When Auction Meets Fixed Price:

A Theoretical and Empirical Examination of Buy-it-Now Auctions

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Abstract

Recently fixed pricing and auctions have been brought together in a new pricing format that offers bidders the option of prematurely ending an auction at a fixed price. The growing popularity of auctions presents an interesting pricing decision for managers: whether to sell at a fixed price, in a regular auction, or through a buy-it-now auction. This paper studies eBay's buy-it-now auction and answers the following research questions: why is fixed price used at traditional auctions, will buy-it-now increase the seller's profit, how is an optimal price determined, and how is the buy-it-now decision influenced by key factors such as the customer's cost of participating in the auction, the seller's reserve price, and the number of potential customers. Our results show that when customers make endogenous participation decisions according to their participation costs, buy-it-now auctions can increase both customers' utility and sellers' profit. Endogenous participation has important implications for seller's pricing decisions such as price formats and levels. Depending on the level of the posted price, the resulting price format could be either fixed price, buy-it-now auction or pure auction. Therefore, the seller needs to be careful and take into account market conditions when posting a price at auctions. We empirically test the model assumptions and predictions using data collected from eBay.

Key Words: Auctions, Pricing Research, Game Theory

JEL Classification: **D44, M30**

1 Introduction

Auctions are quintessentially a variable pricing format, but recently the largest online auction sites have introduced a fixed-pricing element known as a buy-it-now option. This feature allows a bidder to immediately stop an ongoing auction and purchase the item at a fixed, posted price set by the seller. This feature is known as "Buy-it-Now", "uBuy", and "Buy-Now" at eBay, uBid, and Yahoo, respectively. For clarity we refer to all these services generically as buy-it-now. These buy-it-now features have become an important component of online auctions. eBay introduced its service in 2000. Subsequently it has been adopted by 45% of eBay's U.S. auctions by the end of its first year (ZDNet, 2002). These buy-it-now auctions accounted for 28% of gross merchandise sales at eBay by the end of 2003 (eBay, 2003). Empirical work by Park and Bradlow (2005) also indicates that the buy-it-now feature is an important element in auction design.

Economists advocate the efficiency of auctions over fixed price markets when auction costs are low (Wang 1993). The principle being that auctions match products to customers with the highest valuation in the market. However, the buy-it-now feature effectively limits the maximum auction price which would seem to lower the profitability of auctions for sellers. This leads to several perplexing problems: why would sellers choose to adopt buy-it-now auctions, when should they be used, and how should an optimal buy-it-now price be set? To answer these questions we develop an analytical model of auctions. Our results show that the buy-it-now option can actually increase the expected profits from an auction if customer participation costs are substantial. As we show in this research, customer endogenous participation due to costs not only plays a crucial role in determining the outcome of buy-it-now auctions, but also has important pricing and profitability implications.

Our model considers a potential customer's optimal bidding strategy, willingness to pay, probability of winning, and the costs of participating in an auction. We believe the participation and transaction costs borne by the consumer are critical in explaining consumer behavior concerning online auctions, and we consider costs that are born during the auction and those associated with completing the transaction. The customer's participation during the bidding process include direct time costs, opportunity costs associated with waiting for the auction to close, transaction costs associated with bidding, and the customer's cognitive effort expended in the bidding process. Our conjecture is that the cognitive effort expended by a customer in deciding whether to accept a buy-it-now offer is less taxing than a customer who must decide upon a bid. Additionally after the close of the auction the customer may incur additional transaction costs to complete the auction order, such as the time involved in paying for the order and may incur opportunity costs associated with waiting for delivery.

The importance of these participation costs is that they can lower the number of bidders, which in turn reduce the seller's profit. The buy-it-now option reduces participation costs for buyers and attracts buyers who might otherwise bypass the auction. To illustrate this problem consider a customer bidding for an ideal gift for her friend's birthday coming up in one week. The buy-it-now auction presents two options: to bid and wait for the auction to close in three days to find out if she will win, or to purchase at the posted price right away (provided her valuation is higher than the price). If she chooses to bid, then there is a chance that she may not win, in which case she has to start over and bid again or buy at another store. In this example the auction participation cost borne by the customer includes the opportunity cost associated with waiting, checking emails to see if she has been outbid, potentially having to pay an extra charge for quick shipping if she loses and must buy elsewhere, and the possibility of not having a gift on time for her friend's birthday.

Participation costs can include the opportunity costs associated with not winning but the role it plays is different from that of impatience and risk averse. Conceptually transaction costs, impatience and risk aversion can be three independent attributes describing a customer's intrinsic characteristics. A very impatient customer can have a very low transaction cost, and a high-transaction cost customer can be risk seeking. What differentiates transaction costs from patience and risk aversion is that transaction costs lead to endogenous participation decisions, while impatience and risk aversion assume that customers still participate; only their utility function is discounted either by time or a risk parameter. Therefore we consider the auction participation cost is a more general concept and believe that is motivated by behavioral considerations.

Our model indicates that the buy-it-now option allows the auctioneer to lower the transaction costs to potential buyers, which increases the potential selling price. Hence, buy-it-now options are most valuable when a seller faces a customer base with participation costs. The use of buy-it-now auctions also helps address the question of when sellers should use auctions versus fixed, posted price formats found in most retail contexts. An interesting finding from examining the implications of auction participation cost on bidder behavior and seller's profit is that posting a price at the auction could potentially lead to any of the three drastically different price formats: a fixed price, a buy-it-now auction, or a regular auction. Which format would eventually take place depends on the price level. If sellers set a buy-it-now price that is ridiculously high then these formats are immaterial since the buy-it-now option is never exercised. If the buy-it-now price is set too low then consumers never purchase through a bidding process, but instead always exercise the buy-it-now option. Additionally, the simplicity of the fixed, posted price found in most retail environments may generate higher profits than the hybrid auction format which buy-it-now pricing affords. We characterize the price range for each of the price formats and broaden the pricing decision at online auctions: the seller needs to consider all three possible scenarios and choose the price *format* and *level* that lead to the optimal profit. We also consider two model extensions: a positive buy-it-now cost and heterogeneous participation costs. Both generalize the theoretical model and provide additional useful insights.

In order to test key aspects of our theory we conduct an empirical analysis using real auction data from eBay. We conduct our analysis using two methods. First, we examine the adoption of buy-it-now auctions by sellers at an aggregate level and see if their behavior is consistent with our theoretical predictions. Second, we test the assumptions of our consumers' equilibrium strategies using a choice model. We predict which auction a consumer chooses given the set available at the time they made their bid. Auction choices reveal the trade-offs customers make between different auction formats and attributes.

Our paper contributes to both the existing auction and marketing literatures (we refer interested readers to Pinker et al 2003 for a review of existing research on choosing between the fixed price format and the auction). To our knowledge this work is the first to incorporate participation or transaction costs to explain the use of buy-it-now auctions and additionally to empirically test such a theory. In the past, consumer reactions to marketing elements at auctions have been largely ignored (Chakravarti, et al. 2002). Although consumers' limited-information acquisition-ability in a traditional retail setting has gained considerable acceptance (e.g. Ratchford 1982; Mehta, Rajiv and Srinivasan, 2003), in marketing literature consumer uncertainty about the surplus associated with the choice of bidding or a buy-it-now purchase has not been studied. This paper not only sheds light on the trade-off faced by consumers but also the profit-maximizing pricing strategy for sellers who use auctions as a channel to sell their products.

2 Background

Formal research on buy-it-now auctions has just begun to understand the reasons why sellers post prices at auctions. From a bidder's point of view, Mathews (2003) argues that the buy-it-now option is beneficial to impatient bidders who discount future utility. Impatience refers to the notion that customers want their product in a shorter span of time than others. Although an important factor, impatience alone does not reflect the limited cognitive resources that most consumers appear to apply in making decisions (Ratchford 1982, Mehta, Rajiv and Srinivasan 2003). Budish and Takeyama (2001) investigate a two-bidder model and show that if the customers are risk averse then the seller has an incentive to use a buy-it-now option. More recently Kirkegaard and Overgaard (2007) consider a model in which sellers each offer one item for auction in a sequential manner. They find that the seller has incentive to use the buy-it-now price in early auctions if the bidder has multi-unit demand and more similar items will be offered for sale in the future by other sellers.

From a seller's perspective, Hidvégi, Wang and Whinston (2006) study the bidding strategies at ascending English auctions with buy-it-now options, and conclude that risk-averse sellers should prefer setting buy-it-now prices at auctions while risk-neutral sellers should be indifferent between a regular auction and an auction with a buy-it-now option when the price is appropriately set. Although some auctioneers could be risk averse, many large retailers that have opened eBay stores, like Dell, IBM, Sun Microsystems, and Sony are unlikely to be risk averse. Hence, we feel that risk aversion is not a universal explanation. Zeithammer and Liu (2006) study why sellers use pure auction or the posted price selling (fixed price and buy-it-now auctions combined) analyzing eBay's camera category data. The authors find seller's heterogeneity such as the inventory mix plays an important role.

Additionally, recent online auction studies have empirically and experimentally documented such transaction costs that are an important component of our participation costs. Bajari and Hortascu (2003) empirically study common-value auctions at eBay for mint and proof sets of U.S. coins. They found the average cost of bidding in those categories to be $3.20 (with a standard error of $1.48, and a mean book value of $50.10). Additionally, in a field experiment for nearly $10,000 worth of sports cards, List and Lucking-Reiley (2002) found that cognitive costs influence subjects' strategic behavior. Hence, we believe participation costs should be reflected in a model of bidder behavior, which contrasts with past work on buy-it-now auctions which has focused on risk aversion and impatience as potential explanations.

Previous theoretical research has assumed full and/or exogenous participation of bidders when examining buy-it-now auctions (Budish and Takeyama 2001, Mathews 2003). What differentiates our work from these previous ones is that we consider bidder's participation cost as endogenous. We believe this modeling feature not only makes behavioral sense but also offers new insights on seller's profit implications of using this new dynamic pricing model. Specifically, our model shows that depending on the price level posted at the auction, a regular auction, a buy-it-now auction and a fixed price format could all emerge as the resulting price format. Our research determines the price range in which each of these formats could take place. Therefore, we take a broader view of pricing decisions including both the price format and price level.

Our model begins by endogenizing the participation decision, which leads to a participation threshold. Additionally, our work incorporates a buy-threshold. Definition and derivation of the two thresholds are discussed in Section 3.1. Compared to the conditional strategies proposed by prior research,[1] we believe our model better reflects consumer behavior and more realistically models typical online auction formats such as eBay. For example, the bidding strategy we derive in the next section allows customers to arrive sequentially and make a one-time decision upon arrival.

