1. Let $n \in \mathbb{N}$, let $G$ be a subgroup of $S_n$ of order 2010, and let $\alpha \in G$ be an odd permutation. Prove that $G$ contains exactly 1005 odd permutations.

Consider the map $T_{\alpha} : G \to G$, $x \mapsto \alpha x \alpha^{-1}$.

It is a bijection and sends even permutations to odd ones, and vice versa $\Rightarrow G$ has as many even permutations as odd ones.

(This is analogous to the proof of Theorem 5.7.)

2. Consider the following element of $S_8$, written in the cycle notation:

$$\sigma = (143758)(257)(638)$$

(a) Write it as a product of disjoint cycles.

(b) What is the order of $\sigma$?

(c) Is $\sigma$ odd or even? Justify your answer.

In the table format: \[ \sigma = [\begin{array}{ccccccc}
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
4 & 8 & 1 & 3 & 5 & 7 & 2 & 6
\end{array}] \]

$\Rightarrow \sigma = (143)(2867)$

$|\sigma| = \text{lcm}(3,4) = 3 \cdot 4 = 12$

$\sigma$ is odd since 3-cycles are even and 4-cycles are odd.