Section 4.1
Value-at-Risk (VaR) Basics
Overview

Background and history
Define Value-at-Risk (VaR)
More VaR details
Background and history

Define Value-at-Risk (VaR)

More VaR details
Risk measures

• Quantify risk
• Examples
  ⇒ Variance
  ⇒ Beta
  ⇒ Duration
  ⇒ Value-at-Risk (VaR)
  ⇒ Expected shortfall, Expected tail loss, CVaR
Value-at-Risk (VaR)

- Probabilistic worst case
Value-at-Risk (VaR)

- Probabilistic worst case
- Almost “perfect storm”
Value-at-Risk (VaR)

- Probabilistic worst case
- Almost “perfect storm”
- 1 in 100 year flood level
VaR advantages

- Risk $\rightarrow$ single number
VaR advantages

• Risk → single number
• Firm wide summary
  → No problems with derivatives and other complex contracts
VaR advantages

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- Firm wide summary
  $\Rightarrow$ No problems with derivatives and other complex contracts
- Relatively model free
VaR advantages

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- Easy to explain
VaR advantages

- Risk $\rightarrow$ single number
- Firm wide summary
  $\Rightarrow$ No problems with derivatives and other complex contracts
- Relatively model free
- Easy to explain
- Handles deviations from normal distributions
Late 1980’s: trading portfolios
  ⇒ Trading desk: FX
  ⇒ High frequency
  ⇒ High liquidity
History

• Late 1980’s: trading portfolios
  ⇒ Trading desk: FX
  ⇒ High frequency
  ⇒ High liquidity

• JP Morgan, 1990’s
  ⇒ 4:15 and VaR
  ⇒ RiskMetrics, 1994
     Now part of MSCI
VaR methods

- Delta normal
- Historical
- Monte-carlo
- Bootstrap
Define Value-at-Risk (VaR)
Definition: \( \text{VaR}(p) \) is the **smallest** value such that the probability that your losses, \( Q \), **exceed** \( -\text{VaR}(p) \) is less than or equal to \( p \).

\[
\Pr(Q < -\text{VaR}(p)) \leq p \tag{4.1.1}
\]
• This definition of VaR can get confusing/complicated
• Easier when distributions are continuous  
  (At least in the region where VaR is defined.)
• Inequalities become equalities
• Often most relevant to the real world
Value-at-Risk (VaR): continuous

Definition (continuous): $\text{VaR}(p)$ is the value such that probability that your profits/losses, $Q$, exceed $-\text{VaR}(p)$ is $p$.

$$\Pr(Q \leq -\text{VaR}(p)) = p \quad (4.1.2)$$

Sometimes you will see this defined in reverse:
**Value-at-Risk (VaR): continuous**

**Definition (continuous):** VaR\((p)\) is the value such that probability that your profits/losses, Q, exceed \(-VaR(p)\) is \(p\).

\[
\Pr(Q \leq -\text{VaR}(p)) = p \tag{4.1.2}
\]

Sometimes you will see this defined in reverse:

**Definition (continuous):** I am confident with probability \(1 - p\) that my portfolio will not lose more than \(-\text{VaR}(p)\).

\[
\Pr(Q \geq -\text{VaR}(p)) = (1 - p) \tag{4.1.3}
\]
Value-at-Risk (VaR): continuous

- VaR is easier with continuous distributions. Start here when understanding VaR.
- Many textbooks simply start with this assumption. (This includes Daníelson.) It makes the initial understanding much easier, and applies to most practical cases.
Now, some math

\( f_q(x) = \) probability density for P/L
\( F_q(x) = \) Cumulative distribution function for P/L (CDF)

\[
\Pr(Q \leq -VaR(p)) = p
\]

\[
p = \int_{-\infty}^{-VaR(p)} f_q(x) \, dx \quad (4.1.4)
\]

\[
p = F_q(-VaR(p)) \quad (4.1.5)
\]
\( \text{VaR} \rho = 0.05, \ Q \ is \ N(0, \sigma = 10) \)
How do we really calculate VaR?
VaR parameters

- Horizon (1 day)
- Prob of loss (p)
VaR example

- Portfolio value today: 100
- Time horizon: 1 month
- Distribution of value over month
  - Normal distribution
  - Mean 100
  - Std. 10
- Loss probability, $p = 0.05$
Portfolio value density

Portfolio value in 1 month, mean = 100, Std. = 10
Portfolio $p = 0.05$ quantile

Portfolio value in 1 month, today = 100, mean = 100, Std. = 10
• Profit/Loss (P/L) = Q = (future value) − 100
• Loss at 0.05 = 83.6 − 100 = −16.4
VaR calculation

- Profit/Loss (P/L) \( = Q = \) (future value) \( - 100 \)
- Loss at 0.05 \( = 83.6 - 100 = -16.4 \)
- Flip sign (loss) : 16.4
• Profit/Loss (P/L) = \( Q = (\text{future value}) - 100 \)
• Loss at 0.05 = 83.6 - 100 = -16.4
• Flip sign (loss) : 16.4
• \( \text{Pr}(Q \leq -\text{Var}(0.05)) = \text{Pr}(Q \leq -(16.4)) = 0.05 = p \)
VaR calculation

- Profit/Loss (P/L) = \( Q = \) (future value) \(- 100 \)
- Loss at 0.05 = 83.6 \(- 100 = -16.4 \)
- Flip sign (loss) : 16.4
- \( \Pr(Q \leq -\text{VaR}(0.05)) = \Pr(Q \leq -(16.4)) = 0.05 = p \)
- Basic method
  - Find \( p \) quantile for portfolio value
  - Find the portfolio loss at this value
  - Flip sign of loss to a positive number
Portfolio $p = 0.01$ quantile

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Portfolio value in 1 month, today = 100, mean = 100, Std. = 10
VaR calculation

- Profit/Loss (P/L) = $Q = (\text{future value}) - 100$
- Loss at $0.01 = 76.7 - 100 = -23.3$
- Flip sign (loss) : 23.3
- $\Pr(Q \leq -VaR(0.01)) = \Pr(Q \leq -(23.3)) = 0.01 = p$
More VaR details
VaR details

- Two annoyances
- Setting VaR parameters
Two annoyances

- Sign: Can VaR be negative?
- $p$ versus $1 - p$
Can VaR be negative

- VaR defined as a loss
- VaR(0.05) of 10 means **Loss** will only exceed 10 with probability 0.05
  Confusing: −10 might have been a better value for this (net gain)
- Technically, a negative VaR means the $p$ worst case is actually a gain (see next figure)
- In the real world, people can mess this up, and it can get confusing
Negative \( \text{VaR}(0.05) = -(106.8 - 100) = -6.8 \)
\( p \) \textbf{versus} \( (1 - p) \)

- VaR sometimes reported with confidence level \( = (1 - p) \)
- For \( p = 0.05 \), this would be 0.95
- We are 95\% confident that losses won't exceed \( \text{VaR} \) value
- Many people use this convention
- Big examples
  - Jorion’s book, Value-At-Risk
  - My old lecture notes (and exams)
Choosing VaR parameters

- Holding period
  - Risk environment/problem
  - Portfolio changes/liquidity
Choosing VaR parameters

- Holding period
  - Risk environment/problem
  - Portfolio changes/liquidity

- Confidence level
  - How far into tail?
  - Better estimate of extreme losses
  - Data quantity
VaR uses

- Benchmark comparison
- Potential loss measure
- Set capital cushion levels (reserves)
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