1. Consider a simple growth model in which preferences over consumption, $c_t$, and leisure $1 - n_t$ for a representative household are given by:

$$
\sum_{t=0}^{\infty} \beta^t [\ln c_t + \theta \ln(1 - n_t)]
$$

and production is solely a function of hours worked:

$$
y_t = n_t^\alpha.
$$

Assume the consumption good is subject to a cash-in-advance constraint:

$$
P_t c_t \leq M_t
$$

where $M_t$ denotes the beginning of period $t$ money holdings of the household and the relative price of goods to money is $P_t$.

The money supply grows at a fixed rate

$$
M_t^s = (1 + \pi_t) M_0^s
$$

with $\pi M_t^s$ new cash injected into the economy each period via lump-sum transfers. The household is thus subject to a budget constraint:

$$
M_{t+1} = M_t + \pi M_t^s + P_t[y_t - c_t].
$$

(a) For a given $\pi$ find the stationary equilibrium in which $c_t$, $y_t$, and $n_t$ are constant and $P_t$ grows at the rate $\pi$.

(b) Derive an expression for the ratio of the marginal utility of consumption to the marginal utility of leisure in terms of the parameters of the model. Provide an economic interpretation of this expression.

(c) In this model what is the relationship between output and inflation? Is this result consistent with observed long-run cross-country findings reported in McCandless and Weber (1995)?
2. Consider a simple growth model in which preferences for a representative household are:

\[
\sum_{t=0}^{\infty} \beta^t \ln c_t
\]

and the technology is:

\[c_t + k_{t+1} = k_t^\alpha\]

with time \(t\) consumption denoted by \(c_t\) and the time \(t\) capital denoted by \(k_t\). Assume the consumption good is subject to a cash-in-advance constraint:

\[P_t c_t \leq M_t\]

where \(M_t\) denotes the beginning of period \(t\) money holdings of the household and the relative price of goods to money is \(P_t\).

The money supply grows at a fixed rate

\[M_t^s = (1 + \pi)^t M_0^s\]

with \(\pi M_t^s\) new cash injected into the economy each period via lump-sum transfers. The household is thus subject to a budget constraint:

\[M_{t+1} = M_t + \pi M_t^s + P_t [w_t + r_t k_t - c_t - k_{t+1}]\]

In equilibrium, profit maximization by firms assures \(w_t = (1 - \alpha) k_t^\alpha\) and \(r_t = \alpha k_t^{\alpha-1}\), but households take these two prices as given.

(a) For a given \(\pi\) find the stationary equilibrium in which \(c_t\) and \(k_t\) are constant and \(P_t\) grows at the rate \(\pi\). What are the values of of \(c\), \(k\) and real balances, \(\frac{M}{P}\), at this equilibrium?

(b) How is the ratio of consumption to investment affected by anticipated inflation? Explain. Given this model, can the Federal Reserve stimulate capital formation by committing itself to a permanently low growth rate of the money supply?