Tauchen and Pitts (1983), Bessembinder and Seguin (1993). Brock and LeBaron (1993) and Anderson (1996) build models to explain "Mixed Distribution Hypothesis". Their models are based on microstructure theories that the arrival of information leads to a correlation between the trading volume and the volatility of stock returns. Therefore, the causality between the trading volume and the return volatility is spurious. Our paper suggests that the trading volume can be determined by the change in the estimated return volatility (e.g., in some technical and quantitative trading strategies). After the market crash in 1987, the daily volatility has been changing more rapidly. Hence, investors would like to rebalance their portfolios more significantly compared to the period before 1987. This leads to a higher trading volume. Lakonishok and Smidt (1989) document the effect of past stock price patterns on current trading volumes and show the causality between the price and volume. They find that the past winner stocks have higher trading volumes. Our paper is different from theirs in that we document the effect of change of stock return variance on the trading volumes.

The contribution of our paper is two-fold. First, our model is the first to link the volatility of stock return variance to capital gains/losses realizations. The predictions of our model are confirmed by the U.S. capital gains/losses realization data. Second, we demonstrate that higher volatility of stock return variance leads to higher trading volume.

The rest of this paper is organized as follows. Section 2 presents a two-period
portfolio-selection model and derives main predictions. Section 3 describes our data and reports empirical tests. Concluding remarks are offered in Section 4.

2 Model

In this section, we propose a two-period portfolio-selection model with tax to explain the larger the amount of short term capital loss, the higher the ratio of short-term capital gain to the total net capital gain after 1987. Since the volatility of daily return variance increases after 1987, the investors need to adjust the weights in stocks more significantly in responding to the volatile changes in return variance. Consequently, they generate both higher gains and losses in the short-term (the “volatility effect”). However, a rational investor may be reluctant to realize the gains short term since the higher tax bracket makes the short-term capital gains less attractive. But the investor is willing to realize the short-term losses because it can increase the tax benefits of the long-term gains (the “tax effect”). The combination of the two effects can explain the following three facts observed before and after 1987 in the United States. 1) the short-term gains realization (total wealth adjusted) does not change significantly; 2) since the short-term losses realization increases and if the level of long-term net gains (total wealth adjusted) are relatively stable over time, the long-term net gains minus short-term losses plus short-term gain (i.e., total net capital gains) decreases; 3) the ratio of short-term gains to total net gains increases.

There is a representative investor who has two accounts. The first account

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4 The short-term gains are taxed at the ordinary income tax rate, which is currently 35 percent in the highest tax bracket, while the long-term net gains minus the sum of net short-term capital losses and any long-term capital loss carried over from the previous year are taxed at the long-term capital gain tax rate, which is 15 percent in the highest tax bracket. Therefore, investors have incentive to realize short-term losses but not short-term gains considering the tax effects.
consists of buy-and-hold portfolios. The returns of these buy-and-hold portfolios will be realized in the long run; hence, the proceeds are long-term gains. In addition, we assume the returns of these portfolios are not affected by the market risk\(^5\). More specifically, we assume that the net gains of the first account are the net long-term gains and are denoted by \( P \). The second account consists of actively managed portfolios. These portfolios are subject to the adjustment of the weights in the relatively short period. The proceeds of these transactions are the short-term gains or losses. In this account, the investor chooses portfolios by following a standard portfolio-selection model to maximize the quadratic utility.

Suppose there are two assets in the market, one risk free asset and one risky asset. The value of the risk free asset does not change over time, i.e. the return is 0\( (r_f = 0) \). The risky asset has an expected return \( \mu \) \((\mu > 0)\) and the standard deviation of the return \( \sigma \) in each period. In order to simplify the model, we assume that the return of the risky asset can be either \( \mu + \sigma \) or \( \mu - \sigma \) with equal probabilities. The expected return is fixed so that \( \mu \) is constant. We assume that the variance of the stock return \( \sigma^2 \) is varying over time, which can be either \( \bar{\sigma}^2(1 + A) \) or \( \bar{\sigma}^2(1 - A) \) with equal probabilities\(^6\). Hence, the mean and variance of the return variance is \( \bar{\sigma}^2 \) and \( \bar{\sigma}^4A^2 \). So the volatility of return variance is higher when \( A \) is higher.

There are two periods, in period \( t \) \((t=1,2)\). One can think of 1 period as short term (shorter than one year) and 2 periods as long term. In each period, the investor maximizes the quadratic utility by choosing the weight of the risky asset \( w \) in portfolio:

\(^5\)In general, the buy-and-hold portfolios are safer than actively managed portfolios. We assume that these portfolios have no risk for simplicity.
\(^6\)We assume \( 0 < A < 1 \) so that there is no negative variance.
\[
\max_{w(t)} E(r_p(t)) - \frac{1}{2} \text{var}(r_p(t)),
\]

where \( r_p(t) \) is the return of the portfolio formed by the risk free asset and the risky asset with weights \( 1 - w(t) \) and \( w(t) \), respectively. Then

\[
E(r_p(t)) = w(t) \mu + (1 - w(t))r_f, \text{var}(r_p) = w(t)^2 \sigma(t)^2.
\]

The maximization problem yields

\[
w = \frac{\mu}{\sigma(t)^2}.
\]

At this moment, we assume that there is no tax on any transaction. We will relax this assumption later.

Now suppose there are two economies: H and L (e.g., H represents the economy after 1987 and L represents the economy before 1987), which are identical in terms of economic agent, utility function and parameters \( \mu \) and \( r_f \), except for the variance of the return of the risky asset. Suppose that the return variance is more volatile in economy H, i.e. \( \sigma_H > \sigma_L \).

