Math 10b  Self-Quiz on Section 5.4

1. Let \( g(x) = \int_{\pi}^{x} \sqrt{\tan^{-1} t} \, dt \).

   (a) Find \( g'(x) \).
   (b) Find \( g''(x) \).

2. Let \( g(x) = \int_{\frac{\pi}{2}}^{\frac{\pi}{2}x} \frac{\cos t}{t} \, dt \). Find \( g'(x) \).

3. Let \( g(x) = \int_{1}^{x} (\sec(\pi t^2) + 3) \, dt \). Find the equation of the line tangent line to the graph of \( g(x) \) at \( x = 1 \).

4. Let \( F(x) = \int_{0}^{x} f(t) \, dt \), where \( f \) is the function shown below.

   (a) Find \( F'(4) \).
   (b) Rank the following six numbers from smallest to largest:
   \(-1, \ 0, \ 1, \ F(1), \ \text{and} \ F(6)\).
   (c) How many zeros does the function \( F(x) \) have in the interval \([0, 7]\)? Briefly explain.
   \textbf{Note:} this question doesn’t ask what the zeros are, but rather just how many there are.
   (d) Is \( F(x) \) concave down anywhere in the interval \((0, 7)\)? If so, on what interval(s)?

5. Let \( F(x) = \int_{-1}^{x} \frac{t^2 - 3t}{t^2 + 1} \, dt \). Where in the interval \([-1, +\infty)\) is \( F(x) \) decreasing?

6. Do problem #18 on page 373.

7. Is the following statement is true or false? If it’s true, briefly explain why. If it’s false, give an example (a graph or a formula) that shows that it’s false.

   Suppose that \( f \) is a continuous function that is negative and increasing on \([0, +\infty)\). Then the function \( F(x) = \int_{0}^{x} f(t) \, dt \) is also increasing on \([0, +\infty)\).

   \textbf{Remember:} In mathematics, “True” means that the statement \textbf{must} be true. “False” means that the statement \textbf{may} be false.