Course Overview

1. General Filtering Thoughts
2. Two-Sided Filters
3. One-Sided Filters
4. Local Trends
General Filtering Thoughts

- Time series tools that can:
  - Estimate Trends
  - Smooth Out Noise
  - Adjust for Seasonality
  - Make Forecasts
  - Separate the Cycle from the Trend

- Somewhat *ad hoc*

- Common, simple rules used in business forecasting

- Related to technical analysis in finance
Two-Sided Filters

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Moving Average Filter

- Uses Data Before & After $time = t$
- Smooths Data Using Future & Past Information
- Can Choose Information Set
- Can Weight Observations Equally
- Can Choose Alternative Weights
- Not Useful for Forecasting
- Can Improve Understanding of the Data
Moving Average Filter

Equally Weighted

\[ y_t^* = \frac{1}{2m + 1} \sum_{j=-m}^{m} y_{t+j} \]  \hspace{1cm} (1)

Alternative Weighting

\[ y_t^* = \sum_{j=-m}^{m} \omega_j y_{t+j} \]  \hspace{1cm} (2)

Weights should have the property:

\[ \sum_{j=-m}^{m} \omega_j = 1 \]  \hspace{1cm} (3)
Global Surface Temperature Anomaly

[Graph showing the trend of surface temperature anomaly from 1850 to 2050.]
Let’s use an equally weighted moving average filter to reduce some of the noise. Try $m = 2$

```stata
use temperature
tsset year
tssmooth ma tempma = tempa, window(2 1 2)
tsline tempa tempma
```
Global Surface Temperature Anomaly

Surface Temp (C) Anomaly

\[ x(t) = \text{tempa: window}(2 \ 1 \ 2) \]
Global Surface Temperature Anomaly

Still pretty noisy. Try $m = 5$

tssmooth ma temp_ma = tempa, window(5 1 5)
tsline tempa temp temp_ma
Global Surface Temperature Anomaly

Surface Temp (C) Anomaly ma: \( x(t) = \text{tempa: window(5 1 5)} \)
Low Frequency Filters

- Can be used to remove the trend
- Allows the trend to change
- Common Macroeconomic Approach
  - Ignore Growth
  - Concentrate on Business Cycle
  - Output Gap
Examples
- Hodrick / Prescott Filter
- Baxter / King
- Butterworth

Type: help filter
The HP Filter:

$$\min_{y_t^*} \sum_{t=1}^{T} (y_t - y_t^*)^2 + \lambda \sum_{t=2}^{T-1} [(y_{t+1}^* - y_t^*) - (y_t^* - y_{t-1}^*)]^2$$  \hspace{1cm} (4)$$

This filter is approximately a two-way moving average with weights subject to a damped harmonic

- $\lambda$ is a positive smoothing parameter
- Smaller $\lambda$, penalizes variability in the cyclical component
- Larger $\lambda$, penalizes variability in the growth component
There is some debate on choosing values for $\lambda$. Here are some suggested values

- $\sqrt{\lambda} = \sigma_1 / \sigma_2$
- Quarterly Data: $\lambda = 1600$
- Annual Data: $\lambda = 100$
- Monthly Data: $\lambda = 14400$

For more, see the original paper posted on Latte

The Model:

\[ \log(\text{GDP}) = \text{Trend} + \text{Cycle} \]
\[ = \text{Growth} + \text{Cycle} \] (5) (6)

HP Filter in Stata

```stata
use gdp
tsfilter hp gdpcycle = lgdp, trend(gdptrend)
```

Note: \( \lambda = 1600 \) is default value
Log GDP & HP Trend
Log GDP & HP Trend

tsline lgdp gdptrend if dateq > tq(2000q1)
HP Cycle

-0.06
-0.04
-0.02
0
0.02
0.04

lgdp cyclical component from hp filter

1947q3 1964q3 1981q3 1998q3 2015q3
dateq

Patrick Herb (Brandeis University)
Filtering Time Series
ECON/FIN 250: Spring 2016
Much of modern macroeconomics applies the HP filter to many series to separate the trend from the cycle:

- GDP
- Consumption
- Investment

Then looks at the comovement between the cyclical components.
One-Sided Filters

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Moving Average Again

\[ y_t^* = \frac{1}{m} \sum_{j=m}^{1} y_{t-j} \]  

(7)

use \texttt{unemploy}
\texttt{tssmooth ma unma = unrate, window(5,0,0)}

\[ y_t^* = \frac{1}{5} \sum_{j=1}^{5} y_{t-j} \]  

(8)
Exponential Weighted Moving Average (EWMA)

- Average of past values
- Decreasing (exponentially) weights
- Related to many ideas / models in economics
  - Adaptive Expectations (Friedman)
  - Rational Expectations
  - Autoregressive Models
  - Riskmetrics
  - GARCH Models
Exponential Weighted Moving Average (EWMA)

EWMA

\[ y_t^* = \alpha y_t + (1 - \alpha) y_{t-1}^* \]  \hspace{1cm} (9)

