Problem Set 5
Due in Class, Friday, October 13

1. Go to the FRED database at the St. Louis Federal Reserve [http://research.stlouisfed.org/fred2/](http://research.stlouisfed.org/fred2/) (or just Google FRED and St. Louis Fed). Download the following data.

   - Real Gross Domestic Product, 1 decimal (quarterly)
   - Real Private Nonresidential Fixed Investment (quarterly)
   - Civilian Employment: Sixteen Years and Over (monthly)

Convert the employment data from monthly to quarterly (either point sample or take averages). Make sure that the time series all cover the same time periods (i.e. begin and end at the same dates). Also make sure you are working with real and not nominal series. Note that in the NIPA, flow data (e.g. GDP, consumption, and investment) are reported at annual rates. You will need to divide by 4 to get quarterly data in quarterly rates.

Because data for the capital stock are difficult to find, we’ll construct our own series from investment. Assume depreciation, $\delta$, is 0.025 per quarter, and that $K_0 = \frac{1}{0.025}I_{1/1947}$.

Construct a time series for $K_t$ using the standard recursion:

$$K_{t+1} = (1 - \delta)K_t + I_t.$$  

Using the data for labor, capital, and GDP, calculate the log of the Solow residual, assuming capital’s share is 0.36 using the standard growth accounting formula.

$$\ln A_t = \ln Y_t - \alpha \ln K_t - (1 - \alpha) \ln N_t$$

Detrend the log of GDP and the Solow residual using the HP filter with smoothing parameter 1600. You can do this in Matlab with the function posted on the course web page `hptrend.m`. Note that `hptrend.m` calls two other functions, `pentle.m` and `trile.m`, so make sure those functions are in the same workspace as `hptrend.m`.

(a) Plot detrended GDP along with the detrended Solow residual. Report the cross-correlation of the detrended Solow residual with deviations GDP.

(b) Report the standard deviation of the log deviations from trend of GDP for the period prior to 1984 and the period after 1984. Do you see evidence that the volatility of the business cycle has moderated since 1984?

(c) Extra Credit – If you know how to run a regression Regress the detrended Solow residual on its one-period lag, i.e. estimate an AR(1) process:

$$x_{t+1} = \rho x_t + \varepsilon_{t+1}.$$  

Report your values for $\rho$ and the standard deviation of $\varepsilon$. Real business cycle theory interprets the Solow residual as reflecting shocks to technology. In this case, what is the economic interpretation of the standard deviation of $\varepsilon$? Is this interpretation quantitatively plausible?
2. Consider the following non-stochastic growth model in which the representative household is endowed with one unit of time each period and with an initial capital stock, $k_0$. It chooses sequences of consumption $c_t$, labor $h_t$, and saving $k_{t+1}$ to maximize:

$$\sum_{t=0}^{\infty} \beta^t \left[ \ln c_t + \nu \ln (1 - h_t) \right]$$

subject to:

$$c_t + k_{t+1} = w_t h_t + r_t k_t$$

where $w_t$ and $r_t$ are the time $t$ wage and capital rental rates respectively. Assume the household takes these prices as given. Capital depreciates completely each period.

(a) How does an increase in $w_{t+1}$ (holding all other prices constant) effect the relative allocation of leisure across periods $t$ and $t + 1$ (i.e. the ratio of $1 - h_t$ to $1 - h_{t+1}$)?

(b) How does an increase in $r_{t+1}$ (holding all other prices constant) effect the relative allocation of leisure across periods $t$ and $t + 1$? Explain the economics underlying your answer.

(c) How does an increase in $r_{t+1}$ (holding all other prices constant) effect the relative allocation of consumption across periods $t$ and $t + 1$? Explain the economics underlying your answer.

(d) Does an increase in the future interest rate lead to the same correlation between consumption and hours worked that we see in the data?

Now suppose the household is endowed with a Cobb-Douglas production technology, so its time $t$ budget constraint is:

$$c_t + k_{t+1} = A_t k_t^\alpha h_t^{1-\alpha}$$

(e) How does an increase in $A_{t+1}$ effect the relative allocation of leisure across periods $t$ and $t + 1$? Explain your answer in terms of the marginal products of capital and labor.

3. Ljungqvist-Sargent, page 133, exercise 5.1

4. Ljungqvist-Sargent, page 134, exercise 5.3