In the discussion above, we assume that in the two economies the risky asset’s returns have the same mean of variance. But the empirical work in the next section indicates that the economies before and after 1987 have significantly different means of the variance, i.e.

\[
\bar{\sigma}_H > \bar{\sigma}_L.
\]

But the empirical evidence also implies that
Then, we can show that all the following results hold as long as equation (2) is satisfied, even if the mean of the variance is different between the two economies.

If we ignore the tax effect, then the difference in the two economies leads to the difference in the standard deviation of portfolio weight adjustment in the second period $(w_i(2) - w_i(1))$ for economy $i$, $i = H, L$:

**Theorem 2.1** Suppose there are two economies, both of which have a representative investor who maximizes the quadratic utility by selecting the weight of portfolio, i.e. maximize the utility from equation 1. The two economies are identical except that the volatility of return variance in economy $H$ is higher, then the standard deviation of the weight adjustment in the second period (defined as $\text{std}(w(2) - w(1))$) is higher for economy $H$.

The proof of all the theorems and propositions are in the appendix. One immediate conclusion from this theorem is: if the volatility of return variance is larger, the adjustment in weight of the risky asset between the two periods will be larger, e.g. if $A_H > A_L$, given

$$\frac{A_H}{\sigma_H^2} \geq \frac{A_L}{\sigma_L^2},$$

(2)

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\[ |w_H(2) - w_H(1)| = (\mu \frac{2A_H}{\sigma^2 (1 - A_H^2)}), \quad |w_L(2) - w_L(1)| = (\mu \frac{2A_L}{\sigma^2 (1 - A_L^2)}). \]

Since \( A_H > A_L \), it is clear that the change of weight in risky asset is higher in economy H.

Intuitively, since there is no tax, the larger adjustment in the weight of risky asset in economy H will lead to a higher trading volume in the risky asset, thus will lead to higher realized short-term capital gains or loss in each period. To derive this result, we need to make some further assumptions.

The key assumption in this paper is that \( \mu = o(1) \) and \( \sigma = o(1) \). This assumption implies that for any \( i \) and any possible outcomes of return \( r_i(1) \), \( r_i(1) = o(1) \). Therefore

\[ |1 - \frac{1}{1 + r_i(1)}| = o(1) \left| \frac{1}{1 + A_i} - \frac{1}{1 - A_i} \right|. \]

The intuition of this assumption is that the change in price in each period is close to zero and is much smaller than the the volatility of the return variance for both economies.

Another assumption is that \( \frac{\mu}{\sigma^2} | = O(1). \) This assumption guarantee that the weight for stock in the portfolio is not negligible.

Suppose that the prices of risk free and risky assets are both 0 at time \( t = 1 \), thus, the investor in economy \( i \) (\( i=H,L \)) will buy \( \mu/\sigma_i^2(1) \) shares of risky asset. Then the value of portfolio at time \( t = 2 \) is

\[ 1 - \frac{\mu}{\sigma_i^2(1)} + (1 + r_i(1)) \frac{\mu}{\sigma_i^2(1)}, \]

where \( r_i(1) \) is the return of the risky asset at \( t = 1 \) and it can be either \( \mu + \sigma_i(1) \) or
\( \mu - \sigma_i(1) \) with equal probabilities. Since at time \( t = 2 \), the standard deviation of return changes to \( \sigma_i(2) \), so the weight in risky asset becomes \( \frac{\mu}{\sigma^2_i(2)} \). Hence, the change in number of shares of risky asset at time \( t = 2 \) is

\[
\frac{\mu^2}{\sigma^2_i(1)\sigma^2_i(2)}(1 - \frac{1}{1 + r_i(1)}) + \left( \frac{\mu}{\sigma^2_i(2)} \right) \left( \frac{1}{1 + r_i(1)} \right) - \frac{\mu}{\sigma^2_i(1)}.
\]

Rearrange the formula above, we have the change in number of shares of risk asset expressed as

\[
\frac{\mu^2}{\sigma^2_i(1)\sigma^2_i(2)}(1 - \frac{1}{1 + r_i(1)}) + \left( \frac{\mu}{\sigma^2_i(2)} \right) \left( \frac{1}{1 + r_i(1)} \right) - \frac{\mu}{\sigma^2_i(1)}.
\]

As one can see from the rearrangement, the first part denotes the changes in shares due to the change in price, we call it price effect. The second part denotes the changes in shares that are resulted from the change in the return variance, we call it volatility effect.

Using the two assumptions above, one can derive the following proposition:

**Proposition 2.2** If the assumptions above are satisfied, suppose \( \sigma_i(2) \neq \sigma_i(1) \), then for any \( i \)

\[
\left| \frac{\mu^2}{\sigma^2_i(1)\sigma^2_i(2)}(1 - \frac{1}{1 + r_i(1)}) \right| \sim o(1) \left| \frac{\mu}{\sigma^2_i(2)} \frac{1}{1 + r_i(1)} - \frac{\mu}{\sigma^2_i(1)} \right|.
\]

From this proposition, under the assumption that the price is changing much smaller than the return variance, the price effect is much smaller than the volatility effect when there is a volatility change at time \( t = 2 \). Thus, the changes of number of shares in
risky asset are primarily due to the changes in the volatility of the asset.

The short-term gain is defined as the realized capital gain through interim trading.

Let \( \frac{\mu}{\sigma_i^2(1)} \equiv S_i(1) \),

and \( \frac{\mu^2}{\sigma_i^2(1)\sigma_i^2(2)} \left( 1 - \frac{1}{1 + r_i(1)} \right) + \frac{\mu}{\sigma_i^2(2)(1 + r_i(1))} \equiv S_i(2) \).