3 A Theoretical Model

In this section, we develop a theoretical model to study the strategies of the seller and customers at a buy-it-now auction. Our model is set up as a two-stage sequential game

(Figure 1). In the first stage, the seller decides whether to incorporate a buy-it-now option with knowledge about the number of potential customers (n), an estimate of the customers' bidding costs (c), and the seller's own reservation price for the product (r). In the second stage, the customers observe the seller's decision, and make decisions about participation and purchase.

[----- Figure 1 About Here -----]

To clarify our terminology, we use bidding to imply that a customer has chosen not to exercise a buy-it-now option but to participate in the usual auction process. If at the close of the auction the bidder has placed the high bid, that winning bidder becomes a buyer, since there is only one product there is only one buyer. If a customer exercises a buy-it-now option then the customer becomes a buyer and buys the product at the seller's stated buy-it-now price. The assumptions we make in this model are stated as follows.

Assumption 1: Auction Setup. We assume a single-period auction for a risk-neutral seller who uses a second-price sealed bid auction to sell one item. This follows the convention in recent online auction literature, for example, Bajari and Hortacsu (2003) show that bidding at eBay's regular auction can be regarded equivalent as bidding in a second price sealed bid auction. We assume that the seller owns the item and exogenously sets the reservation price. Without a loss of generality, we assume that seller's reservation price is zero in the following analysis. A more detailed discussion on the seller's positive reserve price, both open and secret is in Section 3.2 of this paper. Additionally, we assume that once a bid has been placed that exceeds the reservation price the buy-it-now option disappears or effectively the auction is converted back to a regular auction. This follows eBay's implementation of the buy-it-now auction.

The seller's decision is whether to incorporate a buy-it-now option into a traditional second-price sealed bid auction. There are n potential customers in the market who are aware of the auction and may be interested in purchasing the item, where $n \geq 2$.[2] We assume that this is a private-value auction in that each customer knows their own valuation for the auctioned item perfectly, but only knows the distribution from which the other customers' valuations are drawn. For simplicity, we assume that these valuations are independently and identically drawn from a uniform distribution on the interval [0,1].[3] We denote consumer *i*'s valuation as v_i, hence, v_i ~Uniform[0,1]. In summary, the common knowledge of customers includes the number of potential customers at the auction, the customers' valuation distribution, the participation cost, and the seller's reservation price posted at the auction.

Assumption 2: Customers' auction participation cost at auctions. We assume that there are transaction costs borne by the customer and refer to these as participation costs. There are two types of participation costs: those experienced during the bidding process and a second set incurred if the buy-it-now option is exercised. The first set of participation costs, represented by *c*, refer to the opportunity cost associated with waiting for the auction to close, transaction costs associated with bidding, and the customer's cognitive effort and angst expended in the bidding process. Later in section 3.4 we allow customers to have heterogeneous participation costs, c_i ~Uniform[0,1]. The second type of participation costs, represented by *k*, refers to the costs incurred by the customer in exercising the buy-it-now option. For clarity we will refer to these latter participation costs as completion costs. The costs *c* and *k* is assumed to fall between the upper and lower bounds of the valuation distribution, i.e. $c, k \in [0,1)$. This is a mild assumption since we assume that after incurring the participation cost a customer can still derive some positive surplus. It is a trivial case when the auction participation cost is higher than the upper bound of the valuation distribution, because no one will participate in the auction. A customer only incurs their participation cost (*c*) upon submitting a bid to the auction. This contrasts with the entry cost present in other auction research which requires a customer to pay a fee to know their valuation. We believe this assumption better represents the online auctions observed in practice. Additionally, the customer only incurs the completion cost (*k*) when the buy-it-now option is exercised.

Assumption 3: Customer's purchase decisions are endogenously made. Because customers incur a cost when they engage in bidding, it is natural to assume that they follow a rule when making "bid" or "buy" decisions. We assume that they solve for "cutoff points" or thresholds internally by calculating the expected utility for each choices and act optimally given their own valuations. Specifically, we solve for two thresholds, the "participation threshold", s_a, and the "buy threshold" s_b, which will be discussed further in the following section.

3.1 Consumers' Bidding Strategy

We use backward induction to solve this game. The first problem that needs to be solved is the bidders' optimal bidding strategy in the second stage. For tractability, we limit our attention to pure strategies. Additionally, for clarity in our presentation we first consider the case without completion costs (*k*=0), and then relax this assumption in section 3.3.

We begin by studying a customer's decision process when she arrives at a regular auction. We index the customer by subscript *i*. Before the auction ends, the customer has the option of submitting a bid. In a private-value second price sealed bid auction, the dominant bidding strategy is to submit one's true valuation conditional on participation (Vickrey, 1961; Milgrom and Weber, 1982). If customer *i* participates (by submitting a bid) and wins then her utility is the surplus (excess value above second highest bid) less the auction participation cost *c*. If the customer participates and loses then her utility is decreased by the auction participation cost *c*:

(1) $$U_i = (v_i - t)^+ - c,$$

where $(x)^+$ refers to the positive part of the *x*, *t* denotes the second highest bid among all bidders.

However, because of the participation cost *c*, some bidders, particularly those with low-valuations, would face a negative utility if they participate. The bidding strategy is relevant only when the bidder participates. Hence bidders' participation strategy needs to be characterized first. We define the participation threshold (s_a) as the valuation at which the customer is indifferent between bidding and not bidding. In the following analysis, the "participation-threshold" is used to refer to the borderline valuation, above which a customer will participate in the bidding process (or the expected utility of bidding exceeds zero). An equivalent way of understanding this threshold is to consider when a valuation equals s_a, a customer can win the item only if she is the only bidder (Riley and Samuelson 1981, Samuelson 1985, Menezes and Monterio 2000). In general, a customer's expected utility of bidding given the number of bidders is

(2) $$EU_i(v_i) = v_i[F(s_a)]^{n-1} + (n-1)\int_{s_a}^{v_i}(v_i - z)[F(z)]^{n-2} f(z)dz - c\ .$$

After integrating by parts, (2) becomes

(3) $$EU_i(v_i) = s_a[F(s_a)]^{n-1} + \int_{s_a}^{v_i}[F(x)]^{n-1} dx - c\ ,$$

where *F(.)* denotes the cumulative distribution function of the valuations. The first part of equation (2) represents the case when all other bidders' values are less than the threshold s_a. In other words, this is the case where the customer with a value equal to the threshold wins the auction. The second part captures the more general case, the expected gain from bidding when a customer is facing at least one other opponent whose valuations is above the threshold s_a. Specifically $[F(z)]^{n-2}$ represents the probability that n-2 bidders have a valuation lower than the second price; and the integration calculates the expected surplus of bidding v_i when the second price is z (a random variable with a support on $[s_a, v_i]$). The participation threshold by definition solves the zero-profit condition defined by equation (3):

(4) $$EU_i(v_i = s_a) = 0$$

Under the uniform assumption we made on customers' valuations, this reduces to

(5) $$s_a = c^{1/n}\ .$$

If upon arrival the buy-it-now option is available, the bidder may execute this option and immediately purchase the item for the stated price. The buy-option is available if no other customers have submitted a bid or have already executed the buy-it-now option. When the buy-it-now option is present, a customer must *ex ante* evaluate both options before making the purchase decision. She has to evaluate the expected gains from both the "buy" and "bid" options. In this case, a buy-threshold s_b defines the valuation at which the customer is indifferent between the two options, conditional on participation. Therefore, the buy-threshold solves the following equation:

(6) $$EU_i^{bid}(v_i = s_b) = EU_i^{buy}(v_i = s_b)\ .$$

Again using the uniform assumption and the results of the participation threshold ($s_a = c^{1/n}$) the expected utility of submitting a bid for an eligible bidder with a valuation $v_i > s_a$ simplifies to:

(7) $$EU_i^{bid}(v_i) = \frac{v_i^n - c}{n}\ .$$

It is straightforward to see that the expected utility from purchasing the product directly at the buy-it-now price B for this bidder is

(8) $$EU_i^{buy}(v_i) = v_i - B\ .$$

Therefore the buy-threshold defined in (6) solves

(9) $$\frac{s_b^n - c}{n} = s_b - B$$

Lemma 1 *The consumer's expected surplus curves for buying and bidding intersect at most once (in the non-negative domain).*

[----- Figure 2 About Here -----]

Note that the expected bidding surplus is convexly increasing with a slope less than or equal to 1.

(10) $$\partial \frac{v_i^n - c}{n} / \partial v_i = v_i^{n-1} \le 1$$

The buying surplus is a straight line with the slope being exactly equal to one. Figure 2 plots customer's expected surplus from bidding (the dashed line) and buying (the straight line) over the range of possible valuations. The intersection of the bidding surplus curve and the zero-utility line (the horizontal line) defines the participation threshold, and the intersection of the utility of bidding and buying curves indicates the buy-threshold. The above analysis on the thresholds and Figure 2 are useful in understanding consumer behavior at an auction with a fixed price. The position of these two curves guides the optimal action that a customer should take: to bid or to exercise the buy-it-now option. The customer's optimal strategy in the buy-it-now auction is a two-threshold strategy, summarized in the following theorem:

Theorem 1 *In a buy-it-now auction, the following symmetric equilibrium exists for the potential bidders, which involves two thresholds, the buy-threshold (s_b) and the participation-threshold (s_a):*

(1a). If $B > s_a$: A customer with a valuation $v_i \in [s_b, 1]$ will exercise the buy-it-now option and buy at the posted price upon arrival. If $v_i \in [s_a, s_b)$ then the customer will place a bid. The customer will leave the auction without submitting a bid if $v_i \in [0, s_a)$.

(1b). If $B < s_a$: A customer with a valuation $v_i \in [B, s_a]$ will exercise the buy-it-now option and buy at the price upon arrival. The customer will leave the auction without submitting a bid if $v_i \in [0, B)$.

(2). When the buy-it-now price is not available: the auction becomes a standard second-price sealed bid auction, in which the optimal bidding strategy for a customer is to bid their true valuation conditional upon their participation (e.g. Vickrey, 1961; Milgrom and Weber, 1982).

Theorem 1 prescribes the optimal bidding strategy in a buy-it-now auction. In section 3.2 we discuss the conditions in which the buy-it-now auction materializes. In summary, each customer utilizes a rational expectation to anticipate the strategies of other customers and computes *ex ante* utilities from the available options.
The two thresholds in (5) and (9) offer the following insights:

Proposition 1 *At a buy-it-now auction, the participation-threshold s_a increases in the auction participation cost c and the total number of potential bidders n. The buy-threshold s_b, however, decreases in n and the auction participation cost c. It increases in the posted buy-it-now price B.*

Ceteris paribus, as the auction participation cost increases, it becomes more costly to bid and the buy-it-now option becomes more attractive. For illustration purposes, in both plots in Figure 3 the buy-it-now price is set at 0.7. Figure 3(a) illustrates that when participation costs increase from 0.1 to 0.4, the participation threshold increases from 0.63 to 0.83. Notice that if $c=0.4$ and the buy-it-now price is 0.7, then the customer will not bid but instead will either exercise the buy-it-now option and buy outright or not buy at all. Similarly, as the number of potential customers increases (in Figure 3(b), n increases from 5 to 8) competition in the bidding process intensifies. As a result, the participation threshold required for bidding increases, and fewer potential customers can afford to bid (s_a increases from 0.63 to 0.75). In both examples, the buy-threshold reduces to the price set by the seller.

