1. Set $x^3 - 4x^2 + 3x = 0$, getting $x = 0$, $x = 1$ and $x = 3$. So the graph of $f(x) = x^3 - 4x^2 + 3x$ intersects the $x$-axis at the points $x = 0$, $x = 1$ and and $x = 3$. A sign analysis shows that $f(x) > 0$ on $(0, 1)$ and $f(x) < 0$ on $(1, 3)$. So the area is

$$\int_{0}^{1} (x^3 - 4x^2 + 3x) \, dx - \int_{1}^{3} (x^3 - 4x^2 + 3x) \, dx = \frac{5}{12} - \left(-\frac{32}{12}\right) = \frac{37}{12}.$$ 

2. Set $xe^{-x^2} = 0$, getting $x = 0$ and $e^{-x^2} = 0$. The second equation has no solutions, so $f(x) = xe^{-x^2}$ crosses the $x$-axis at $x = 0$ only. A sign analysis shows that $f(x) < 0$ on $[-1, 0)$ and $f(x) > 0$ on $(0, \sqrt{\ln 3}]$. So the area is

$$-\int_{-1}^{0} xe^{-x^2} \, dx + \int_{0}^{\sqrt{\ln 3}} xe^{-x^2} \, dx.$$ 

Evaluate these integrals using substitution, with $u = -x^2$ and $du = -2x \, dx$. The result is

$$-\left(\frac{1}{2e} - \frac{1}{2}\right) + \frac{1}{3} = \frac{1}{2} - \frac{1}{2e} + \frac{1}{3} = \frac{5}{6} - \frac{1}{2e}.$$

3. Solve $x^2 = 2 - x^2$, getting $x = -1$ and $x = 1$. So the curves intersect at the points $x = -1$ and $x = 1$. A sign analysis shows that $2 - x^2 > x^2$ on $(-1, 1)$, so the area is

$$\int_{-1}^{1} \left[(2 - x^2) - (x^2)\right] \, dx = \int_{-1}^{1} (2 - 2x^2) \, dx = 2x - \frac{2x^3}{3}\bigg|_{-1}^{1} = (2 - \frac{2}{3}) - (-2 + \frac{2}{3}) = \frac{8}{3}.$$ 

4. Solve $\sin x = \cos x$. The only solution in the interval $[0, \pi]$ is $x = \frac{\pi}{4}$. A sign analysis shows that $\cos x > \sin x$ on $[0, \frac{\pi}{4})$ and $\sin x > \cos x$ on $(\frac{\pi}{4}, \pi]$. So the area is

$$\int_{0}^{\pi/4} (\cos x - \sin x) \, dx + \int_{\pi/4}^{\pi} (\sin x - \cos x) \, dx,$$

which equals

$$\left(\sin x + \cos x\right|_{0}^{\pi/4} + \left(-\cos x - \sin x\right|_{\pi/4}^{\pi})$$

$$\left(\sin x + \cos x\right|_{0}^{\pi/4} - \left(\cos x + \sin x\right|_{\pi/4}^{\pi})$$

$$= \left(\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) - (0 + 1)\right) - \left((-1 + 0) - \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)\right)$$

$$= \left(\frac{2}{\sqrt{2}} - 1\right) + \left(\frac{2}{\sqrt{2}} + 1\right) = \frac{4}{\sqrt{2}},$$

which can also be written as $2\sqrt{2}$.

5. The region is shown below:
(a) To compute the area with respect to $x$, we must divide the region into two parts.

\[
\text{Area} = \int_0^2 \frac{1}{2} x^2 \, dx + \int_2^6 \left( -\frac{1}{2} x + 3 \right) \, dx = \left[ \frac{x^3}{6} \right]_0^2 + \left( -\frac{x^2}{4} + 3x \right)_2^6 \\
= \frac{4}{3} + \left( (\,-9 + 18) - (\,-1 + 6) \right) = \frac{4}{3} + 4 = \frac{16}{3}.
\]

(b) First rewrite the two functions as functions of $y$. Solving $y = -\frac{1}{2} x + 3$ for $x$ gives $x = 6 - 2y$. Solving $y = \frac{1}{2} x^2$ for $x$ gives $x = \pm \sqrt{2y}$; choose $x = \sqrt{2y}$ since the region is in the first quadrant.

The interval of integration with respect to $y$ is $[0, 2]$, and $6 - 2y > \sqrt{2y}$ on this interval. So

\[
\text{Area} = \int_0^2 \left[ (6 - 2y - \sqrt{2y}) \right] dy = \int_0^2 \left[ 6y - y^2 - \frac{1}{2} \cdot \frac{2}{3} (2y)^{3/2} \right] dy \\
= \int_0^2 \left[ 6y - y^2 - \frac{1}{3} (2y)^{3/2} \right] dy = 12 - 4 - \frac{8}{3} = \frac{16}{3}.
\]

6. First rewrite $2y = 3 - x$ as a function of $y$: $x = 3 - 2y$. Then find the points of intersection by solving $3 - 2y = y^2$, getting $y^2 + 2y - 3 = 0 \Rightarrow (y + 3)(y - 1) = 0 \Rightarrow y = -3$ and $y = 1$. Note that on the interval $(-3, 1)$, $3 - 2y > y^2$. So the area is

\[
\int_{-3}^1 \left[ (3 - 2y) - y^2 \right] dy = \int_{-3}^1 \left[ 3y - y^2 - \frac{y^3}{3} \right] dy = \frac{32}{3}.
\]

7. (a) The graphs of $b(t)$ and $d(t)$ intersect at $t = -12$, which is outside the interval $[0, 6]$. Note that $b(t) > d(t)$ for all $t \geq 0$. So the area between the two graphs is

\[
\int_0^6 \left( b(t) - d(t) \right) \, dt = \int_0^6 \left( 16 + 2t - (4 + t) \right) \, dt = \int_0^6 (12 + t) \, dt = 12t + \frac{t^2}{2} \bigg|_0^6 = 90.
\]

This area represents the change in the town’s population after 6 years. Since $b(t) > d(t)$, we know that the town’s population increased during this time. More specifically, the population has increased by 90 people.

(b) Initially there were 5000 people in the town, so after 6 years there are 5090 people.