If \( S_i(1) > S_i(2) \), then the short-term gain is

\[ STG_i = ((S_i(1) - S_i(2))r_i(1),0)^+; \]

the loss is

\[ STL_i = ((S_i(1) - S_i(2))r_i(1),0)^-. \]

If \( S_i(1) < S_i(2) \), then the short-term gain is

\[ STG_i = ((S_i(2) - S_i(1))r_i(2),0)^+; \]

the loss is

\[ STL_i = ((S_i(2) - S_i(1))r_i(2),0)^-; \]

To be more specific, if the investor sells shares at time \( t = 2 \), then the investor will either gain or lose \( (S_i(1) - S_i(2))r_i(1) \). If the investor buys shares at time \( t = 2 \), then the gains and loss will be realized at time \( t = 3 \), the amount of the gain or loss will be \( (S_i(2) - S_i(1))r_i(2) \).

Using this definition, we have our main result:

**Theorem 2.3** In a two-period model, assume that: 1) economy H and L have the same economic agent with the same utility function, the same parameters of market
except that economy \( H \) has a larger volatility of variance of risky return;\(^7\) 2) the risky return in the first period \( r_i(1) \) \((i=H,L)\) satisfies \( r_i(1) = o(1) \); 3) the mean and average variance of risky return, \( \mu \) and \( \bar{\sigma}^2 \), satisfy \( \left| \frac{\mu}{\bar{\sigma}^2} \right| = O(1) \). The expected short-term capital gain is higher in economy \( H \) than in economy \( L \), i.e.

\[
E(STG_H) > E(STG_L).
\]

Similarly, the short-term loss is also higher in economy \( H \) than in economy \( L \), i.e.

\[
\]

Next step, we include the tax effect. Suppose that there is no tax for long-term gains and the tax rate for short-term gains is higher for larger short-term gains because of different tax brackets. To simplify, assume that there is no tax for both economies if there is a relatively small amount of gains due to small adjustment in portfolio weights; if there is a larger amount of gains, the gains will be subject to a specific tax rate, i.e. if the short-term gains \( STG \) satisfies

\[
STG \leq \frac{\mu}{\bar{\sigma}^2} \left( \frac{1}{1-A_L} - \frac{1}{1+A_L} \right) \left( 1 + \mu + \bar{\sigma} \sqrt{1+A_L} \right),
\]

no tax is imposed on the gains; if the gains are larger, the tax rate \( t \) satisfies

\[
\left( \frac{1}{1-A_H} - \frac{1}{1+A_H} \right) (1-t) = \frac{1}{1-A_L} - \frac{1}{1+A_L}.
\]

Under this assumption, we have the following theorem:

**Theorem 2.4** In a two-period model, assume that: 1) economies \( H \) and \( L \) have

\(^7\)As is mentioned above and shown in details below, this assumption can be relaxed and the relaxation does not hurt our results as long as equation 2 holds.
the same economic agent with the same utility function, the same parameters of market except that economy H has a larger volatility of risky return variance; ² 2) the risky return in the first period \( r_t(1) \) \( (i=H,L) \) satisfies \( r_t(1) = o(1) \); 3) the mean and average variance of risky return, \( \mu \) and \( \sigma^2 \), satisfy \( \frac{\mu}{\sigma^2} = O(1) \); 4) the tax rate \( t \) satisfies equation 3. The expected short-term capital gains are approximately equal in economy 1 and in economy 2, i.e.

\[
E(STG_{H}) = E(STG_{L})(1 + o(1)).
\]

Meanwhile, the short-term losses are higher in economy H than in economy L, i.e.

\[
E(STL_{H}) > E(STL_{L}).
\]

From this theorem, one can see that there is a tax benefit in economy L since the short-term gains are smaller through the adjustment in the weight of the risky asset. Although volatility of return variance has a positive effect on short-term gains, higher tax rate has a negative effect. These two effects approximately offset each other. Therefore, there is no significant change in short-term gains. However, since the losses is not affected by the tax rate, the short-term losses are higher in economy H. In addition, since the short-term losses can increase the tax benefit of the long-term gains, one should expect that the investor will at least have the incentive to realize the losses in both economies.

Furthermore, from Theorem 2.4 and the assumption that the long-term gains are not affected by the market risk and are constant in both economies, one can show that the

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² Similar to Theorem 2.3, this assumption can be relaxed and the relaxation does not hurt our result as long as equation 2 holds.
The ratio of short-term gains to total net gains is higher in economy H.

**Theorem 2.5** Under the same assumptions in Theorem 2.4, define the total net gains \( TNG = E(STG_i) - E(STL_i) + P \) and the ratio of short-term gains to total net gains as

\[
STR_i = \frac{E(STG_i)}{TNG},
\]

then the economy with higher volatility of stock return variance has a higher ratio of short-term gains to total net gains, i.e. for \( i = H \) or \( L \), we have \( STR_H > STR_L \).

Note that in this proposition, we define the total net gains as \( E(STG_i) - E(STL_i) + P \).\(^9\) This implicitly assumes \( P > |E(STG_i) - E(STL_i)| \) to keep the total net gains positive.\(^10\)

One implication of this model is to explain the emergence of hedge funds after 1987. The higher \( A \) provides investors the incentive to either make larger adjustments in the weight of stocks or conduct trading with higher frequency. The needs for high frequency trading trigger the emergence of hedge fund, which further leads to larger short-term gains or losses.

### 3 Data and Empirical Test

Given that there is always a tax effect in the real world, we test the assumptions

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\(^9\)\( P \) is the total long term net gain and \( E(STG_i) - E(STL_i) \) is the total short term net gain.