[----- Figure 3 About Here -----]

3.2 The Seller's Profit Maximization Problem

Now we consider the game's first stage: the seller's decision about whether to include a buy-it-now option and if so what price. The seller's decision is based upon the customers' strategy discussed in the previous section. Additionally, we assume that if the seller selects a buy-it-now auction, then this option disappears after the first bidder. This mirrors eBay's implementation of a buy-it-now format, since we assume that the seller's reserve is equal to 0.

Another insight from Figure 3 is that various pricing strategies can lead to different action spaces for customers. The seller not only needs to pick a buy-it-now price, but also needs to consider the resulting selling format. For example, if the buy-it-now price is set lower than the participation threshold, then the "bid" option disappears from a customer's decision space. This happens since a customer with a valuation greater than the participation threshold would not want to bid because the potential gain from bidding never exceeds directly exercising the buy-it-now option. For customers with a valuation lower than the posted price, they will not buy since this generates negative surplus. Bidding is not an option either since it requires an even higher threshold. Therefore, a price set lower than the participation threshold will result in a "pure" fixed-post format similar to the usual retail environment. If on the other hand the buy-it-now price is set too high then bidding is preferred.

A buy-it-now auction only occurs when customers perceive a chance for the auction to change format, namely, a chance for bidding as well as the opportunity to exercise a buy-it-now option. These cases are shown in Figure 4. The seller gets an expected revenue equal to that of a regular auction when posting a price $B > p_2$, and a revenue of the standard fixed-price, if $B \le p_1$. The revenue associated with the buy-it-now price is achieved when $p_1 < B < p_2$. According to Lemma 1 consumer surplus, the expected surplus curves of purchasing outright from the exercise of a buy-it-now option and bidding intersect at most once. Therefore, we can solve for the bounds on the pricing solution to identify when a buy-it-now auction occurs:

(11) $$p_1 = c^{1/n}$$

(12) $$p_2 = 1 - \frac{1-c}{n}$$

[----- Figure 4 About Here -----]

The above analysis suggests that posting a fixed price can result in different selling formats, hence different expected revenue functions. The seller needs to consider the impact of the selected price on the auction format and adopts the price and format that yields the highest profit. Now we consider each of these three formats and determine the optimal pricing decision for the seller. As we noted earlier, any buy-it-now price lower than the participation-threshold, i.e. $B < c^{1/n}$, yields a pure, fixed-price strategy. Furthermore if $B > 1 - \frac{1-c}{n}$, the price posted at the auction has no effect since bidding gives customers higher expected utility than buying for all possible valuations. Consequently, the buy-it-now auction turns into a regular auction. Therefore, the seller's expected revenue is a step function:

(13) $$\pi = \begin{cases} \pi_c & B \in [0, c^{1/n}] \\ \pi_b & B \in (c^{1/n}, 1 - \frac{1-c}{n}] \\ \pi_a & B \in (1 - \frac{1-c}{n}, 1] \end{cases}$$

We denote π_c, π_b and π_a as the expected revenue from fixed-price, a buy-it-now auction, and a regular auction respectively. We derive the profitability of each type as follows.

The expected revenue of a monopoly seller facing n customers in a fixed-price setting is

(14) $$\pi_c = B(1 - B^n).$$

This function is concave and has a unique optimum. The expected revenue of a regular auction with endogenous participation equals:

(15) $$\pi_a = n(n-1)\int_{s_a}^{1} v(1 - F(v))[F(v)]^{n-2} f(v)dv$$

If customers' valuations follow a uniform distribution this simplifies to:

(16) $$\pi_a = \frac{(n-1)[1-(n+1)c + nc^{(n+1)/n}]}{n+1}$$

Next we examine the revenue from a buy-it-now auction. Unlike in a regular auction, where the bidder's arrival sequence does not alter the bidding strategy or the seller's expected profit, the bidder's arrival sequence is of vital importance in a buy-it-now auction because after the reserve price is met the buy-it-now option disappears. A customer's action potentially alters the format of the auction. For instance, by submitting a bid at an auction that the seller initially offered with a buy-it-now auction, the auction is converted into a regular auction. Alternatively if the consumer exercises the buy-it-now option then the auction ends and is no longer available to other customers. As a result, whether the buy-it-now auction will end with a purchase at the fixed price or proceed as regular auction, namely the realization of the price format of a buy-it-now auction depends on the arrival sequence of bidders, particularly the first bidder's valuation.

Therefore, to obtain the seller's expected profit we need to condition upon the first customer's action.[4] Customers are indexed as $i=1, 2, \ldots, n$. If the first customer quits the auction without doing anything, i.e. $v_1 < s_a$, then the second customer faces the same decision: to buy at the buy-it-now price, to bid, or to not participate. Note that $v_1 < s_a$ is a sufficient condition for the first customer to choose not to participate. The price B must be higher than the participation threshold in order for buy-it-now auction to occur. This process continues until the auction receives a customer who either submits a bid or ends the auction by exercising the buy-it-now option, or the auction ends without receiving any bids from any of the *n* potential customers. Knowing this process, the seller can compute expected revenue for each customer conditional upon the previous one. We denote V_i as the seller's expected revenue starting from customer *i*. His *ex ante* expected revenue from a buy-it-now auction π_b is just V_1. The recursive nature of the process means that the seller anticipates expected revenue $\{V_1, V_1, ..., V_n\}$ from the customer $\{1, 2, ..., n\}$, where

(17)
$$\begin{aligned} V_1 &= s_a V_2 + (s_b - s_a)\pi_a + (1 - s_b)B \\ V_2 &= s_a V_3 + (s_b - s_a)\pi_a + (1 - s_b)B \\ &\vdots \qquad\qquad \vdots \\ V_n &= s_a \cdot 0 + (s_b - s_a)\pi_a + (1 - s_b)B \end{aligned}$$

Solving (17) recursively gives the seller's *ex ante* buy-it-now auction revenue:

(18)
$$\pi_b = V_1 = (1 - c)/(1 - s_a)[\ (s_b - s_a)\pi_a + (1 - s_b)B\ ]$$

The seller's problem is to choose an optimal buy-it-now price B^* to maximize total expected revenues:

(19)
$$B^* = \arg\max\left(\max_{B < p_1} \pi_c,\ \max_{p_1 \le B < p_2} \pi_b,\ \max_{B > p_2} \pi_a \right) .$$

In order to decide on the optimal buy-it-now price, the seller first computes the optimal price in each of the three intervals of *B*: fixed-price interval [$0, p_1$], buy-it-now auction interval (p_1, p_2), and auction-only interval [$p_2, 1$]. Note that in the auction-only interval the expected revenue π_a is a constant, and no price level has to be determined. Then the seller picks the pricing decision that yields the greatest expected revenue from the three. Figure 5 shows an example (n=4, c=0.02) in which the buy-it-now price is set at 0.68 and achieves the optimal expected revenue for the seller. As bidding becomes more costly, the seller's profit from using pure auction will decrease because of customers' endogenous participation, and offering a buy-option becomes more appealing to the seller.

By using a buy-it-now price, the seller can recoup part of the revenue loss due to customer's participation cost by allowing customers purchasing directly. Consequently, consumers extract higher surplus from auction, and the seller gets higher expected profit.

[----- Figure 5 About Here -----]

A numerical analysis showing the impact of the buy-it-now price on seller's expected revenue is presented in the Appendix of this paper. The results show that for most cases when participation cost is low, the buy-it-now auction generates higher expected revenue than fixed price selling because of the efficiency of the auction in extracting customer surplus. When the participation cost is high, the seller can strategically set the buy-it-now price to get expected revenue as high as that from fixed price selling most of the time. It hence protects the seller from getting low auction revenue. An interesting result can occur when the number of bidders is small and costs are null. Namely, that the hybrid format of the buy-it-now auction is superior to the traditional auction. The reason is that the traditional auction does poorly in markets with few buyers, while posted prices (and the hybrid buy-it-now auction) do quite well. (We discuss this further in the Appendix, see especially the discussion around Table A1.1).

It is worth noting that the outcome of an optimal strategy for the seller could be a fixed-price selling format. Different prices lead to different corresponding selling formats, and different revenue levels. Therefore, the seller needs to pick the optimal price based on the market situation (the size of the customer pool and the participation cost of bidding) that maximizes his expected revenue, and set the corresponding buy-it-now price or fixed-price as appropriate. Table 1 outlines the seller's pricing options and the conditions under which each would be implemented.

[----- Table 1 About Here -----]

Due to the dynamic nature of the buy-it-now auction, i.e. the auction format is determined by the action of the first bidder (which terminates the buy-it-now option) or the first buyer (which ends the auction), it is useful to study the properties of the expected revenue function over the corresponding range, (p_1, p_2), and its relationship to the parameters of the model.

Proposition 2 *The expected revenue of a buy-it-now auction is concave and has a unique optimum in the buy-it-now price B.*

Maximizing the expected profit defined in (18), we obtain the optimal buy-it-now price, denoted as B^*:

(20) $$B^* = \frac{2+\pi_a}{3} - \frac{2(1-c)}{3n} < 1 \quad .$$

The optimal buy-it-now price is less than the upper bound of customer valuation distribution. If the price is set too high, the buy-it-now auction would be converted to a regular auction. This could lead to a less desirable revenue outcome, especially when bidding is costly.

Proposition 3 T*he optimal buy-it-now price increases in the number of potential customers n.*

A comparative static analysis on the optimal buy-it-now price with respect to n shows that as the number of potential customers increases then the optimum buy-it-now price increases. The intuition is that as demand increases, it is less likely that the product will go unsold and the seller can potentially gain more profit from an auction than selling it at a fixed price. Hence the seller relies less on the buy-it-now option to achieve the optimal revenue. Nonetheless, when n increases, a customer's buy-threshold decreases--holding everything else constant--the customer is more likely to buy. To prevent the item being sold at a lower price than it necessary, the seller charges a higher buy-it-now price.

Proposition 4 *As the auction participation cost of bidders (c) increases, it is not always better for the seller to increase the buy-it-now price.*

The increase in the auction participation cost has two effects: it raises the participation threshold and decreases the buy-threshold for potential customers. By posting a fixed-price at the auction, the seller can increase the chance of a sale knowing that the expected revenue from selling in the standard auction format is low. This intuition is that the high auction participation cost directly reduces the seller's expected auction revenue. Lowering the buy-it-now price reduces the chance that a buy-it-now auction is turned into a regular auction by the first bidder which leads to less expected profit.

