Solutions to Class Handout on Modeling with Functions

1. Let $w$ be the width of the rectangle. Since the length of the rectangle is 4 times longer than its width $w$, we get the following picture:

![Rectangle Diagram]

(a) Let $a$ be the area of the rectangle. Note that $w$ is the independent variable and $a$ is the dependent variable. The formula for the function is

$$a = w \cdot (4w) \Rightarrow a = 4w^2.$$

(b) Let $p$ be the perimeter of the rectangle. Note that $w$ is the independent variable and $p$ is the dependent variable. Since $p = w + w + 4w + 4w$, the formula for the function is:

$$p = 10w.$$

2. Let $w$ be the width of the garden and let $l$ be the length. Let $a$ be the area of the garden. Note that $w$ is the independent variable and $a$ is the dependent variable. The formula for the function is

$$a = w \cdot l.$$

Here $a$ is a function of two independent variables, so we must eliminate $l$. The perimeter of the rectangle is 150, so $2w + 2l = 150$. Solving this for $l$ gives $l = 75 - w$. So our final answer is

$$a = w(75 - w) \Rightarrow a = 75w - w^2.$$

3. (a) The cost = $3 \cdot 8 + 3 \cdot 8 + 2 \cdot 10 + 2 \cdot 10 = $88

(b) The cost = $3 \cdot 4 + 3 \cdot 4 + 2 \cdot 14 + 2 \cdot 14 = $80

(c) Let $w$ be the width of the garden and let $l$ be the length. Let $C$ be the cost of fencing in the garden. Note that $w$ is the independent variable and $C$ is the dependent variable. The cost of fencing the garden can be written as follows:

$$C = 3 \cdot w + 3 \cdot w + 2 \cdot l + 2 \cdot l = 6w + 4l$$ dollars.

Here $C$ is a function of two independent variables, so we must eliminate $l$. The fence is 36 feet long, so perimeter of the rectangle is 36, so $2w + 2l = 36$. Solving this for $l$ gives $l = 18 - w$. So the final answer is

$$C = 6w + 4(18 - w) \Rightarrow C = 2w + 72.$$

4. Let $s$ be the length of the side of the base of the container and let $h$ be its height. Let $v$ be the volume of the container. Note that $s$ is the independent variable and that $v$ is the dependent variable. The volume of the container is $v = s^2h$.

Here $v$ is a function of two independent variables, so we must eliminate $h$. Since the surface area of the container is 144, $2s^2 + 4sh = 144$. Solving this for $h$ gives

$$h = \frac{144 - 2s^2}{4s}.$$

So the final answer is

$$v = s^2 \left( \frac{144 - 2s^2}{4s} \right) \Rightarrow v = \frac{144s - 2s^3}{4}.$$
5. Let \( r \) be the radius of the base of the cylindrical can and let \( h \) be the height of the can. Let \( a \) be the surface area of the can. Note that \( r \) is the independent variable and that \( a \) is the dependent variable. The surface area of the can is \( a = 2\pi r^2 + 2\pi rh \).

We must eliminate \( h \). The volume of the can is 100, so \( \pi r^2 h = 100 \); solving for \( h \) gives \( h = \frac{100}{\pi r^2} \). So our final answer is

\[
    a = 2\pi r^2 + 2\pi r \cdot \frac{100}{\pi r^2} \Rightarrow a = 2\pi r^2 + \frac{200}{r}.
\]

6. (a) Let \( t \) be the time that has passed since the car left Boston and let \( d \) be the distance travelled by the car since it left Boston. Note that \( t \) is the independent variable and \( d \) is the dependent variable. The formula for the function is

\[
    d = 70t.
\]

(b) Let \( D \) be the distance between the truck and Boston. Note that \( t \) is the independent variable and \( D \) is the dependent variable. When the truck starts moving, it is 170 miles west of Boston; as it moves, its distance from Boston decreases. The function is

\[
    D = 170 - 60t.
\]

7. Let \( t \) be the time that has passed since the cars left the intersection, and let \( d \) be the distance between them. Note that \( t \) is the independent variable and \( d \) is the dependent variable. After \( t \) hours, the car traveling north has traveled a distance of \( 45t \) miles and the car traveling east has traveled a distance of \( 30t \) miles. So we get the following picture:

From the Pythagorean theorem, we get

\[
    d^2 = (45t)^2 + (30t)^2.
\]

So the formula for the function is

\[
    d = \sqrt{(45t)^2 + (30t)^2}.
\]

8. (a) The value \( v \) of the house is the dependent variable and time \( t \) is the independent variable. We’re told that the function is linear, so we must find the equation of the line.

Let \( t = 0 \) be the time the house was purchased (i.e., eight years ago). Then the points \((0, 242000)\) and \((8, 282000)\) are on the line. So

\[
    m = \frac{282,000 - 242,000}{8 - 0} = 5000.
\]

We already know that the \( v \)-intercept is 242000, so \( b = 242,000 \). So the formula for the function is

\[
    v = 5000t + 242,000.
\]

(b) To find the natural domain, find the set of values of \( t \) for which this function makes sense. Time can’t be negative, so \( t \geq 0 \). In theory, the value of the house can keep increasing at this rate for ever. So the domain is \([0, +\infty)\).
9. (a) The blood glucose level $g$ depends on the time $t$ that has passed since the test began. So $t$ is the independent variable and $g$ is the dependent variable. We’re told that the function is linear (for the first 30 minutes of the test), so we must find the equation of the line.

At the start of the test, the blood glucose level is 90 units, so the point $(0, 90)$ is on the line. After 15 minutes, the blood glucose level is 130 units, so the point $(15, 130)$ is also on the line. So $m = \frac{130 - 90}{15 - 0} = \frac{8}{3}$. We already know that the $g$-intercept is 90, so $b = 90$. So the formula for the function is $g(t) = \frac{8}{3}t + 90$.

(b) We’re told that the function is linear for the first 30 minutes of the test, so the domain is the set \{ $t : 0 \leq t \leq 30$ \}. We can also write this as $[0, 30]$.

(c) The slope is $\frac{8}{3}$, so $\frac{\text{change in blood glucose level}}{\text{change in time}} = \frac{8}{3}$. This says that for each three minutes that pass, the blood glucose level increases by 8 units.