\( ^{10} \)Since the long term gain dominates the short term gain in the real data, this assumption is consistent with the data.
of Theorem 2.4 and Theorem 2.5 as well as their predictions based on the real data. We have total capital gains and long-term capital gains realized annually by US individual taxpayers for the period of 1970-2008 from Department of the Treasury. For the same period, we collect daily value weighted market returns from CRSP. We further divide the whole sample period into two sub-samples: 1970-1986 and 1987-2008. The reason to use the year of 1987 as a watershed is that it witnesses the largest one-day stock market crash in history. And perhaps more importantly, the stock market crash of October 1987 fundamentally changed the dynamics of stock return volatility. Prior to 1987, implied volatilities of equity options (both on individual stocks and on stock indices) were much less dependent on strike price. Since the crash, the volatility surface of equity options has become skewed and this has become a persistent feature of the U.S. equity options market. Rubinstein (1994) refers to this phenomenon as "crashophobia". Investors are concerned about the possibility of another crash similar to October 1987, and they price options accordingly. Although we investigate the volatility of stock variance in this study, not the implied volatility from a stock options, the significance of the crash of 1987 and its impact on the psychology of market participants makes the year of 1987 a natural watershed of our sample period.

3.1 Test of Model Assumptions

We treat the two sub-periods, 1970-1986 and 1987-2008, as two economies, and test whether the assumptions in Theorem 2.4 and Theorem 2.5 hold in our data.

For Theorem 2.4 to hold, we need the following assumptions: 1) the economies H
and L have the same economic agent with the same utility function, the same parameters of market except that economy H has a larger volatility of risky return variance; 2) the risky return in the first period  \( r_i(1) \) (i=H,L) satisfies  \( r_i(1) = o(1) \); 3) the mean and average variance of risky return,  \( \mu \) and  \( \sigma^2 \), satisfy  \( |\frac{\mu}{\sigma^2}| = O(1) \); 4) the tax rate  \( t \) satisfies equation 3.

To examine whether the assumptions are valid, we use a GARCH(1,1) model

\[
\sigma_i^2 = \alpha_0 + \alpha_1 r_{i-1}^2 + \beta \sigma_{i-1}^2
\]  

(4)

to estimate the daily variance of stock returns. The estimates of  \( \alpha_0, \alpha_1 \) and  \( \beta \) are 0, 0.90 and 0.09, respectively. Secondly, we conduct a t-test to see whether the two sub-periods have the same population means of daily returns and daily return variance. A summary of the test is exhibited in Table 1. According to the p-values, we can not reject the null hypothesis that the population mean of daily returns is the same for both sub-periods, but can reject the null hypothesis that the population volatility of daily returns is the same for both sub-periods. Thus assumption 1) is only partially satisfied in the real data. However, as explained in the last section, the violation of the assumption of constant mean of variance will not hurt the model as long as the variance of return variance divided by the fourth power of the mean of return variance is not larger in the economy with lower-volatility variance, as is stated in equation 2. To test this condition, we calculate the variance and mean of daily variance for each month in the two sub-period, respectively. And for each month, we calculate the value of variance of return variance and divide it by the fourth power of the mean of return variance, which gives us the monthly value of  \( \frac{\sigma_i^2}{\sigma_i^4} \) in equation 2. Finally, we conduct a t-test to test the null
hypothesis that the population mean of $\frac{\sigma^2}{\sigma^2}$ in the earlier period is not larger than the later period. As is shown in Table 1, the large p-value implies that we can not reject the null hypothesis. So the model is not hurt by the larger volatility of returns in the later period.

The sample mean of returns is 0.00043 for the earlier sub-period and 0.00039 for the later sub-period, which are small enough to be treated as $o(1)$. And the sample realization of $|\frac{\mu}{\sigma^2}| = O(1)$ are 7.03 and 8.07 for the two periods, respectively, which can be reasonably treated as $O(1)$. In this way, assumptions 2) and 3) are satisfied in our data.

3.2 Test of Model Predictions

Figure 1 shows the trend of volatility during the whole sample period of 1970-2008. In the period before 1987, the trend is smoother than the period after that year. Two largest volatilities appear in 1987 and 2008, and several other large volatilities are clustered around 1997-2002. We calculate the sample variance of daily variance as $0.03*10^{-7}$ for period 1970-1986 and $0.65*10^{-7}$ for 1987-2008. An F test on the null hypothesis that the earlier period has a non-smaller volatility of variance provides a p-value of almost 0 and suggests a rejection of the null hypothesis that the volatility of daily variance are the same for the two sub-periods. Furthermore, the trend of monthly values of $A$ is plotted in Figure 2.\footnote{Empirically, we calculated the sample standard deviation of return variance divided by the sample mean of return for each month, and the value of $A$ in that month is calculated as the ratio of these two sample moments.} A t-test on the null hypothesis that the earlier period...
has non-smaller value of A yields a p-value of 0.01, providing statistical evidence that the value of A significantly has increased in the later period compared to the earlier one.

