Math 5a        Solutions to Worksheet on Section 2.1

I. Question on records for the one-minute mile.

1. Is the record time a function of the year? Why or why not?

   Solution: If the record time is a function of the year, then the year is the input and
   the record is the output. But this is not a function, since there are years in which
   two world records were set. For example, the year 1954 is matched with two outputs:
   3:59.4 and 3:58.0. This violates the definition of a function.

2. Is the year a function of the record time? Why or why not?

   Solution: If the year is a function of the record time, then the record time is the input
   and the year is the output. Clearly each record is set in one and only one year, so each
   record is matched with one and only one year. So this correspondence is a function.

II. Let \( f(x) = x^2 - 5x + 4 \).

1. What is the independent variable in this function?

   Solution: \( x \) is the independent variable

2. What is the dependent variable in this function?

   Solution: \( f(x) \) is the dependent variable

3. Find the following:

   (a) \( f(0) \)  \hspace{1cm} Solution: \( f(0) = 0^2 - 5 \cdot 0 + 4 = 4 \)
   (b) \( f(-2) \)  \hspace{1cm} Solution: \( f(-2) = (-2)^2 - 5(-2) + 4 = 18 \)
   (c) \( f(\frac{1}{2}) \)  \hspace{1cm} Solution: \( f(\frac{1}{2}) = (\frac{1}{2})^2 - 5 \cdot \frac{1}{2} + 4 = \frac{7}{4} \)
   (d) \( f(\sqrt{3}) \)  \hspace{1cm} Solution: \( f(\sqrt{3}) = (\sqrt{3})^2 - 5 \cdot \sqrt{3} + 4 = 7 - 5\sqrt{3} \)

4. In part (b) of the previous question, you should get \( f(-2) = 18 \). Which number is the input in this statement? Which is the output?

   Solution: \(-2\) is the input, \(18\) is the output.

III. The piecewise function \( f(x) = \begin{cases} 49 \text{ cents} & \text{if } 0 < x \leq 1 \\ 71 \text{ cents} & \text{if } 1 < x \leq 2 \\ 93 \text{ cents} & \text{if } 2 < x \leq 3 \end{cases} \) gives the cost (in cents) of mailing a first-class letter that weighs \( x \) ounces, where \( x \leq 3 \).

1. Find \( f(2) \).

   Solution: Since \( 2 \) is in the interval \((0, 2]\), \( f(2) = 71 \).

2. How much does it cost to mail a first class letter that weighs 2.75 ounces?

   Solution: \( f(2.75) = 93 \text{ cents} \), so it costs 93 cents to mail a first class letter that weighs 2.75 ounces.

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IV. Let \( f(x) = x^2 - 4 \). Find the following, and then simplify. Note: in this problem, \( a \) and \( h \) are arbitrary, unspecified real numbers.

1. \( f(a) \)
   Solution: \( f(a) = a^2 - 4 \)

2. \( f(a + h) \)
   Solution: \( f(a + h) = (a + h)^2 - 4 = a^2 + 2ah + h^2 - 4 \)

3. \( f(a + h) - f(a) \)
   Solution: \( f(a + h) - f(a) = [(a + h)^2 - 4] - (a^2 - 4) = a^2 + 2ah + h^2 - 4 - (a^2 - 4) = 2ah + h^2 \)

4. \( \frac{f(a + h) - f(a)}{h} \)
   Solution: \( \frac{f(a + h) - f(a)}{h} = \frac{[(a + h)^2 - 4] - (a^2 - 4)}{h} = \frac{a^2 + 2ah + h^2 - 4 - (a^2 - 4)}{h} = \frac{2ah + h^2}{h} = 2a + h \)

V. Find the domain of each of the following functions.

1. \( f(x) = \frac{1}{x^2 - 5x - 6} \)
   Solution: We can’t have zero in the denominator, so we must find all values of \( x \) that make the denominator zero and then exclude them from the domain. We solve \( x^2 - 5x - 6 = 0 \), getting \((x + 1)(x - 6) = 0 \) \( \Rightarrow x = -1, x = 6 \). So the domain of \( f(x) \) is all real numbers except \(-1\) and \(6\). If we write this in set-builder notation, we get \( \{ x : x \neq -1, x \neq 6 \} \).

2. \( f(x) = \sqrt{4 - x} \)
   Solution: We can’t have a negative number under the square root the domain of \( f(x) \) will be all values of \( x \) that make the denominator \( \geq 0 \). To find this set, we solve the inequality \( 4 - x \geq 0 \) for \( x \). The solution set is \( x \leq 4 \), so the domain of \( f(x) \) is all real numbers \( x \leq 4 \). If we write this in set-builder notation, we get \( \{ x : x \leq 4 \} \).

3. \( f(x) = \sqrt[3]{x + 5} \)
   Solution: We can take the cube root of any real number. So \( x \) can take on any value, so the domain of \( f(x) \) is all real numbers.