Proposition 5 *The optimal buy-it-now price decreases in the seller's reserve.*

In the above analysis, we have assumed a zero-reserve price for the auctioned good. Although many of the sellers at eBay do not employ a reserve price in order to encourage participation, the use of a reserve price is a common practice for those sellers who want to ensure that their winning bid meets a certain level. For instance a seller who wishes to cover their acquisition cost of the auctioned item. Now we examine how the reserve price influences the seller's decision regarding the buy-it-now price.[5] If the seller uses a reserve price at the auction, then he maintains the right not to sell the item if the highest bid is lower than the reserve price. At eBay, a seller can set a reserve price in two ways: use a starting bid or a secret reserve. The starting bid is equivalent to the open reserve studied extensively in the traditional auction theory. It is "open" because it is observable to all the potential bidders. Alternatively, the seller can use a secret reserve price. The amount of the secret reserve is not revealed to the public. However, the bidders know that there is a secret reserve need to be met in order to be a winner. At eBay, the status of the reserve being met or not is public. This reserve option has just begun to receive research attention (e.g. Katkar and Lucking-Reily, 2001; Bajari, and Hortascu, 2003).

First consider the open reserve case. Following the existing auction literature, if the auction participation cost of bidding is zero, a bidder's weakly dominant strategy in an auction with a positive starting bid (an open reserve) is to bid her true valuation if it is higher than the reserve price. If however, the seller's reserve is strictly positive, then in order to participate in the bidding process, not only must the bidder's valuation be greater than the reserve price, but she also needs to make sure that after paying for the auction participation cost that the expected gain from bidding remains positive. Therefore, when the open reserve is strictly positive, the participation threshold becomes $s_a' = r + c^{1/n}$, and it increases in r . The seller's expected profit is similar to (15) except that the lower bound of the integration will be raised from s_a to s_a'.

Therefore, we can see that the participation threshold increases in the open reserve (the starting bid). The effect of the open reserve is similar to the auction participation cost (in Proposition 4) in that it increases the customers' participation threshold. Comparing to the regular auction, the seller prefers to use a buy-it-now option when he has a high reserve price, everything else being equal. The reason is that as the reserve goes up, fewer people can participate in this auction due to the reduced gain from bidding. As a result, the seller's *ex ante* expected auction profit decreases. Offering a buy opportunity reduces the chance of no sale and the seller with a high reserve price needs to adjust the price downwards.

In addition to setting a positive starting bid to ensure non-negative profit, the seller can set a secret reserve price, which is assumed to come from the same distribution as that of the valuations of the customers (also see Bajari and Hortascu, 2003). The value of the secret reserve is unobservable, therefore, *a priori*, customers cannot calculate the participation threshold the same way as in the open reserve case. However, they know that there is one more opponent they are bidding against: the seller. Consequently the only difference brought by the secret reserve case is that there is one more bidder in the game, the main results derived previously still follow.

3.3 Positive Cost for Exercising the Buy-it-Now Option

So far we have assumed that exercising the buy-it-now option is costless to customers. The cost of bidding, *c*, thus can be interpreted as the incremental cost of bidding relative to the exercising the buy-it-now option. Our conjecture is that bidding is more cognitively effortful than exercising the buy-it-now option. However this need not be the case and in this section, we analyze the more general case where the customer bears a positive completion cost (k>0) and study the managerial implications on the optimal pricing format and expected profit.

Because exercising the buy-it-now auction entails a positive cost of k for the customer, the expected utility of purchasing right away becomes

(21) $$EU_i^{buy}(v_i) = v_i - B - k ,$$

and the new buy-threshold ($\tilde{s}_b$) is a function of both c and k

(22) $$\frac{\tilde{s}_b^n - c}{n} = \tilde{s}_b - B - k \ .$$

Proposition 6 Customer's cost of exercising the buy-it-now option k increases the buy-threshold $\tilde{s}_b$.

We now analyze what implications a positive k has on seller's optimal price format and expected profit. The seller's pricing decision and the expected profit is considered in two mutually exclusive cases depending on the magnitude of k.

First, when $k \in [0, 1 - c^{1/n} - \frac{1-c}{n}]$, the sellers expected profit function is still a piece-wise function containing three separate function forms:

(23) $$\pi = \begin{cases} \tilde{\pi}_c & B \in [0, c^{1/n}] \\ \tilde{\pi}_b & B \in (c^{1/n}, 1 - \frac{1-c}{n} - k], \\ \pi_a & B \in (1 - \frac{1-c}{n} - k,\ 1] \end{cases}$$

where $\tilde{\pi}_c = B(1 - (B+k)^n)$; and $\tilde{\pi}_b = (1-c)/(1-s_a)[\ (\tilde{s}_b - s_a)\pi_a + (1-\tilde{s}_b)B]$. In other words, when the cost of exercising the buy-it-now option is reasonably small (less than $1 - c^{1/n} - \frac{1-c}{n}$) then the buy-it-now auction format could still possibly take place, along with the regular auction and fixed price formats. Note that when k =0 the above results reduces to those derived earlier (equations 11~13). Moreover, the results derived in the previous sections pertaining to the buy-threshold and the buy-it-now auction still hold under this condition.[6]

Proposition 7 When $k \in [0, 1 - c^{1/n} - \frac{1-c}{n}]$, the seller's expected profit function is a piecewise function specified in equation (23). The buy-it-now auction expected profit function $\tilde{\pi}_b$ is concave in *B*; the optimal buy-it now price is $B^* = \frac{2(1-k)+\pi_a}{3} - \frac{2(1-c)}{3n} < 1$. Customer's cost of exercising the buy-it-now option *k* decreases the optimal buy-it-now price.

Similar to our analysis in Section 2.2 we know that the only pricing range in which a buy-it-now auction can take place is between $\tilde{p}_1 = c_1^{1/n}$ and $\tilde{p}_2 = 1 - \frac{1-c}{n} - k$. An increase in *k* not only makes the buy-it-now option more expensive for the customer to use, but also narrows the pricing ranging from. Hence a high *k* decreases the optimal buy-it-now price, given that the buy-it-now auction takes place.

The second case we consider is $k > 1 - c^{1/n} - \frac{1-c}{n}$. Under this condition, the expected utility of purchasing directly will be larger than bidding for all valuations. In this case, a fixed price format would result instead of a buy-it-now auction when the seller posts a fixed price at the auction. The expected seller's revenue will be $\tilde{\pi}_c = B(1-(B+k)^n)$. The optimal price does not have a closed-form solution, but can be easily obtained numerically.

Proposition 8 When $k \in (1 - c^{1/n} - \frac{1-c}{n}, 1]$, posting a fixed price at the auction would lead to a fixed price format. The optimal selling price maximizes $\tilde{\pi}_c = B(1-(B+k)^n)$.

When $k \in (1 - c^{1/n} - \frac{1-c}{n}, 1]$, the pricing range for buy-it-now auction ceases to exist. This is because such a high *k* pushes $\tilde{p}_2$ to the left of $\tilde{p}_1$, i.e. $\tilde{p}_2 < \tilde{p}_1$. This also implies that in the positive expected utility domain, for all customer valuations the expected utility of purchasing outright will be higher than bidding. Therefore, if the seller posts a fixed price at an auction, effectively he is setting up a fixed price selling format instead of a buy-it-now auction.

Thus, we have extended the model by incorporating a positive buy-it-now cost. We find that all our previous results hold when the cost of exercising the buy-it-now auction is low to moderate. An interesting result surfaces from analyzing the second case: a high *k* leads to a fixed price format, instead of a buy-it-now auction or a pure auction. The intuition is that a high buy-it-now cost reduces the pricing space that allows the buy-it-now auction to happen. Once k is higher than $1-c^{1/n}-\frac{1-c}{n}$, the buy-threshold ($\tilde{s}_b$) will be pushed lower than the participation threshold (s_a). Hence, the bidding format will never materialize. In order to price optimally, the seller must first know which of the above two cases he is facing.

3.4 Heterogeneous Participation Costs

The extension of our current model to a case with heterogeneous costs across customers is conceptually straightforward, but analytically intractable. Green and Laffont (1984) have shown that when the auction participation cost is heterogeneous across customers, there exists a unique bidding equilibrium such that the customer will bid her true valuation if and only if her expected utility of bidding is greater than the cost of bidding. This is the same as what we derived in the homogenous cost case. What is different in this case is that now the threshold is binding at various points due to the heterogeneous costs, instead of one. As a result, we are unable to find an analytical solution for the expected utility of bidding even if the simplest distribution case is assumed (e.g. uniform [0,1]). However, we are able to characterize the function shape of the expected utility of bidding as $\frac{\partial EU_{bid}}{\partial v_i} \geq 0; \frac{\partial^2 EU_{bid}}{\partial v_i^2} \geq 0; \text{ and } 0 \leq EU_{bid} \leq v_i$, illustrated in Figure 6 (cf. Green and Laffont, 1984). Note that this shape is the same as in the homogenous cost case (e.g. Figures 2, 3 and 4). We speculate that our main results will not change even with the heterogeneous participation costs, because Lemma 1 and Theorem 1 would still hold, which are the key to deriving the subsequent results.

Based on this previous finding, we conducted a simulation study to explore to what extent allowing for heterogeneity changes our results. We find that first, participation cost's negative effect on pure auction profit is less severe when costs are heterogeneous than homogenous. Although some customers may have a high participation cost, which prevents them from participating, from the seller's point of view as long as there are customers who have high valuations and participate, the expected profit should not suffer much. This is because auction's expected profit is determined by the second highest bidder's valuation. Second, related to the mitigated negative effect of participation costs on auction profit, the fixed price format is always dominated. Comparing auction and posted-price selling, auction is more powerful in extracting customer's surplus than the posted price (Wang, 1993). Here we assumed the cost of running an auction for the seller is zero. Third, similar to the homogeneous-cost case, buy-it-now auctions still can dominate the other two pricing formats, especially when n is small. When n is small, auction's power of extracting surplus cannot be fully utilized; the fixed-price component complements the auction yielding higher profit for the seller. This intuition is the same as in the homogenous-cost case in our paper.

To summarize, generalizing the customer's auction participation cost to be heterogeneous does offer some new findings, but the major insights and intuition from our basic model do not change. Detailed simulation procedure and results are reported in a technical appendix.

4 Empirical Analysis

In this section we employ actual data from eBay to see if there is evidence supporting the model's assumptions and findings. Because the theoretical model is an abstraction from the actual eBay marketplace from which the empirical data are generated, it is impossible to match the theoretical model to the empirical model perfectly. Hence, the main purpose of this empirical analysis is to test our propositions concerning seller and bidder behavior, particularly to test if the comparative statics for the buy-it-now price can be supported by the data and to identify evidence for endogenous participation. Unfortunately we are not able to directly observe many of the variables in our model and must rely upon proxies to perform these tests. We take two quite different approaches to analyze the data to reduce our reliance upon a single methodology. We start by examining the data at an aggregate level to see if the seller's buy-it-now decision is influenced by the variables as predicted by our theoretical model. Secondly, we estimate an individual-level choice model to measure the consumer's revealed preferences for various auction formats and attributes.