As predicted by Theorem 2.4, in a world where non-zero tax is imposed on short-term capital gains, a larger volatility of daily stock return variance in the second period implies bigger short-term losses and a similar level of short-term gains compared to the first period. Furthermore, Theorem 2.5 predicts that given a more volatile stock return variance, the ratio of short-term capital gains to total net capital gains is higher. As is shown in columns 1) and 2) of Table 3, we have the series of net long-term gains plus net short-term gains and the series of net long-term gains in excess of short-term losses for the whole period of 1970-2008. Considering the fact that the total amount of capital gains is highly correlated to the investors' initial wealth, we calculate the daily values of market index using value weighted market returns and use their values on the first transaction day in each year as a proxy for initial wealth. Based on the series of market index, we derive the market value adjusted version of columns 1) and 2) by dividing the raw numbers by the value of market index observed at the beginning of each year and get columns 3) and 4). Since the market value adjusted numbers eliminate the wealth effect, they are thus used for later empirical tests.\(^{12}\) Because the net short-term gains are the short-term gains in excess of the short-term loss, the difference between columns 3) and 4) gives us the short-term capital gains. If we assume that net long-term gains are constant over time, column 4) is essentially some constant minus short-term gains. Using data in columns 3) and 4), we conduct a t-test on the null hypotheses that the population mean of short-term losses, short-term gains and short-term gains as a proportion of total net gains as follows:

\(^{12}\)From now on, we refer to the market value adjusted values when we talk about various capital gains or losses.
are the same during the two sub-periods, 1970-1986 and 1987-2008, respectively. As is summarized in Table 2, the short-term losses significantly increase in the second sub-period (assuming constant net long-term gains), and so do the ratio of short-term gains to total net gains. While there is no statistical evidence that the short-term gains change between the two periods. All these facts are consistent with the predictions of Theorem 2.4 and Theorem 2.5.

Figure 3 shows the trend of short-term losses, short-term gains and the ratio of short-term gains to total net gains. After the year of 1987, there is an obvious rise in both the short-term losses (assuming a constant net long-term gains) and the ratio of short-term gains to total net gains. In the meanwhile, the short-term capital gains do not exhibit a significant difference before and after 1987.

3.3 The Impact of Taxes on Short-Term Gains

The tax rate for long-term capital gains is generally lower than that for short-term capital gains. However, this relative tax advantage of long-term capital gains is smaller during recent years. One argument is that it is the loss of tax advantage of long-term capital gains that induces the investors to realize more short-term capital gains, which further increases the ratio of short term capital gains to total net gains. However, there are two pieces of evidence that are against this argument. First, we show in Section 3.2 that the short-term capital losses increase in recent years, but the short term capital gains do not change significantly. Second, in the two sub-periods, the relative tax benefit of long-term capital gains is similar, while the ratio of short term capital gains to the total net gains is significantly different. Table 4 presents the tax rates for these two types of
capital gains and their ratio during our sample period of 1970-2008. We take the tax rate for long-term capital gains as the denominator in the ratio. Hence a larger ratio implies a loss of tax advantage of long-term capital gains. Figure 4 plots the trend of this ratio over time. One can see that although the relative tax advantage of long-term capital gains is smaller in the later period after 1978, there are two sub-periods that witness similar values of the ratio of these two types of tax rates.

We then test whether the ratio of short-term capital gains to the total net gains increases from the first sub-period to the second one regardless of the fact that the long-term capital gains do not lose its tax advantage in the second sub-period compared to the first one. Table 5 summarizes the results. Firstly, column 1 shows the testing result for the null hypothesis that the ratio of tax rate for long-term capital gains to that for short-term capital gains is not larger in the second sub-period compared to the first one. A p-value of 0.91 implies a failure to reject the null hypothesis and provides evidence that there is no loss of tax advantage of long-term capital gains in the second period. In the contrast, the volatility of stock return variance is significantly increased in the second period, with a p-value of almost 0 in column 2. As for the comparison of capital gains between the two sub-periods, one can see that there is a significant increase from the first sub-period to the second one for both the short-term losses and the ratio of short-term gains to the total net gains. Therefore, controlling for the relative tax benefit for the two types of capital gains, the predictions of Theorem 2.4 and Theorem 2.5 are still consistent with the real data.

4 Conclusion
Historically, the capital gains tax has attracted less scholarly attention than the dividend tax. And among the studies of capital gains tax, almost all of them have focused exclusively on the long-term capital gains tax. This paper fills the gap in the literature by examining whether the volatility of stock variance affects an investor's intertemporal decision of realizing capital gains or losses. We build a two-period two-economy portfolio-selection model and illustrate that higher volatility of stock return variance can lead to higher short-term capital losses realizations and higher trade volumes. Most prior studies have focused the impact of capital gains taxes on stock returns or trading volume. Our theoretical model demonstrates that real causation can run in the opposite direction - it is possible that the changes in the second moment of stock variance can affect intertemporal capital gains/losses realization decisions and trading volumes.
References


Feldstein M. J. Slemrod and S. Yitzhaki, 1980, The effects of taxation on the selling of


Proof of the Theorems

Proof of Theorem 2.1. Since in economy H, \( w_H(t) = (\mu)/\sigma^2_H(t) \) for \( t = 1, 2 \), then

\[
std(w_H(2) - w_H(1)) = (\mu)std\left(\frac{\sigma^2_H(1) - \sigma^2_H(2)}{\sigma^2_H(1)\sigma^2_H(2)}\right).
\]

To calculate \( STD_H \equiv std\left(\frac{\sigma^2_H(1) - \sigma^2_H(2)}{\sigma^2_H(1)\sigma^2_H(2)}\right) \), noticing that \( \sigma_H(t) \) has the two-point distribution and is independent over time, we have

\[
STD_H = \frac{A_H}{(1 - A^2_H)\bar{\sigma}^2}.
\]

Similarly,

\[
STD_L = \frac{A_L}{(1 - A^2_L)\bar{\sigma}^2}.
\]

Since \( \mu \) is constant and the same for both economies and \( A_H > A_L \), one has

\[
std(w_H(2) - w_H(1)) > std(w_L(2) - w_L(1)).
\]