Recursive substitution reveals declining weights

\[

ty_t^* = \alpha y_t + (1 - \alpha)(\alpha y_{t-1} + (1 - \alpha)y_{t-2}^*) \\
y_t^* = \alpha y_t + \alpha(1 - \alpha)y_{t-1} + (1 - \alpha)^2 y_{t-2}^* \\
\vdots \\
y_t^* = \alpha y_t + \alpha (1 - \alpha)y_{t-1}^* + \alpha (1 - \alpha)^2 y_{t-2} + \ldots + (1 - \alpha)^t y_0^* \]  \hspace{1cm} (10)
Exponential Weighted Moving Average (EWMA)

\[ y_{t-j} \]

Weight \( \alpha = 0.3 \), \( \alpha = 0.2 \), \( \alpha = 0.1 \)

Patrick Herb (Brandeis University)
Estimate the unemployment rate trend

use unemploy
tssmooth exp unewma = unrate, parms(0.1)

- This sets $\alpha = 0.1$
- Stata can find the optimal $\alpha$ by excluding the parms() options
  - Be careful with this!
Exponential Weighted Moving Average (EWMA)

Filtering Time Series

Patrick Herb (Brandeis University)
The Optimality of the EWMA

Suppose your time series comes from the following process:

\[ x_t = x_{t-1} + e_t \quad x_t = \text{random walk} \]
\[ y_t = x_t + \eta_t \quad e_t, \eta_t = \text{white noise} \]

- You do not observe \( x_t \)
- This implies \( y_t \) is a random walk plus noise
- In this case, EWMA is the optimal forecast
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Double Exponential Weighted Moving Average (DEWMA)

Filter Once

\[ y_t^* = \alpha y_t + (1 - \alpha) y_{t-1}^* \]  \hspace{2cm} (11)

Filter Again

\[ y_t^{**} = \alpha y_t^* + (1 - \alpha) y_{t-1}^{**} \] \hspace{2cm} (12)

- The double filter can follow local trends
- The trend can keep going up when the observed \( y_t \) is going down
- Getting initial values for \( y_0^* \), \( y_0^{**} \) can be tricky
Double Exponential Weighted Moving Average

Estimate the unemployment rate trend using the double exponential weighted moving average

```
use unemploy
tssmooth dexp unema = unrate, parms(0.1)
tsline unrate unema
```
Double Exponential Weighted Moving Average

\[ \text{dexp parms(0.1000) = unrate} \]
Holt-Winters Smoother

\[ y_t^* = a_{t-1} + b_{t-1} \]  
\[ a_t = \alpha y_t + (1 - \alpha)(a_{t-1} + b_{t-1}) \]  
\[ b_t = \beta(a_t - a_{t-1}) + (1 - \beta)b_{t-1} \]
Holt-Winters Smoother

Estimate the unemployment rate trend using the Holt-Winters smoother with parameters $\alpha = 0.1, \beta = 0.2$

```
use unemploy
tssmooth hwinters unhw = unrate, parms(0.1 0.2)
tsline unrate unhw
```
Holt-Winters Seasonal Smoother

- Seasonal version of Holt-Winters
- $S_t$ is a repeating seasonal adjustment
- This is the multiplicative version (there is an additive version)
- $s$ specifies the period of the seasonality
  - monthly data would have $s = 12$

\[
y_t^* = (a_{t-1} + b_{t-1})S_t \quad (16)
\]
\[
a_t = \alpha \frac{y_t}{S_{t-s}} + (1 - \alpha)(a_{t-1} + b_{t-1}) \quad (17)
\]
\[
b_t = \beta(a_t - a_{t-1}) + (1 - \beta)b_{t-1} \quad (18)
\]
\[
S_t = \gamma \frac{y_t}{a_t} + (1 - \gamma)S_{t-s} \quad (19)
\]
Estimate the unemployment rate trend using:
- Seasonal Holt-Winters smoother
- Not seasonally adjusted unemployment rate monthly data
- Choose parameters $\alpha = 0.1$, $\beta = 0.2$, $\gamma = 0.05$

```
use unemploy

tssMOOTH shwINTERS unhws = unratensa, //
   parms(0.1 0.2 0.05) period(12)

tSline unratensa unhws if datem > tm(1990m1)
```
Seasonal Holt-Winters & Unemployment Rate NSA

![Graph showing seasonal Holt-Winters and unemployment rate NSA.](image)

- **Unemployment Rate** NSA
- **Datem**
- **Shw parms(0.100 0.200 0.050) = unratensa**

Patrick Herb (Brandeis University)
\[ y_t = (\text{Trend}) + (\text{Seasonal}) + (\text{Cycle}) + (\text{Noise}) \] (20)

- Filters can help separate parts
- Filters can help smooth out the noise
- Not necessarily predictive
- Sometimes difficult to estimate
  - Initial value problems
- Also, a little *ad hoc*
  - Not clear what the data generating process is