4.1 Data

We collected auction data from eBay during the period April 1 to May 20, 2003 for the following four product categories: Apple iPod MP3 player 10 GB (including both old and new items), Lexar memory stick 128 MB (only new items), KitchenAid 525W Mixer (only refurbished), and KitchenAid KSM103 Professional Mixers (all new). In the first two categories, sellers consist of both individual sellers and eBay's power sellers. KitchenAid is the only seller for the two mixer products. At the time our data was collected, the average retail prices for the four categories were respectively \$400, \$60, \$399 and \$329. We queried eBay many times per hour since the buy-it-now price disappears during the course of the auction once the seller's reserve price is met. Additionally, the reserve status, original duration, and the feedback ratings of both buyers and sellers can change during the auction. If the data was collected only after the auction close, we would not fully recover this information.

During one and a half months, we collected without interruption a total of 273 auctions for iPod players, 328 auctions for Lexar memory sticks, 292 auctions for KitchenAid 525W mixers, and 176 auctions for KitchenAid KSM103 professional mixers. The data set contains detailed auction information such as complete bidding history, the auction starting date and time, end date and time, buy-it-now prices, bidder ID, feedback ratings for both the sellers and buyers, the minimum bid, the number of bids received, and the reserve status (i.e., whether the auction has a secret reserve, and if so whether it has been met). We also have the item condition information (new or used) for the four product categories.

Table 2 presents the descriptive statistics of some variables that are proxies for the parameters in the theoretical model. The original auction duration (e.g. 3-day, 5-day) set by the seller is used as a proxy for the number of potential customers, because the longer the duration, the more customers are eventually aware of the auction. On average, the auction duration specified by the sellers is about 5 days. The true seller's reserve price is the maximum of the starting bid and the secret reserve of an auction. The data collection method enabled us to recover the secret reserve from the bid price when the message "the reserve price is met" is shown. The customer's feedback rating measures the experience a customer has at eBay. Later on in our analysis we use it as an inverse proxy for the auction participation cost (e.g., List and Lucking Reiley, 2002): more experience means more familiar with the auction rules and lower participation cost. Across the four categories (iPod, Lexar, KA-525, KA-KSM103), the sellers' adoption of buy-it-now auctions varies greatly: 34% for iPod auctions, 21% for Lexar memory sticks, and 78% for the two KitchenAid mixers.

[----- Table 2 About Here -----]

4.2 Testing Seller's Buy-it-Now Decision

We first test the theoretical findings of the comparative statics of the buy-it-now price. We pool auction data collected from the four product categories and examine the pricing decisions made by the sellers at an auction level. Our theory predicts that the optimal buy-it-now price increases in the number of potential customers (n), decreases in seller's reserve price (r) and customer's participation cost (c).

We do not directly observe n, r, or c, hence we replace them with proxy measures. If we assume there is some arrival process for consumers that are independent across consumers, then the length of the auction should serve as a proxy for the potential number of customers. We can observe the seller's reserve price if the auction is complete, otherwise we use the starting bid as a proxy for the reserve price since it is not observed. Finally, we hypothesize that there is likely to be some type of power law of practice at work at online auctions similar to other online sites (Johnson, Bellman, and Lohse 2003). The power law implies that as experience increases the cost of participating in the auction will decrease. One measure of experience that is readily observable is the number of auctions that a customer has participated in, which is highly correlated with the customer's feedback rating. Therefore we employ the inverse of the customer's feedback rating as a proxy to customer participation costs. None of these proxies is perfect, and our hope is that in the future more accurate measures of these constructs will be available for a more powerful test.

To test the propositions from the comparative static analysis on the buy-it-now prices (propositions 3, 4, and 5), we use a lognormal model to reveal the relationship between the buy-it-now prices (B) and the key proxy variables. Because this analysis is only relevant when buy-it-now option is offered, we perform the following regression using data of buy-it-now auctions only.

(24)
$$f(B' \mid \beta, \sigma, X) = (2\pi)^{-1/2}(B'\sigma)^{-1}\exp\{-\frac{1}{2\sigma^2}(\ln(B') - X\beta)^2\}$$

where X is a vector of proxies for n, c, r. The buy-it now price B' is normalized by the retail price of the product, $B' = B / retail\ price$, to account for the price variations across the four product categories.

The results show that all the estimates (standard errors in parentheses) have the expected sign and are all statistically significant (Table 3). This provides strong evidence that sellers' behavior is consistent with the findings of our theoretical model.

[----- Table 3 About Here -----]

4.3 Predicting Consumer Choice of Auctions

Now we turn to test the key assumption we make in the theoretical model, that is, customers make an endogenous decision on whether to participate in an auction. In our data set, we observe the time at which each bid is placed. This time together with the auction starting and ending times allows us to infer the list of auctions that were available when the bidder made the decision of which auction (option) to select (choice set Ω is constructed such that if a buy-it-now price available to the bidder, then she has two options for that auction: to bid and to buy at the buy-it-now price). Using a multinomial logit model formulation we estimate the parameters of the auction covariates such as the "time-to-end" of an auction, buy-it-now price, starting bid, and the auction duration set by the seller.

These variables are chosen to serve as the proxy respectively for the participation cost (due to waiting or monitoring), buy-it-now price, reserve price, and the number of potential customers as specified in the theoretical model. Note that instead of using the maximum of starting bid and secret reserve price as the seller's reserve price like we did in the last analysis, we use the starting bid variable together with a dummy variable containing three levels "no reserve", "reserve met" and "reserve not met" to control for the seller's reserve effect. This is because when bidders arrive at the auction all that is observable to her is the starting bid for each auction and whether a secret reserve is met for that auction if there is one. She cannot tell the level of secret reserve price unless it has been met. Other variables not included in our theory but that we control for include seller's rating, quality of the product (used or new), and the secret reserve option.

Let $i=1, 2, \ldots, I$ denote the bidder and let $j=1, 2, \ldots, J$ denote the option in the choice set Ω, the probability that bidder i chooses option j is:

$$P(Y_{ij} = 1) = \frac{\exp(X_j \beta)}{\sum_{j' \in \Omega_i} \exp(X_{j'} \beta)} \tag{25}$$

where X_j is the auction covariates for the j^{th} option when bidder i is making her decision, and β is the vector of the parameters associated with these covariates.

The parameter estimates of this reduced form model are presented in Table 4. The results across four product categories consistently show evidence of endogenous participation. The original duration set by the seller at the beginning of the auction is found significant and negatively related to the choice probability of an auction. This variable is used to capture the number of the potential bidders at an auction (i.e. competition among customers). The longer the duration, the more bidders will arrive, hence the higher the participation threshold, everything else being equal. If the "full and exogenous participation" assumption were true then we would not observe a negative relationship between the auction choice probability and the original duration.

[----- Table 4 About Here -----]

Another result that lends support to our theory about auction participation cost is that the "time-to-end" of an auction is also significantly and negatively related to the probability of an auction being chosen. "Sniping" behavior is an often-observed behavior at online auctions (Wilcox, 2000; interested readers are referred to Roth and Ockenfels, 2002 for a more in-depth discussion on late bidding). It occurs when many bidders place bids close to the end of the auction. We believe this interesting behavior may be partly due to the participation costs associated with bidding. Should these costs vanish, we would not observe such a strong preference for auctions almost ready to close, especially for the private value auctions considered here.

The minimum bid variable, used to directly test the reserve price effect, is again shown to significantly decrease the choice probability. The preference toward the buy-it-now option is tested using three-level dummy variables: regular auction and a buy-it-now auction with a buy-price lower or higher than the average of the buy-it-now prices in the data set. The results show that customers welcome the buy-it-now option for the KitchenAid auctions. This is consistent with the fact that the auction participation cost in this category is higher than the others as we conjecture from Table 2. Customers at iPod and Lexar auctions prefer bidding over exercising the buy-it-now option.

5 Summary and Conclusions

This paper examines the effects of the buy-it-now price for private value, second-price sealed bid auctions. Our research findings have important implications for understanding consumer behavior in the competitive environments found within auctions. We show that an increase in customer's auction participation costs can reduce the seller's revenue under a traditional auction format, although such effect is not as strong when the costs are heterogeneous than constant across customers. The buy-it-now feature offers the seller a flexible marketing tool to reduce the inefficiency of the traditional auction. It can increase both the seller's profit and the customers' utility. Auction participation costs are clearly an important consideration for sellers in deciding which auction format to employ. In contrast most auction research ignores customer participation costs and assumes full participation. Moreover, our empirical findings complement and confirm the findings of our theoretical model.

Our findings indicate that the seller's choice of using a buy-it-now auction option depends upon being able to predict a customer's auction participation costs, the number of customers, and the seller's own reserve price to configure a profitable auction. The results of this model can also help auction companies, like eBay, improve their own pricing policy for sellers who wish to use the buy-it-now feature. Currently, sellers are charged five cents for using this option (except for the auto category). We recommend that auctioneers like eBay should price this option differently across various product categories to increase transactions and in turn increase total revenue.

Our research is not without limitations. First, in our model we do not consider seller's cost for using the buy-it-now option and assume that the seller's reservation price is exogenously given. Hence, the seller's strategic behavior in choosing other auction design parameters such as a reserve price and auction duration in conjunction with a buy-it-now price is assumed away. It would be interesting to determine the optimal auction design with buy-it-now prices that maximizes firm's profit. Unfortunately, we have not been able to find an analytically tractable framework for this analysis.

Second, we do not explore the potential signaling effect of the buy-it-now price in this paper, however, it is possible that a fixed price at auction can convey and signal product quality information to the buyers, provided that the price is credible (Qiu, Popkowski Leszczyc and He, 2005). Our findings suggest that the signaling effect is somewhat limited at eBay auctions. The dynamic nature of eBay's buy-it-now auction limits this signaling effect largely to the first qualifying bidder. In addition, the price range for signaling is also limited, because a high price may lead to a regular auction instead of a purchase. In order to fully exploit the signaling effect of the buy-it-now price, the seller needs to know the upper bound of price range in which the fixed price or the buy-it-now auctions could take place.

Third, in our empirical analysis we ignored the endogeneity in setting the buy-it-now prices, in order to be consistent with the theoretical model. Potentially an empirical analysis that follows a structural approach for constructing consumer and seller decisions could overcome this issue.