If the mean of the variance is not the same for two economies but equation 2 holds, then one can define \( (\bar{\sigma} - \mu)^2 = \frac{\bar{\sigma}^2}{A_L} \) and \( A_H \) and \( A_L \) by following two equations respectively

\[
\frac{A_H\bar{\sigma}^2}{1 - A^2_H} = \frac{A_H}{1 - (A_H)^2} \quad \frac{1}{1 - A^2_L} = \frac{A_L}{1 - (A_L)^2}.
\]

one can show that \( A_H > A_L \) when equation 2 is satisfied. In this case, one can consider the economy H as a economy with the same mean of volatility \( (\bar{\sigma} - \mu)^2 \) with economy L, but with higher volatility of stock return variance(\( A_H > A_L \)).
Hence, by using the definition of \((\bar{\sigma})^2\), \(A_H\) and \(A_L\), one can show that

\[
|w_i(2) - w_i(1)| = \frac{\mu}{(\bar{\sigma})^2}\left(\frac{2A_i}{1-(A_i)^2}\right),
\]

where \(i = H\) or \(L\). Therefore, the proof follows as above since \(A_H > A_L\).

**Proof of Proposition 2.2.** If \(\sigma_i(2) \neq \sigma_i(1)\),

\[
\left| \frac{\mu}{\sigma_i^2(2)} \frac{1}{1+r_i(1)} - \frac{\mu}{\sigma_i^2(1)} \right| = \left| o(1) + 1 \left( \frac{\mu}{\bar{\sigma}} \left( \frac{1}{1+A_i} - \frac{1}{1-A_i} \right) \right) \right|,
\]

the equation holds since \(\left| \frac{\mu}{\bar{\sigma}} \right| = O(1)\), and \(A_H, A_L\) are constant.

**Proof of Theorem 2.3.** We consider four different cases, each of which occurs with the probability of \(\frac{1}{4}\). And \(i\) refers to the two types of economies, \(H\) or \(L\).

Case 1: Suppose \(\sigma_i^2(1) = \bar{\sigma}^2(1-A_i)\), and \(\sigma_i^2(2) = \bar{\sigma}^2(1+A_i)\). If \(r_i(1) = \mu + \bar{\sigma}\sqrt{1+A_i}\), then

\[
S_i(2) - S_i(1) = \left( \frac{\mu}{\sigma_i^2(2)} \frac{1}{1+r_i(1)} - \frac{\mu}{\sigma_i^2(1)} \right)(1+o(1))
\]

\[
= \frac{\mu}{\bar{\sigma}^2} \left( \frac{1}{1+\mu + \bar{\sigma}\sqrt{1+A_i}} \frac{1}{1+A_i} - \frac{1}{1-A_i} \right)(1+o(1)).
\]

Since \(\mu + \bar{\sigma}\sqrt{1+A_i} = o(1)\); hence, \(S_i(2) - S_i(1) < 0\) and the short-term gain is realized at time \(t = 2\), the amount is
\[ STG_i = -\frac{\mu}{\sigma^2}(\frac{1}{1 + \mu + \sigma^2} \frac{1}{1 + A_i} - \frac{1}{1 - A_i}) (1 + o(1)) (1 + \mu + \sigma \sqrt{1 + A_i} - 1). \]

Since \( A_H > A_L \), we have \( STG_H > STG_L \) because both

\[-\left( \frac{1}{1 + \mu + \sigma^2} \frac{1}{1 + A_i} - \frac{1}{1 - A_i} \right) \quad \text{and} \quad \mu + \sigma \sqrt{1 + A_i} \quad \text{increase as} \quad A_i \quad \text{increases}.\]

If the mean of the variance is not the same for two economy but equation 2 holds, since

\[ S_i(2) - S_i(1) = (\frac{\mu}{\sigma_i^2} (1 + r_i(1)) \frac{1}{1 + r_i(1)} - \frac{\mu}{\sigma_i^2} (1 + o(1)) \]

\[ = \frac{\mu}{\sigma^2} (\frac{1}{1 + \mu + \sigma^2} \frac{1}{1 + A_i} - \frac{1}{1 - A_i}) (1 + o(1)) \]

\[ = \frac{\mu}{\sigma^2} (\frac{1}{1 + A_i} - \frac{1}{1 - A_i}) (1 + o(1)), \]

redefine \( A_H', A_L' \) as equation 5, then

\[ S_i(2) - S_i(1) = \frac{\mu}{(\sigma')^2} (\frac{1}{1 + A_i} - \frac{1}{1 - A_i}) (1 + o(1)). \]

Since \( A_H' > A_L' \), one can still show that the short-term capital gain is higher with higher \( A_i \). Similarly, one can show that the results hold for all the subsequent analysis (including all the following theorems and propositions).

If \( r_i(1) = \mu - \sigma \sqrt{1 + A_i} < 0 \), then

\[ S_i(2) - S_i(1) = \frac{\mu}{\sigma^2} (\frac{1}{1 + \mu - \sigma \sqrt{1 + A_i}} \frac{1}{1 + A_i} - \frac{1}{1 - A_i}) (1 + o(1)) < 0. \]

In this case, there is a loss at time \( t = 2 \).
\[ STL_i = -\frac{\mu}{\sigma_i^2} \left( \frac{1}{1 + \mu - \sigma \sqrt{1 + A_i}} \right) \left( \frac{1}{1 + A_i} \right)(1 + o(1))(\mu - \sigma \sqrt{1 + A_i}). \]

Case 2: Suppose that \( \sigma_i^2(1) = \sigma^2(1 + A_i) \), and \( \sigma_i^2(2) = \sigma^2(1 - A_i) \). One can show that both short-term gain and loss are higher for economy H, following the same logic in case 1.

