I. Continuity of some familiar functions:

\[ f(x) = \sin x: \text{continuous on } (-\infty, \infty) \]

\[ f(x) = \cos x: \text{continuous on } (-\infty, \infty) \]

\[ f(x) = e^x: \text{continuous on } (-\infty, \infty) \]

\[ f(x) = \ln x: \text{continuous on } (0, \infty) \]

(Same is true of any exponential function \( f(x) = a^x \))

\[ f(x) = \tan x: \text{continuous everywhere except } \ldots -\frac{3\pi}{2}, \frac{-\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \ldots \]

II. Theorem: Combining continuous functions

Suppose \( f(x) \) and \( g(x) \) are continuous at \( x = a \) and that \( c \) is a constant. Then the following functions are also continuous at \( x = a \).

1. \( f + g \)
2. \( f - g \)
3. \( cf \)
4. \( fg \)
5. \( \frac{f}{g} \) if \( g(a) \neq 0 \)
Exercise 1. Determine which of the following functions $h(x)$ are continuous on $(-\infty, \infty)$, and explain why. If a function is not continuous, find the point(s) of discontinuity.

(a) $h(x) = (x^4 - 1) \sin x$  
(b) $h(x) = \frac{x^3}{e^x}$  
(c) $h(x) = \frac{\cos x}{e^x - 4}$

Exercise 2. Remember that if $f(x)$ is continuous at $x = a$, then $\lim_{x \to a} f(x) = f(a)$. Use this fact to find the following limits. Simplify your answers.

(a) $\lim_{x \to \sqrt{e}} (5 + \ln x)$  
(b) $\lim_{x \to \frac{\pi}{4}} (3 \sin x)$

(c) $\lim_{x \to \ln 3} x e^x$  
(d) $\lim_{x \to -1} \frac{2^x}{x^4 + 4}$

III. Theorem: Limits of compositions

Suppose $f(x)$ is continuous at $b$ and $\lim_{x \to a} g(x) = b$. Then $\lim_{x \to a} f(g(x)) = f(b)$, i.e.,

$$\lim_{x \to a} f(g(x)) = f \left( \lim_{x \to a} g(x) \right).$$

In other words, you can take the limit inside the composition.

Example: Find $\lim_{x \to 3} \cos(x^2 - 9)$.

Solution. Here $f(x) = \cos x$ and $g(x) = x^2 - 9$. Note that $\lim_{x \to 3} g(x) = \lim_{x \to 3} (x^2 - 9) = 0$. Since $f(x) = \cos x$ is continuous at $x = 0$, we can apply the theorem:

$$\lim_{x \to 3} \cos(x^2 - 9) = \cos \left( \lim_{x \to 3} (x^2 - 9) \right) = \cos(0) = 1.$$

Exercise 3. Use the theorem to compute the following limits. Simplify your answers.

(a) $\lim_{x \to 0} e^{\sin x + 3}$  
(b) $\lim_{x \to 2} \ln \left( \frac{x^3 e^x}{x + 6} \right)$