Fourth, we leave the role of competition among auctions for future research. To incorporate the competition between concurrent and/or future online auctions, one must allow for forward-looking and forward-seeing bidding. Zeithammer (2006) shows that even in pure auctions, the equilibrium bidding strategy is intractable in closed form. Since our research objective is to illustrate the dynamic nature of buy-it-now auction, i.e. the price format could change as a result of different pricing levels, and the profit implications for the seller, we tried to work with a tractable and stylized model so that the insights of the model are more transparent. More recently, Kirkegaard and Overgaard (2007) make a first attempt in this direction by considering multi-unit demand bidders. They find that future competition from other sellers offering the same product provides a seller an incentive to use the buy-it-now feature in the current auction. However, the sequential auction model did not consider the bidder's participation cost. Given that participation cost is likely to exist and be heterogeneous in reality, it requires additional theoretical or empirical modeling efforts to assess the net effect of competition.

In a recent empirical research using notebook pc data from eBay's Korean site, where buy-it-now prices are permanent, Chan, Kadiyali and Park (2006) find that most sellers (62%) set the buy-it-now prices suboptimally, and the authors suggest that misestimating of the market competition across auctions could be a possible cause. Extending our current model to a competitive environment, we conjecture that, ceteris paribus, the posted price would be used more often compared to that in the monopolistic case. The sellers need to offer it not only to recoup the hassle cost lost in the transaction, but also as a counter measure to the competition. The price level would be lower. In reality, however, whether or not to post a fixed price at auctions is influenced by factors such as seller's inventory mix (Zeithammer and Liu, 2006) and ability to differentiate (e.g. through product offerings, seller's reputation and so on). The price level is affected by the extent to which the posted price enlarges the market size. It is an interesting research question to study to what extent sellers should and could respond to the market competition both theoretically and empirically. We believe the aforementioned issues are worthwhile extensions of the current model and that the future study of consumer behavior at auctions is a fruitful area for both researchers and practitioners.

Notes

1. The previously derived conditional strategies (e.g. Hidvégi et al, 2006) assumed that all the potential bidders are present at the same time and monitoring the auction throughout the whole time while the auction is open and are able to revise their bids immediately once a certain bid level is reached.

2. Alternatively, one can model n as a random variable following a distribution, for example, Poisson. We expect that the model insights will remain the same.

3. For simplicity, uniform distribution assumption is adopted for analysis in this paper. The main results of the paper are general as shown in the Appendix by the generalized proofs and numerical analysis.

4. In our model, n potential customers arrive randomly at an auction. We do not consider the case where customers strategize concerning the sequence of entry in this paper.

5. We do not consider setting the optimal reserve price in this paper, but rather treat the reserve price as exogenous. We refer interested readers to Riley and Samuelson (1981) for more discussion on optimal auction design by setting the reserve price optimally.

6. All previously obtained results still hold when $k \in [0, 1 - c^{1/n} - \frac{1-c}{n})$. Proofs are similar to those in the k=0 case, and are kept in a technical appendix to avoid repetition.

7. References

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Table 1: Seller's Optimal Pricing Format and Prices.

Selling Format and Price Range	Seller's Expected Revenue Function	Optimal Price B*	Optimal Revenue
Fixed price: $B \in [0, c^{1/n}]$	$B(1-B^n)$	$(1+n)^{-1/n}$	$(1+n)^{-1/n}(1-(1+n)^{-1})$
Buy-it-now Auction $B \in (c^{1/n}, 1-\frac{1-c}{n}]$	$\frac{(1-c)}{(1-s_a)}[(s_b - s_a)\pi_a + (1-s_b)B]$	$\frac{2n+\pi_a n - 2 + 2c}{3n}$	$(1-c)(\pi_a + \frac{2\sqrt{2/3}(n-\pi_a n + c - 1)^{3/2}}{3(1-c^{1/n})n\sqrt{n(n-1)}})$
Pure Auction $B \in (1-\frac{1-c}{n}, 1]$	π_a	--	π_a

Table 2: Descriptive Statistic of the Proxy Variables for the Key Parameters.

	Buy-it-now Adoption	Original Duration	Seller's Reserve Max(starting bid, secret reserve)	Bidder Experience median
iPod 10GB	34%	4.89	244.07	49.78
MP3 player	(0.47)	(2.08)	(107.74)	(280.92)
Lexar 128MB	21%	5.06	32.93	36.25
Memory stick	(0.41)	(2.15)	(17.24)	(58.36)
Kitchen Aid	78%	4.31	207.84	15.81
525W mixer	(0.41)	(1.17)	(66.41)	(30.83)
KitchenAid	78%	4.87	167.23	11.62
KSM103 Mixer	(0.41)	(0.94)	(34.87)	(29.60)

Table 3: A Log-Normal Regression on the Buy-it-Now Prices

	Proxy Variable	Result to Test	Predicted Signs for Proxy Variables	Estimates (Std. Err.)
constant				-0.4273*** (0.0284)
n	original auction duration	proposition 3	+	0.0088* (0.0049)
c	(inversely) customer's feedback rating	proposition 4	+	0.0077*** (0.0024)
r	max(starting bid, secret reserve)	Proposition 5	–	-0.0002** (0.0001)
σ				0.1255 (0.1378)

Note: *** Statistical significant at 1%, ** statistical significant at 5%, * statistical significant at 10%.

Table 4: Multinomial Logit Analysis Results

	Result to Test	Predicted Signs	Parameter Estimates (Std. Err.)			
			IPOD	LEXAR	KA-525	KA-KSM
Constant			1.9759*** (0.1101)	2.3966*** (0.1116)	2.0110*** (0.2286)	0.5665 (0.3532)
time to end	s_a increases in *c*: proposition 1	-	-0.4344*** (0.0204)	-0.7681*** (0.0195)	-0.6161*** (0.0209)	-0.4874*** (0.0301)
ln(start bid)	s_a increases in *r*: proposition 1	-	-0.1565*** (0.0091)	-0.0733*** (0.0123)	-0.1791*** (0.0535)	-0.2777*** (0.0466)
Reserve met	control	+/-	-0.5465* (0.2986)	1.1329*** (0.3258)	-1.4734*** (0.2865)	-9.7173 (45.4420)
Reserve not met	control	+/-	0.5013*** (0.0691)	-0.0904 (0.1355)	-0.1349 (0.0984)	1.3359** (0.3169)
ln(seller rating)	control	+	0.0315** (0.0158)	0.0275** (0.0092)	-	-
Quality	control	+	0.0316 (0.0579)	-	-	-
Original duration	s_a increases in *n*: proposition 1	-	-0.0819*** (0.0161)	-0.0530*** (0.0124)	-0.0417* (0.0243)	-0.0256 (0.0373)
BIN_low	s_b increases in *B*: proposition 1	+/-	-0.9836*** (0.0799)	-0.2692*** (0.0800)	0.1610** (0.0640)	0.1381*** (0.0131)
BIN_high	s_a increases in *r*: proposition 1	+/-	-0.4055*** (0.0597)	-0.0587 (0.0595)	-	0.0778 (0.1164)
N			1661	2438	1887	912
LL			-2479.0757	-5413.8228	-2780.2681	-1189.4304
LL_0			-2849.2794	-5961.6414	-3055.6191	-1313.6448
d.f			9	8	6	7
$\chi 2$ *test p-value*			0.0000	0.0000	0.0000	0.0000

Note: *** Statistical significant at 1%, ** statistical significant at 5%, * statistical significant at 10%.

Figure 1: Sequential Moves of the Firm (Seller) and Consumers.

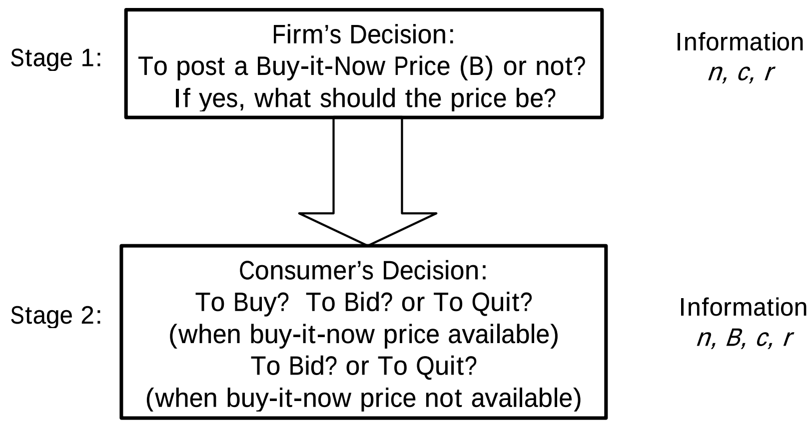


Figure 2: Expected Utility Surplus for Bidding and Buying.

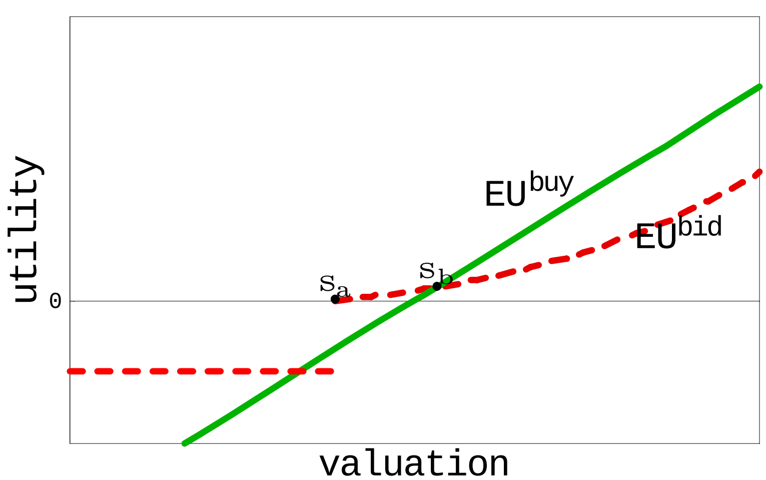


Figure 3: Buy-threshold s_b, Participation-threshold s_a, and Participation cost *c*.