Case 3: Suppose that \( \sigma_i^2(1) = \sigma^2(1 + A_i) \), and \( \sigma_i^2(2) = \sigma^2(1 + A_i) \). Then

\[ S_i(2) - S_i(1) = \frac{\mu}{\sigma_i^2} \left( \frac{1}{1 + \mu + \sigma \sqrt{1 + A_i}} \right) \left( \frac{1}{1 + A_i} \right)(1 + o(1)) = o(1); \]

thus, the short-term gain and loss are small enough to be dominated by their values in Case 1 and Case 2.

Case 4: Suppose that \( \sigma_i^2(1) = \sigma^2(1 - A_i) \), and \( \sigma_i^2(2) = \sigma^2(1 - A_i) \). Similarly to Case 3, the short-term gain and loss are small enough to be dominated by their values in Case 1 and Case 2.

In sum, the expected short-term gain and loss (which are primarily determined by Case 1 and Case 2 are larger in economy H.

**Proof of Theorem 2.4.** The proof is similar to the proof of Theorem 2.3, and i here refers to the two types of economies, H and L.

In Case 1 stated in Theorem 2.3,

\[ S_i(2) - S_i(1) = \left( \frac{\mu}{\sigma_i^2(2)} - \frac{\mu}{\sigma_i^2(1)} \right)(1 + o(1)). \]

If \( r_i(1) = \mu + \sigma \sqrt{1 + A_i} \), there is short-term gain. The tax is zero for economy L, but \( t \)
for economy H. From equation 3, the investor in economy H is not better off if she adjusts weight by \( \frac{\mu}{\sigma^2_2} - \frac{\mu - 1}{\sigma^2_1} = \frac{\mu}{\sigma^2} \left( \frac{1}{1 - A_H} - \frac{1}{1 + A_H} \right) \). We assume that in this case, the investor only adjusts her weight in risky asset by \( \frac{\mu}{\sigma^2} \left( \frac{1}{1 - A_L} - \frac{1}{1 + A_L} \right) \). Since the difference in return between the two economies is \( o(1) \), we have

\[ STG_H = STG_L (1 + o(1)). \]

If \( r_1(1) = \mu - \sigma \sqrt{1 + A_i} \), there is short-term loss. Since there is no tax for both economies, one has \( STG_H > STG_L \) based on the same analysis in Theorem 2.3.

Similarly, one can show that for the case 2 stated in Theorem 2.3, one has

\[ STG_H = STG_L (1 + o(1)), STL_H > STL_L. \]

In Cases 3 and 4, short-term gain and loss are both small(\( o(1) \)). In sum, one has:

\[ E(STG_H) = E(STG_L)(1 + o(1)), E(STL_H) > E(STL_L). \]

**Proof of Theorem 2.5.** From Theorem 2.4, \( E(STG_H) = E(STG_L)(1 + o(1)) \), \( E(STL_H) > E(STL_L) \), it is clear that \( STR_H > STR_L \).
Table 1: Daily Stock Return and Daily Volatility in the Two Sub-Periods

<table>
<thead>
<tr>
<th>variable X</th>
<th>daily return</th>
<th>daily volatility</th>
<th>$\frac{A^2}{\sigma^2}$</th>
</tr>
</thead>
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<tr>
<td>sample mean for</td>
<td>0.00043</td>
<td>0.00007</td>
<td>1.59 * 10^7</td>
</tr>
<tr>
<td>1970-1986</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>sample mean for</td>
<td>0.00039</td>
<td>0.00013</td>
<td>1.86 * 10^7</td>
</tr>
<tr>
<td>1986-2008</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>null hypothesis</td>
<td>$X_1 = X_2$</td>
<td>$X_1 = X_2$</td>
<td>$X_1 \leq X_2$</td>
</tr>
<tr>
<td>p-value</td>
<td>0.85</td>
<td>0.00</td>
<td>0.83</td>
</tr>
</tbody>
</table>

Note:
1) X1 refers to the value of X during the sub-period before 1987, and X2 refers to the value of X during the sub-period after 1987.
2) A * denotes significance at 5% significance level.
Table 2: Capital Gains in the Two Sub-Periods

<table>
<thead>
<tr>
<th>variable X</th>
<th>NLTG</th>
<th>STG</th>
<th>STG / NTTLG</th>
</tr>
</thead>
<tbody>
<tr>
<td>sample mean for 1970-1986</td>
<td>37.32</td>
<td>1.25</td>
<td>0.03</td>
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<tr>
<td>sample mean for 1987-2008</td>
<td>15.00</td>
<td>1.32</td>
<td>0.08</td>
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<td>$X_1 = X_2$</td>
<td>$X_1 = X_2$</td>
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<tr>
<td>p-value</td>
<td>0.00*</td>
<td>0.70</td>
<td>0.00*</td>
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</table>

Note:
1) $X_1$ refers to the value of $X$ during the sub-period before 1987, and $X_2$ refers to the value of $X$ during the sub-period after 1987.
2) The four numbers on capital gains and losses are in billion dollars.
3) NLTG refers to net long-term gains; STG refers to the short-term gain; STG / NTTLG refers to the ratio of short-term gains to the total net gains.
4) A * denotes significance at 5% significance level.
Table 3: Total Capital Gain and long-term Capital Gain

<table>
<thead>
<tr>
<th>Year</th>
<th>1) NLTG+NSTG</th>
<th>2) NLTG-STG</th>
<th>3) ((NLTG + NSTG)_{adj})</th>
<th>4) ((NLTG - STG)_{adj})</th>
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<td>27.57</td>
<td>28.91</td>
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<td>31.35</td>
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<td>1973</td>
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<td>545.20</td>
<td>463.61</td>
<td>36.87</td>
<td>31.35</td>
</tr>
</tbody>
</table>

