Most people who study mathematics do so in order to apply it to real-world problems in physics, biology, economics, and other fields. So it’s important to learn to do applied problems (that is, word problems). The problems in Section 2.6 involve applications of functions.

There is no simple algorithm for these problems, but it helps to remember that there are two things that you must do in each such problem:

1. Find the independent and dependent variables
2. Find the formula for the function that relates them.

Note that you will need to know the following formulae:

- Suppose that a circle has radius $r$. The formula for the area of the circle is $a = \pi r^2$. The formula for the circumference is $c = 2\pi r$.

- Suppose that a rectangle has width $w$ and length $l$. The formula for the area of the rectangle is $a = wl$. The formula for the perimeter is $p = 2w + 2l$.

- Suppose that the base of a rectangular solid is a square with sides of length $s$, and suppose that the height of the solid is $h$.

![Diagram of a rectangular solid](image)

The formula for the volume of the solid $v = s^2 h$. (This is just the area of the base times the height.) The formula for the surface area, assuming the box has a top, is $a = 2s^2 + 4sh$. (This is just the sum of the areas of the six sides: $s^2 + s^2 + sh + sh + sh + sh = 2s^2 + 4sh$.)

- Suppose that the base of a cylinder is a circle of radius $r$, and suppose that the height of the cylinder is $h$.

![Diagram of a cylinder](image)

The formula for the volume of the cylinder is $v = \pi r^2 h$. (This is just the area of the base times the height) . The formula for the surface area, assuming the cylinder has a top, is $a = \pi r^2 + \pi r^2 + 2\pi rh \Rightarrow a = 2\pi r^2 + 2\pi rh$. (To see why, draw a picture; imagine cutting the top and bottom off the solid, and then cutting the cylinder straight up the side and unrolling it into a rectangle.)
**Example 1.** Express the area of a circle as a function of the radius of the circle.

**Solution:** We’re told that the area is a function of the radius, so the area depends on the radius. So the radius is the independent variable; let’s call it \( r \). The area is the dependent variable; let’s call it \( a \). In this example, it’s easy to find the formula relating \( a \) and \( r \):

\[
a = \pi r^2.
\]

We’ve written \( a \) as a function of \( r \), so we’re finished.

**Note:** We can also write \( f(r) = \pi r^2 \).

**Example 2.** The length \( l \) of a rectangle is 3 units more than its width \( w \). Express the perimeter of the rectangle as a function of the width \( w \).

**Solution:** Let \( w \) be the width of the rectangle and let \( p \) be the perimeter of the rectangle. We’re told that the perimeter is a function of the width, so \( w \) is the independent variable and \( p \) is the dependent variable. Since the length of the rectangle is 3 units more than its width \( w \), we get the following picture:

![Diagram of a rectangle with dimensions labeled as w and w+3]  
The formula for the function is

\[
p = 2w + 2(w + 3) \quad \text{or} \quad p = 4w + 6.
\]

**Example 3.** A rectangular garden has a perimeter of 34 feet. Express the area of the garden as a function of the width of the garden.

**Solution:** Let \( w \) be the width of the garden and let \( l \) be the length. Let \( a \) be the area of the garden. Note that \( w \) is the independent variable (we know this because we are asked to express the area as a function of \( w \)) and \( a \) is the dependent variable. The area can be written as follows:

\[
a = wl.
\]

However, this won’t suffice for a final answer, since here \( a \) is a function of two variables, namely \( w \) and \( l \). We must find a way eliminate \( l \). Here’s how. We’re told that the perimeter of the rectangle is 34. So \( 2w + 2l = 34 \). Solving this equation for \( l \) gives \( l = 17 - w \). So we can replace the \( l \) in the formula for our function with the expression \( 17 - w \). We get

\[
a = w(17 - w) \quad \text{or} \quad a = 17w - w^2.
\]

**Example 4.** A magazine publishing company had a profit of $98,000 per year when it had 32,000 subscribers. When it had 35,000 subscribers, it had a profit of $116,000. Assume that the profit is a linear function of the number of subscribers. Write a formula for that function.
Solution: First note that we did this kind of problem in Section 1.10. The only difference is that now the problem will tell us which quantity is the independent variable and which quantity is the dependent variable.

We’re told that profit is a function of the number of subscribers, so the independent variable $x$ is the number of subscribers and the dependent variable $y$ is the profit. From now on, we’re in familiar territory. Since the function relating $x$ and $y$ is linear, we need to find two points on the line. When $x = 32,000$, $y = 98,000$, so the point (32000,98000) is on the line. Similarly, the point (35000,116000) is on the line. So $m = \frac{116,000 - 98,000}{35,000 - 32,000} = 6$. Solving for $b$ gives $b = -94,000$. So the linear function we’re after is

$$f(x) = 6x - 94,000 \quad \text{or} \quad y = 6x - 94,000.$$ 

Comment: When we are talking about linear functions, it is easiest to use the slope-intercept form of the line, and write $y = mx + b$ (or $f(x) = mx + b$).