Generalized continued fractions, such as \( F(x) = \)

\[
\frac{1}{1 - \frac{w_0 x}{(1 - \frac{w_1 x}{(1 - \frac{w_2 x}{(1 - \frac{w_3 x}{(1 - \frac{w_4 x}{\cdots})} \cdots)}}) \cdots)}}
\]

are generating functions of weighted Łukasiewicz paths, like the path below.

In general, Łukasiewicz paths start and end at height 0, never go below the initial height; the up steps can be arbitrarily large, but the down steps always reduce the height by 1.

The Łukasiewicz paths can encode partitions, permutations, \( n \)-ary trees, and more. We will present the encodings, which let us write the ordinary generating function of the encoded objects as a generalized continued fraction.