Note:
1) All the numbers are in billion dollars.
2) Column 1) refers to net long-term capital gains plus net short-term capital gains.
3) Column 2) refers to net long-term capital gains in access of short-term capital losses.
4) Columns 3) and 4) contain values in columns 1) and 2), respectively, divided by the values of market index observed in the first day of each year.
Table 4: Tax Rates for Long and Short-Term Capital Gain

<table>
<thead>
<tr>
<th>Year</th>
<th>1) ST</th>
<th>2) LT</th>
<th>3) LS Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1970</td>
<td>71.75</td>
<td>32.21</td>
<td>0.449</td>
</tr>
<tr>
<td>1971</td>
<td>70</td>
<td>34.25</td>
<td>0.489</td>
</tr>
<tr>
<td>1972</td>
<td>70</td>
<td>36.5</td>
<td>0.521</td>
</tr>
<tr>
<td>1973</td>
<td>70</td>
<td>36.5</td>
<td>0.521</td>
</tr>
<tr>
<td>1974</td>
<td>70</td>
<td>36.5</td>
<td>0.521</td>
</tr>
<tr>
<td>1975</td>
<td>70</td>
<td>36.5</td>
<td>0.521</td>
</tr>
<tr>
<td>1976</td>
<td>70</td>
<td>39.875</td>
<td>0.570</td>
</tr>
<tr>
<td>1977</td>
<td>70</td>
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<td>0.570</td>
</tr>
<tr>
<td>1979</td>
<td>70</td>
<td>28</td>
<td>0.400</td>
</tr>
<tr>
<td>1980</td>
<td>70</td>
<td>28</td>
<td>0.400</td>
</tr>
<tr>
<td>1981</td>
<td>69.125</td>
<td>28/20</td>
<td>0.347</td>
</tr>
<tr>
<td>1982</td>
<td>50</td>
<td>20</td>
<td>0.400</td>
</tr>
<tr>
<td>1983</td>
<td>50</td>
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<td>28</td>
<td>0.727</td>
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<tr>
<td>1988</td>
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<td>28</td>
<td>1.000</td>
</tr>
<tr>
<td>1989</td>
<td>28</td>
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<tr>
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<td>1993</td>
<td>39.6</td>
<td>29.19</td>
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<td>1994</td>
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<td>29.19</td>
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<tr>
<td>1997</td>
<td>39.6</td>
<td>29.19/21.19</td>
<td>0.635</td>
</tr>
<tr>
<td>1998</td>
<td>39.6</td>
<td>21.19</td>
<td>0.535</td>
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<tr>
<td>1999</td>
<td>39.6</td>
<td>21.19</td>
<td>0.535</td>
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<tr>
<td>2000</td>
<td>39.6</td>
<td>21.19</td>
<td>0.535</td>
</tr>
<tr>
<td>2001</td>
<td>39.1</td>
<td>21.17</td>
<td>0.541</td>
</tr>
<tr>
<td>2002</td>
<td>38.6</td>
<td>21.16</td>
<td>0.548</td>
</tr>
<tr>
<td>2003</td>
<td>35</td>
<td>21.05/16.05</td>
<td>0.529</td>
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<tr>
<td>2004</td>
<td>35</td>
<td>16.05</td>
<td>0.459</td>
</tr>
<tr>
<td>2005</td>
<td>35</td>
<td>16.05</td>
<td>0.459</td>
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<tr>
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<td>15.7</td>
<td>0.449</td>
</tr>
<tr>
<td>2008</td>
<td>35</td>
<td>15.7</td>
<td>0.449</td>
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</tbody>
</table>

Note: Column 1) and 2) refer to the tax rate for short-term capital gains and long-term capital gains, respectively. And column 3) refers to the ratio of column 2) to column 1).
Table 5: Tax Rates, Variance of Return Volatility and Capital Gains in the Two Sub-Periods

<table>
<thead>
<tr>
<th>variable X</th>
<th>LSRATIO</th>
<th>VOLofVOL</th>
<th>NLTG</th>
<th>STG / NTTLG</th>
</tr>
</thead>
<tbody>
<tr>
<td>1970-1978</td>
<td>0.526</td>
<td>4.7×10^-9</td>
<td>30.57</td>
<td>0.02</td>
</tr>
<tr>
<td>1998-2008</td>
<td>0.499</td>
<td>8.4×10^-8</td>
<td>13.59</td>
<td>0.08</td>
</tr>
<tr>
<td>null hypothesis</td>
<td>X1 ≥ X2</td>
<td>X1 ≥ X2</td>
<td>X1 = X2</td>
<td>X1 = X2</td>
</tr>
<tr>
<td>p-value</td>
<td>0.91</td>
<td>0.00*</td>
<td>0.00*</td>
<td>0.00*</td>
</tr>
</tbody>
</table>

Note:
1) X1 refers to the value of X during the sub-period of 1970-1978, and X2 refers to the value of X during the sub-period of 1998-2008.
2) LSRATIO refers to the sample mean of the ratio of tax rate for long-term gains to that for short-term gains in each sub-period.
3) VOLofVOL refers to the sample variance of equity return variance in each sub-period.
4) NLTG and STG / NTTLG refer to the sample mean of net long-term gain and the sample mean of the ratio of short-term gains to the total net gains in each sub-period, respectively, which are in billion dollars.
5) A * denotes significance at 5% significance level.
Figure 1: Trend of Daily Return Volatility
Figure 2: Trend of Monthly Values of A
Figure 3: Trend of long-term Gain in excess of short-term Loss, short-term Gain and short-term Gain / Total Net Gain
Figure 4: The Ratio of Tax Rate for long-term Capital Gain to That for short-term Capital Gain