1. Let $A$ be the matrix
\[
\begin{bmatrix}
2 & 4 & 0 \\
-3 & -4 & -2 \\
6 & 13 & -1 \\
0 & 2 & -2
\end{bmatrix}
\]
(a) (4 points) If $A$ is the matrix for a linear transformation $T: \mathbb{R}^n \to \mathbb{R}^m$, what are $m$ and $n$?
(b) (8 points) Find a basis for $\text{im} \ A$.
(c) (8 points) Apply the Gram-Schmidt algorithm to the basis you obtained in part (b) to find an orthonormal basis for $\text{im} \ A$.
(d) (4 points) Compute the orthogonal projection of $\vec{e}_1$ on $\text{im}(A)$.

2. Compute the determinant of each of the following matrices. Indicate clearly the method being used.

(a) (9 points)
\[
\begin{bmatrix}
4 & 2 & -2 \\
3 & -1 & 2 \\
7 & 6 & -1
\end{bmatrix}
\]
(b) (9 points)
\[
\begin{bmatrix}
-4 & 0 & 0 & 0 & 0 \\
3 & 2 & 0 & 1 & 4 \\
6 & 0 & -2 & 3 & -3 \\
-3 & 0 & 1 & 0 & 0 \\
2 & 0 & -1 & 2 & 0
\end{bmatrix}
\]
(Hint: This matrix doesn’t have very many patterns that give nonzero products.)

3. Determine whether each statement is true or false. If it is true, justify it. If it is false, provide a counterexample.

(a) (6 points) If $A$ is an $m \times n$ matrix, then $A^T A$ is symmetric.
(b) (6 points) If $A$ is an $n \times n$ matrix whose columns are linearly independent, then $\det A \neq 0$.
(c) (6 points) If $T$ is a linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^n$ such that $T(\vec{e}_1), \ldots, T(\vec{e}_n)$ are all unit vectors, then $T$ is an orthogonal transformation.
4. Let \( \mathfrak{B} \) be the basis for \( \mathbb{R}^2 \) consisting of \( \vec{v}_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \) and \( \vec{v}_2 = \begin{bmatrix} 3 \\ 2 \end{bmatrix} \). Consider the linear transformation \( T : \mathbb{R}^2 \to \mathbb{R}^2 \) given by:

\[
T(\vec{v}_1) = 8\vec{v}_1 - 5\vec{v}_2 \\
T(\vec{v}_2) = 13\vec{v}_1 - 8\vec{v}_2.
\]

(a) (5 points) Find the matrix for \( T \) with respect to the basis \( \mathfrak{B} \), \( [T]_\mathfrak{B} \).

(b) (5 points) Find the change-of-basis matrix \( S_\mathfrak{B} \).

(c) (5 points) Find the standard matrix of \( T \).

(d) (5 points) Is \( T \) an orthogonal transformation? If it is, verify that its standard matrix is an orthogonal matrix. If it is not, give an example showing why not.

(e) (4 points) Describe \( T \) geometrically.

5. Let \( \vec{v}_1 = \begin{bmatrix} 2 \\ 4 \\ 7 \\ -2 \end{bmatrix} \) and \( \vec{v}_2 = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 4 \end{bmatrix} \). Let \( V \) be the subspace of \( \mathbb{R}^4 \) spanned by \( \vec{v}_1 \) and \( \vec{v}_2 \).

(a) (4 points) Compute the cosine of the angle between \( \vec{v}_1 \) and \( \vec{v}_2 \). Is this angle acute, right, or obtuse?

(b) (4 points) State the definition of the orthogonal complement of \( V \).

(c) (8 points) Describe \( V^\perp \) as the kernel of some matrix, and find a basis for \( V^\perp \).