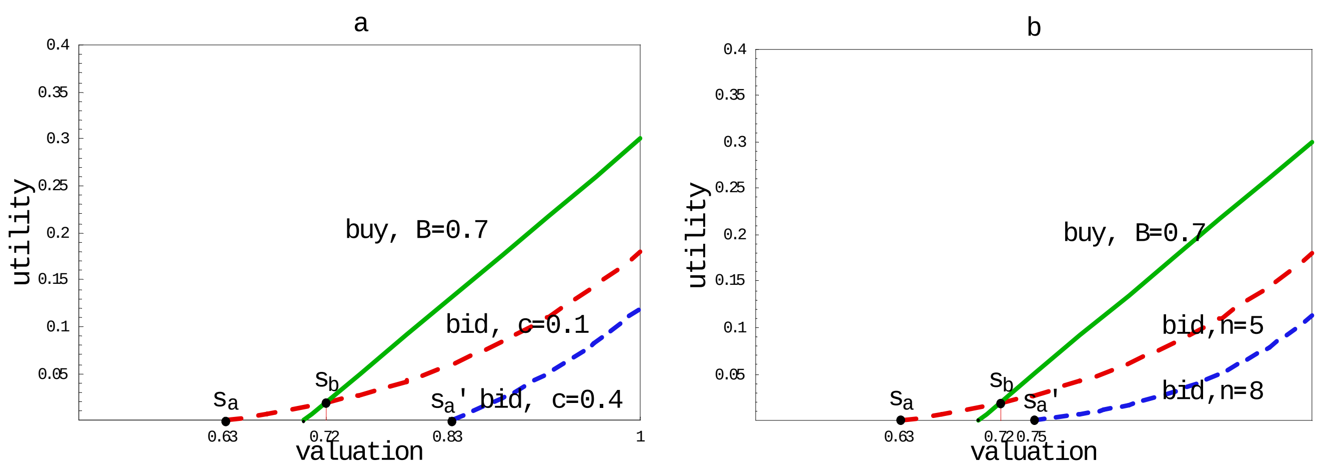


Figure 4: Pricing Formats under Different Pricing Strategies

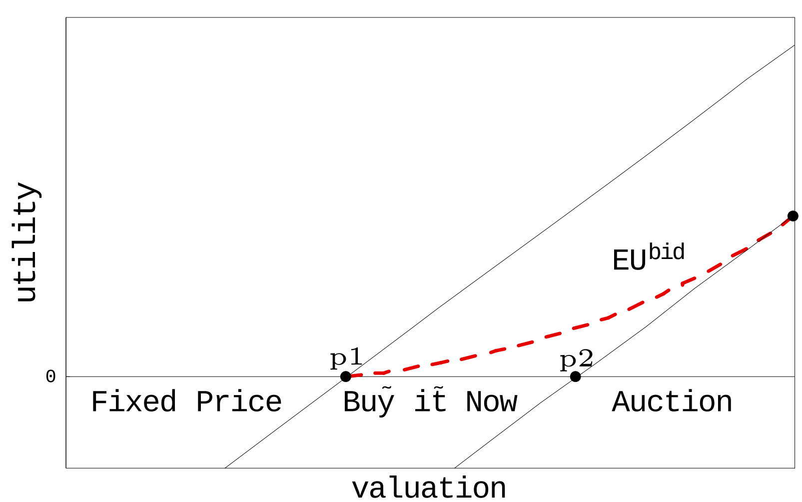


Figure 5: Seller's Expected Revenue.

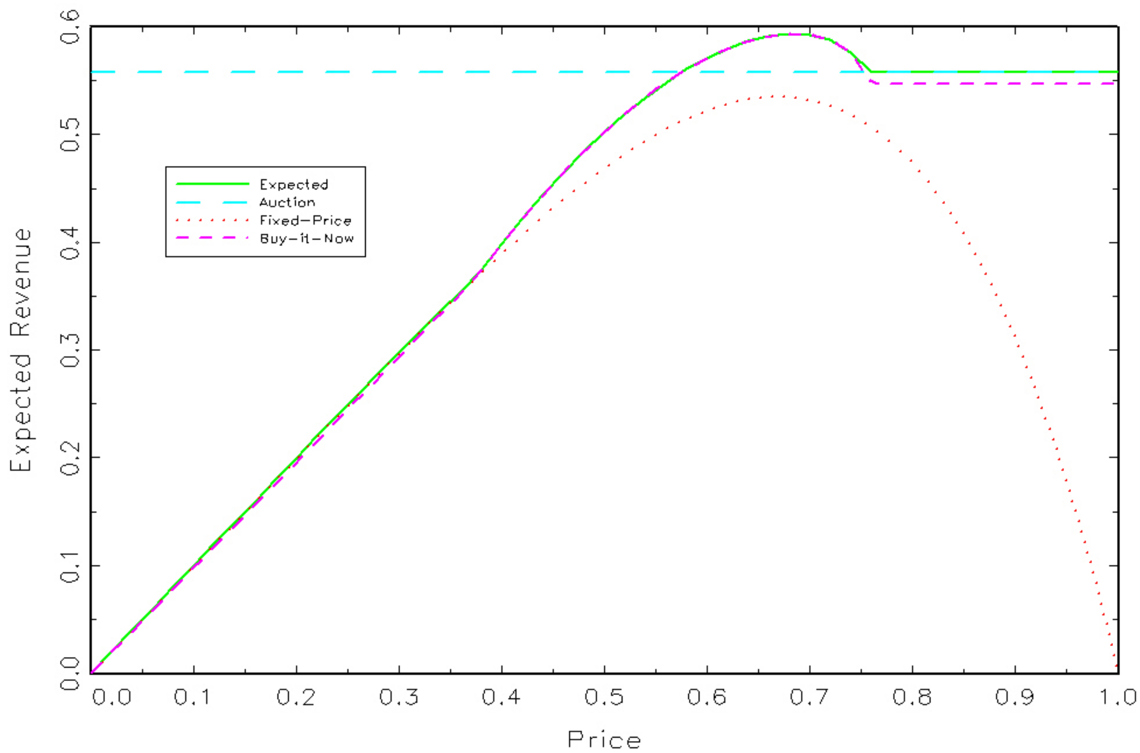
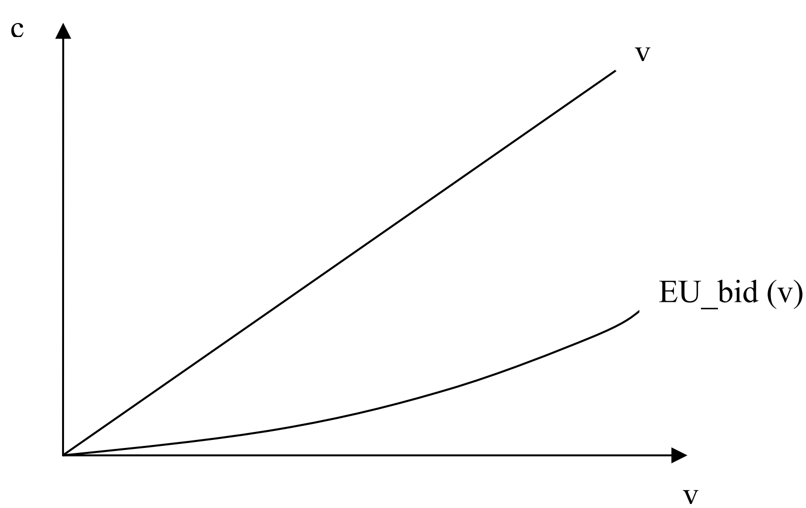


Figure 6. Shape of Expected Utility of Bidding with Heterogeneous Costs



Appendix

Proof for Lemma 1 (for generalized distribution *F*). What we want to show here is $\frac{\partial EU_i(v_i)}{\partial v_i} \le 1$, where $EU_i = s_a[F(s_a)]^{n-1} + \int_{s_a}^{v_i}[F(x)]^{n-1}dx - c$. And $\frac{\partial EU_i(v_i)}{\partial v_i} = F[v_i]^{n-1} \le 1$. ■

Proof for Theorem 1 (for generalized distribution *F*). To prove that the equilibrium bidding strategy prescribed in Theorem 1 exists, we need to show that each customer does not have an incentive to unilaterally deviate. Knowing the number of potential competing bidders at the auction, and the auction participation cost, each customer arrives at the auction computes ex ante utility of bidding, purchasing at the buy-it-now price, and not participating.

First consider $B > s_a$. Suppose a customer with a valuation higher than the buy-threshold chooses to submit a bid. The auction becomes a regular auction. As a result, this customer is forced to bid in order to obtain the product. The resulting utility is $\int_{s_a}^{v_i}\left[F(x)\right]^{n-1}dx$. From Lemma 1 we know it is lower than $v_i - B$. Not participating is not an option either, as it requires the customer to forgo the positive utility, $v_i - B$, derived from exercising the buy-it-now option and receive zero utility instead. Therefore, the consumer will be worse off from the choice of any option other than purchasing directly. If $v_i < s_b$ and the consumer buys the product at the pre-set buy-price, the consumer gains a negative profit (equal to $v_i - B - \int_{s_a}^{v_i}\left[F(x)\right]^{n-1}dx$) from this transaction. Therefore it does not constitute equilibrium. Not participating is weakly dominated too, because the consumer could potentially gain a positive surplus $\int_{s_a}^{v_i}\left[F(x)\right]^{n-1}dx$ from bidding. Any action for a customer whose valuation is lower than s_a makes the consumer worse off, causing a negative utility -*c*.

Second, consider $B < s_a$, then there is a case such that $B < v_i < s_a$. In this case a customer's dominant strategy is to buy directly to gain $v_i - B$. Submitting a bid or leaving the auction without doing anything will result in a negative profit ($-c$ and $-(v_i - B)$ respectively). Therefore they are both strictly dominated. Any action from a customer whose valuation is lower than B will result in a negative profit (-*c* if the customer bids, and $(v_i - B)$ if the customer exercises the buy-it-now option). In addition, if a customer's dominant strategy is to buy at the buy-it-now price, waiting before exercising the buy-it-now option is dominated because subsequent customers may arrive and potentially change the auction format to one less favorable for the customer.

Suppose there is no buy-it-now price when a customer arrives. For a customer to bid when $v_i < s_a$ results in a negative surplus equal to -*c*. If $v_i > s_a$ the customer submits a bid equal to their valuation. The proof of this statement is the same as that of Vickrey (1961) and Milgrom and Weber (1982). ■

Proof for Proposition 1 (with generalized distribution *F*): We first prove the results regarding s_a. Define $g_1 \equiv s_a F(s_a)^{n-1} - c$. By the implicit function theorem, we have:

$$\frac{\partial s_a}{\partial c} = -\frac{\partial g_1 / \partial c}{\partial g_1 / \partial s_a}, \text{ and } \frac{\partial s_a}{\partial n} = -\frac{\partial g_1 / \partial n}{\partial g_1 / \partial s_a}.$$

$$\frac{\partial g_1}{\partial s_a} = \left[F(s_a)\right]^{n-1} + (n-1)s_a\left[F(s_a)\right]^{n-2} f(s_a) > 0;$$

$$\frac{\partial g_1}{\partial c} = -1; \quad \frac{\partial g_1}{\partial n} = s_a\left[F(s_a)\right]^{n-1} \ln\left[F(s_a)\right] < 0;$$

Therefore, $\frac{\partial s_a}{\partial c} > 0; \frac{\partial s_a}{\partial n} > 0.$

Next, define $g_2 \equiv s_b - B - \int_{s_a}^{s_b} \left[F(x)\right]^{n-1} dx$. Again, by the implicit function theorem, we have:

$$\frac{\partial s_b}{\partial B} = -\frac{\partial g_2 / \partial B}{\partial g_2 / \partial s_b}; \quad \frac{\partial s_b}{\partial c} = -\frac{\partial g_2 / \partial c}{\partial g_2 / \partial s_b}; \text{ and } \frac{\partial s_b}{\partial n} = -\frac{\partial g_2 / \partial n}{\partial g_2 / \partial s_b}.$$

$$\frac{\partial g_2}{\partial s_b} = 1 - \frac{\partial \int_{s_a}^{s_b} \left[F(x)\right]^{n-1} dx}{\partial s_b} = 1 - F[s_b]^{n-1} > 0, \quad \frac{\partial g_2}{\partial B} = -1 < 0.$$

