Handout 2 on Section 2.3 – Limits of Rational Functions

Limits of Rational Functions come in Three Flavors:
Suppose $r(x) = \frac{f(x)}{g(x)}$ is a rational function.

1. $\lim_{x \to a} g(x) \neq 0$ 
   - In this case, $\lim_{x \to a} r(x) = r(a)$.

2. $\lim_{x \to a} g(x) = 0$ BUT $\lim_{x \to a} f(x) \neq 0$ 
   - We’ll talk about this case in §2.5.

3. $\lim_{x \to a} g(x) = 0$ AND $\lim_{x \to a} f(x) = 0$ 
   - See below for examples of this.

Examples of Case 3 from above:
(For each example, check that $\lim_{x \to a} g(x) = 0$ and $\lim_{x \to a} f(x) = 0$ by plugging $a$ into the numerator and denominator.)

(i) $\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$

(ii) $\lim_{x \to -3} \frac{x^2 - 3x - 18}{x^2 + 4x + 3}$

(iii) $\lim_{x \to 0} \frac{\sqrt{x + 2} - \sqrt{2}}{x}$
(iv) \( \lim_{x \to 0} \frac{\frac{1}{x+3} - \frac{1}{3}}{x} \)

Some other examples:

(v) \( \lim_{x \to 1} f(x) \) where \( f(x) = \begin{cases} x^3 - 3x & \text{if } x \leq 1 \\ 2x \cdot \frac{x}{x+1} & \text{if } x > 1 \end{cases} \)

(vi) \( \lim_{x \to 0} |x| \)