Fin285a: Computer Simulations and Risk Assessment
Section 4.2
VaR Issues
Overview

Just a quantile
Violates technical risk properties
Can VaR be manipulated?
Just a quantile

Violates technical risk properties

Can VaR be manipulated?

Just a quantile
VaR is just a quantile

- VaR marks quantile point on the distribution
- Does not tell you about left tail beyond VaR
- Might ignore a lot of risk
- Might be sensitive to probability level
VaR ignores left tail
Two portfolios: X straight equity position. Y is hedged with a put option which puts a floor on the losses.

Ignore the cost of this option.

Also, assume the floor is below the VaR(p) quantile cutoff.

Result: VaR is the same in both cases, but risk is quite different. (see next figure)

(See Danielson, page 80, for another example.)
VaR quantile confusion
Violates technical risk properties

Just a quantile

Violates technical risk properties

Can VaR be manipulated?

Violates technical risk properties
Let $h(X)$ be a quantitative measure of risk.

1. Monotonic: $X_i \leq Y_i : h(X) > h(Y)$
2. Homogeneous: $h(cX) = ch(X)$
3. Translation invariance $h(X + c) = h(X) - c$
4. Subadditivity: $h(X + Y) \leq h(X) + h(Y)$
Quick info: bonds and VaR

• Bond
  ➞ 1 year maturity
  ➞ Principal = 100
  ➞ Interest = 5%
  ➞ No default: pays 105
  ➞ Price = 100

• Default probability = 0.0125

• If there is a default, bond returns $[0, 100]$ uniform (recovery)

• Estimate $p = 0.01$ VaR for each bond and portfolio (test subadditivity)
Quick info: bonds and VaR

\[ 100 \times 1.25 \times 10^{-4} = 0.0125 \quad 80 \times 1.25 \times 10^{-4} = 0.01 \]
Quick info: bonds and VaR

\[ \frac{x}{100} \ast p_{\text{default}} = p \]  \hspace{1cm} (4.2.1)

\[ \frac{x}{100} \ast 0.0125 = 0.01, \quad x = 80, \quad \text{VaR} = 20 \] \hspace{1cm} (4.2.2)
Add a second bond

- Now purchase two bonds (principal = 100), A and B
- Interest is again 5%
- Price of each is 100
- Assume that the second bond is linked to the first
- They will never default at the same time
- Probability that each bond defaults (alone) is 0.0125
- Probability of default is $2 \times 0.0125 = 0.025$
- Probability of no default is $1 - 2 \times 0.0125 = 0.975$
Quick info: bonds and VaR

\[ 100 \times 2.5 \times 10^{-4} = 0.025 \quad 40 \times 2.5 \times 10^{-4} = 0.01 \]
Quick info: bonds and VaR

\[ \frac{x}{100} \times p_{\text{default}} = p \]  

\[ \frac{x}{100} \times 0.025 = 0.01, \quad x = 40 \]

- Since only one bond can default at a time:
  - Loss on defaulting bond is \( 100 - 40 = 60 \)
  - Gain on other bond is \( 105 - 100 = 5 \)
  - Net loss is \( 55 \)

- For \( A + B \), \( \text{VaR}_{A+B}(0.01) = 55 \)
- \( \text{VaR}_A(0.01) + \text{VaR}_B(0.01) = 20 + 20 = 40 \)
- \( \text{VaR}_{A+B} > \text{VaR}_A + \text{VaR}_B \): Violates subadditivity
Why is this bad?

- Violates common sense about diversification
- Suggests that splitting stuff up might make it appear less risky
- Is this math all that common?
Can VaR be manipulated?

Just a quantile
Violates technical risk properties
• Can a sneaky trader manipulate VaR?

• Return to single bond example

⇒ Investor holds single bond as defined before

⇒ Investor sells insurance on bond defaults to others for some amount \( z \)

◊ Like a credit default swap

◊ Investor only paid \( z \) when no default occurs

◊ This amount doesn’t matter too much in the problem

⇒ Only insure in states where recovery is less than 80 (of 100)

⇒ Pay out to take other investor back to 80 in all these states
Implementing the strategy

- **Sell** insurance on all losses beyond 20

What happens

1. No default: earn insurance premium with no default, $105 + z$

2. Default ($> 80$): Loss less than 20, no change

3. Default ($< 80$): All defaults recovering less than 80 trigger insurance payments

  ⇒ Investor receives: $X - (80 - X) = 2X - 80$
  
  Portfolio values change: $80 \rightarrow 80$, $0 \rightarrow -80$, range $= [80, -80]$

  ⇒ What is the loss in this range? $-[80 - 100, -80 - 100] = [20, 180]$

  ⇒ Previously it was: $-[80 - 100, 0 - 100] = [20, 100]$
The strategy in a picture

Before insurance

After insurance

Portfolio value

PDF
What’s happening?

- Risk increases
- \( \text{VaR}(0.01) \): no change, why?
  \[
  \begin{align*}
  \Pr(L \leq 20) &= 0.99 \text{ (blue + no default)} \\
  \Pr(L \geq 20) &= 0.01 
  \end{align*}
  \]
- \( \text{VaR} \) only deals with the specific quantile of the loss
- Remainder of the distribution tail can be modified without affecting \( \text{VaR} \)
Can traders move VaR?

- Since VaR represents a single quantile target, risk can be increased without impacting VaR
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- This is **NOT** a good property for a risk measure, or something that regulatory policy should be based on
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- Since VaR represents a single quantile target, risk can be increased without impacting VaR
- This is **NOT** a good property for a risk measure, or something that regulatory policy should be based on
- These examples should be somewhat troubling
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