$\frac{\partial g_2}{\partial c}$ bears the same sign as $\frac{\partial g_2}{\partial s_a}$, since we have proved that s_a is an increasing function of c, and $\frac{\partial g_2}{\partial s_a} = F[s_a]^{n-1} > 0$. Hence, $\frac{\partial g_2}{\partial c} > 0.$

$$\frac{\partial g_2}{\partial n} = -\int_{s_a}^{s_b}\left[F(x)\right]^{n-1} \ln\left[F(x)\right] > 0.$$

Therefore, $\frac{\partial s_b}{\partial B} > 0; \frac{\partial s_b}{\partial n} < 0; \frac{\partial s_b}{\partial c} < 0.$ ■

A Numerical Analysis

The relative performance of buy-it-now auction (BIN) relative to regular auction (RA) and fixed price (FP). Uniform distribution [0,1] case.

	c=0	**c=0.04**	**c=0.08**	**c=0.12**	**c=0.16**	**c=0.2**	**c=0.4**	**c=0.6**	**c=0.8**
n=5	2.1%	7.9%	16.3%	29.2%	46.9%	66.5%	229.6%	704.4%	3346.2%
n=25	0.1%	9.0%	25.6%	43.8%	64.1%	87.3%	280.9%	846.2%	3997.1%
n=45	0.0%	12.4%	29.8%	48.7%	70.0%	94.1%	296.0%	886.2%	4178.1%
n=65	0.0%	14.1%	31.9%	51.2%	72.8%	97.5%	303.3%	905.7%	4275.4%
n=85	0.0%	15.2%	33.2%	52.7%	74.6%	99.5%	307.8%	917.3%	4324.1%

Table A1.1: Percentage Revenue Increase: (BIN-RA)/RA.

Overall, setting an optimal buy-it-now price gives the seller higher expected revenue than using a regular auction. Holding *n* constant, as *c* increases, the percentage of revenue gain increases. An extreme case is when *c=1* based on the uniform [0, 1] distribution, the expected revenue for the regular auction is zero, while the buy-it-now auction's revenue is much higher because of its fixed-price component. Holding *c* constant, the percentage of revenue gain increases in *n* (expect for c = 0).

An interesting special case is c = 0: we see that when *n* is small, buy-it-now auctions may be better than regular auctions. This is because when the number of potential bidders is small, the auction's efficiency in extracting consumer surplus is fair, while fixed price sales are good. The hybrid nature of the buy-it-now auction leads to an overall improvement. The seller can charge a posted price that gains higher expected revenue. For example, when n = 5, by setting B^* = 0.75 the seller can get BIN revenue of 0.681, while RA revenue is 0.667. As *n* increases, buy-it-now auctions are dominated by regular auctions.

	c=0	c=0.04	c=0.08	c=0.12	c=0.16	c=0.2	c=0.4	c=0.6	c=0.8
n=5	14.5%	6.8%	1.4%	-0.5%	-0.1%	0.0%	0.0%	0.0%	0.0%
n=25	8.7%	0.01%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
n=45	6.2%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
n=65	4.8%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
n=85	4.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%

Table A1.2: Percentage Revenue Increase (BIN-FP)/FP.

Table A1.2 shows that when participation cost is low, the buy-it-now auction generates higher expected revenue than the fixed price selling because of the efficiency of the auction. When the participation cost is high, the expected revenue from the buy-it-now auction can be as high as fixed price selling. This is due to the flexible nature of the buy-it-now auction. A buy-it-now auction can reduce to fixed price selling depending on the price set.

The optimal selling format is presented in Table A1.3, where a B represents a buy-it-now auction should be adopted, and an F indicates the cases where fixed price selling should be used.

	c=0	c=0.04	c=0.08	c=0.12	c=0.16	c=0.2	c=0.4	c=0.6	c=0.8
n=5	B	B	B	F	F	F	F	F	F
n=25	B	B	F	F	F	F	F	F	F
n=45	A	F	F	F	F	F	F	F	F
n=65	A	F	F	F	F	F	F	F	F
n=85	A	F	F	F	F	F	F	F	F

Table A1.3: Optimal Selling Format.

To test the robustness of the model, we conducted numerical studies based on exponential and Weibull distributions. The results are consistent with those from the uniform distribution. Numerical results are available upon request. ■

Proof of Proposition 2: To show that the expected profit function for using the buy-it-now feature is strictly concave, we need to show that $\frac{\partial^2 \pi_b}{\partial B^2} < 0$.

$$\frac{\partial^2 \pi_b}{\partial B^2} = \frac{1-c}{(1-s_b^{n-1})(1-s_a)}\left(-2 + \frac{(\pi_a - B)(n-1)s_b^{n-2}}{\left(1-s_b^{n-1}\right)^2}\right)$$

First note that the second term in the brackets $\left(-2 + \frac{(\pi_a - B)(n-1)s_b^{n-2}}{\left(1-s_b^{n-1}\right)^2}\right)$ is decreasing in n . We denote it as m . Therefore, it reaches its maximum when $n = 2$ (n should be no less than two is required by the second-price sealed bid auction) $m = \frac{(\pi_a - B)}{(1-s_b)^2}$, where $s_b = 1 - \sqrt{1 - 2B + c_1}$. If we can show that $-2 + m < 0$ then we can prove concavity of π_b . This is true because

$$m < \frac{\pi_a \mid_{n=2,c=0} - B}{1 - 2B + c} = \frac{1/3 - B}{1 - 2B + c} \leq \frac{1}{3}$$

Hence π_b is concave in B.

Next, we determine the optimal buy-it-now price. In order to get an analytically tractable result for the optimal buy-it-now price we use a second-order Taylor series expansion to approximate s^n around the upper bound of the value distribution:

$$s_b^n = 1 + n(s_b - 1) + \frac{(n-1)n(s_b - 1)^2}{2}.$$

Based upon a comparison with simulated numerical results this approximation is very accurate. The corresponding explicit solution for $s_b = 1 - \frac{\sqrt{2(1-B)n + 2c - 2}}{\sqrt{n(n-1)}}$. $s_b \in [0,1]$ is guaranteed by the buy-it-now condition $B < 1 - \frac{1-c}{n}$ and $n \geq 2$. Therefore, solving the first order condition $\frac{\partial \pi_b}{\partial B} = 0$, we obtain $B^* = \frac{2+\pi_a}{3} - \frac{2(1-c)}{3n} < 1$. ■

Proof of Proposition 3: $\frac{\partial B^*}{\partial n} = \frac{2-2c}{3n^2} + \frac{1}{3}\frac{\partial \pi_a}{\partial n}$. The first term on the right hand side of the equation is positive because $c < 1$. It is left to show that the second term is also positive in order to prove the proposition. $\frac{\partial \pi_a}{\partial n} = \frac{n(2-c(1+n)^2)+c^{1+\frac{1}{n}}(n^2(2+n)-n)-(n^2-1)\ln c}{n(1+n)^2}$, which decreases in c. Namely, this expression reaches its minimum when $c = 1$. We can further show that $\lim_{c\to 1}\frac{\partial \pi_a}{\partial n} = 0$. Therefore, $\frac{\partial \pi_a}{\partial n} > 0, \frac{\partial B^*}{\partial n} > 0$. ■

Proof of Proposition 4: For example, in the π_b region, $\frac{\partial B^*}{\partial c} = \frac{(c^{1/n}-1)n(n-1)+2}{3n}$. Its sign depends on the relationship between *n* and *c*. Furthermore, it can be shown that for most of the cases (especially when *n* is large) the auction participation cost increases and the optimal buy-it-now price decreases. ■

Proof of Proposition 5: When $r>0$, $B^* = \frac{2+\pi_a^r}{3} - \frac{2(1-c)}{3n}$,

where $\pi_a = \frac{(n-1)[1+(c^{1/n}+r)^n][n(c^{1/n}+r-1)-1]}{n+1}$. To prove that $\frac{\partial B^*}{\partial r} < 0$, we only need to show that $\frac{\partial \pi_a}{\partial r} < 0$. Since $r + c^{1/n} \le 1$, it follows that

$\frac{\partial \pi_a}{\partial r} = n(n-1)(r+c^{1/n}-1)^{n-1} < 0$. ■

Proof of Proposition 6: To prove that $\tilde{s}_b$ increases in c_2, we use implicit function theorem and define $\kappa \equiv \tilde{s}_b - B - k - \int_{s_a}^{\tilde{s}_b}\left[F(x)\right]^{n-1} dx$: $\frac{\partial \tilde{s}_b}{\partial k} = -\frac{\partial \kappa / \partial c_2}{\partial \kappa / \partial \tilde{s}_b} = 1 - F[\tilde{s}_b]^{n-1} > 0$. ■

Proof of Proposition 7: Next, we determine the optimal buy-it-now price. In order to get an analytically tractable result for the optimal buy-it-now price we use a second-order Taylor series expansion to approximate $\tilde{s}_b{}^n$ around the upper bound of the value distribution:

$$\tilde{s}_b^n = 1 + n(\tilde{s}_b - 1) + \frac{(n-1)n(\tilde{s}_b - 1)^2}{2}.$$

Based upon a comparison with simulated numerical results this approximation is very accurate. The corresponding explicit solution for $\tilde{s}_b = 1 - \frac{\sqrt{2(1-B-k)n+2c-2}}{\sqrt{n(n-1)}}$. $\tilde{s}_b \in [0,1]$ is guaranteed by the buy-it-now condition $B < 1 - \frac{1-c}{n} - k$ and $n \geq 2$. Therefore, solving the first order condition $\frac{\partial \pi_b}{\partial B} = 0,$ we obtain $B^* = \frac{2(1-k)+\pi_a}{3} - \frac{2(1-c)}{3n} < 1.$ when k=0, B^* reduces to that in equation (20). Moreover, $\frac{\partial B^*}{\partial k} = -\frac{2}{3} < 0.$ ■

Proof of Proposition 8: When $k \in (1 - c^{1/n} - \frac{1-c}{n}, 1)$, the participation threshold $p_1 = c^{1/n} > 1 - k - \frac{1-c}{n} = p_2$. The pricing range for buy-it-now auction format to exist $[p_1, p_2]$ disappears under this condition. And because $p_1 > p_2$, purchasing outright dominates bidding over the entire positive utility domain. In other words, the seller effectively faces a fixed price decision when customers have a large cost for using the buy-it-now auction. The optimal selling price hence is the solution to $\tilde{\pi}_c = B(1-(B+k)^n)$.