Due Dates for Homework Assignments

Section 2.1 (and 1.11) Wednesday October 11
Section 2.2 Monday October 16
Section 2.3 Wednesday October 18
Section 2.4 Thursday October 19
Section 2.5 Monday October 23
Section 2.6 Wednesday October 25
Section 2.7 Thursday October 27
Modeling with Functions Wednesday November 1

Homework Assignments

I. Section 2.1.

Part A. Do the following problems on pages 149–152. Note: Make sure you simplify your answers.

#21, 23, 30, 31, 36–38, 48–56 evens only, 71ab.

Part B. Suppose that the state income tax in a midwestern state is assessed as follows:
Income that is ≤ $20,000 is taxed at a rate of 2.5%; income that is greater than $20,000 but less than $100,000 is taxed at a rate of 7%; income ≥ $100,000 is taxed at a rate of 10%.

1. Write a piecewise function that gives the state tax rate $R$ as a function of income $I$.
2. How much state income tax does a resident of this state who earning an income of $99,000 have to pay? How much tax does a person earning an income of $100,000 have to pay?

II. Section 2.2

Part A. Do the following problems on pages 159–62.

#4, 36, 38, 50, 51abc.

Part B. Graph the following functions. Be sure to label your axes carefully.

1. $f(x) = \begin{cases} -2, & \text{if } x \leq -2 \\ |x|, & \text{if } -2 < x \leq 1 \\ \sqrt{x}, & \text{if } x > 1 \end{cases}$
2. $g(x) = \begin{cases} \frac{1}{x}, & \text{if } x \leq -1 \\ x^3, & \text{if } -1 < x < 1 \\ 4, & \text{if } x \geq 1 \end{cases}$

III. Section 2.3

Part A. Do the following problems on pages 168–71. In Problem 5e, write your answer using interval notation.

#5, 7, 8, 44.

Part B. Consider the function $f(x)$ whose graph is shown on the next page. Use the graph to answer the following questions.
1. What is $f(-2)$? What is $f(0)$?

2. For what values of $x$ is $f(x) = 0$?

3. On what interval(s) is $f(x)$ positive? On what interval(s) is $f(x)$ negative?

4. On what interval(s) is $f(x)$ increasing? On what interval(s) is $f(x)$ decreasing?

Graph of $f(x)$:

IV. Section 2.4.
Part A. Do the following problems on pages 177–79.

#6, 8, 11, 16, 19, 22, 25.

Part B. The drain on a water tank is opened and water starts flowing out. The volume of water in the tank $t$ minutes after the valve is opened is given by the function $V(t) = 50 - 15t + t^2$ gallons. Find the average rate at which the volume of water changes from $t = 1$ minutes to $t = 3$ minutes. Your answer should be negative. Why does this make sense?

V. Section 2.5.
Part A. Do the following problems on pages 187–190.

#58, 60, 64.

Part B. Make an accurate sketch of the graph of each of the following two functions:

1. $y = -2(x + 1)^3$
2. $y = 1 - \sqrt{x - 1}$

Part C. The annual sales of a company can be modeled by the function $f(t) = 3 + .02t^2$, where $t$ represents years since 1995 and $f(t)$ is measured in millions of dollars. Suppose you want $t$ to represent years since 2005, rather than years since 1995. What transformation could you apply to the function $f(t)$ to accomplish this? Write the new function $g(t)$ that results from this transformation. (You don’t need to simplify $g(t)$.)
VI. Section 2.6.

Part A. Do the following problems.

1. Let \( f(x) = \sqrt{x+3} \) and let \( g(x) = x^2 - 4 \). Find the following. You do not have to simplify your answers.

   a. \( (f + g)(x) \)  
   b. \( (fg)(x) \)  
   c. \( (f - g)(1) \)  
   d. \( \left( \frac{g}{f} \right)(2) \)  
   e. \( \left( \frac{f}{g} \right)(2) \)  
   f. \( (f \circ g)(x) \)  
   g. \( (g \circ f)(x) \)  
   h. \( (f \circ f)(1) \)  
   i. \( (g \circ g)(13) \)  

2. Let \( f(x) = \frac{x - 1}{x} \) and \( g(x) = x + 5 \). For what value(s) of \( x \) so that \( (f \circ g)(x) = 0 \)?

Part B. Do the following problems on pages 196–8.

#26ab, 27–32, 61, 66.

Part C. For each of the following functions, find two functions \( f \) and \( g \) such that \( h(x) = (f \circ g)(x) \). (You may not use \( f(x) = x \) or \( g(x) = x \).)

   a. \( h(x) = (x - 7)^5 \)  
   b. \( h(x) = \sqrt[3]{2x^3 - 1} \)  
   c. \( h(x) = \frac{1}{\sqrt{3x^2 + 5}} \)  
   d. \( h(x) = \frac{2}{x^5 - 4x} \)  
   e. \( h(x) = |7x - 2| + 5 \)

VII. Section 2.7. Do the following problems on pages 204–06.

# 7, 8, 12–14, 20, 22ab, 28, 40, 41, 43, 79, 81, 86.

Hint for homework problems #12–14, 20: it’s easiest to do these problems graphically, using the horizontal line test.

VIII. Modeling with Functions.

1. A rectangular box with a square base is constructed out of wood. The volume of the box is 12 cubic feet.

   (a) Express the surface area of the box as a function of the length of the side of the base. Assume that the box has a top.

   (b) The top and the bottom of the box are made of birch, which costs $4.00 per square foot. The four sides of the box are made of pine, which costs $2.50 per square foot. Express the total cost of making the box as a function of the length of the side of the base. (Assume no wood is wasted during construction.)

2. A cylindrical soda can holds 25 cubic centimeters of liquid. Express the surface area of the cylinder as a function of the radius of the base.

OVER FOR REST OF ASSIGNMENT →
3. A car leaves an intersection and travels due south at a constant speed of 30 mph. Two hours later, a second car leaves the same intersection and travels due east at a constant speed of 40 mph. Find the distance between the two cars $t$ hours after the second car left the intersection. Note: you do not have to simplify your answer.

4. A hot-air balloon rises vertically into the air at a rate of 3 meters per second. A camera is mounted on the ground 50 meters away from the point of lift-off (see below). Express the distance between the camera and the balloon as a function of the number of seconds that have passed since the balloon lifted off.

5. The inside of a cylindrical thermos is covered with glass on its bottom and its sides. The total surface area of the glass is $300\pi$ square cm. Express the volume of the thermos as a function of the radius of its base.

6. A computer workstation was purchased for $10000. Thereafter, its value $v$ depreciated as a linear function of time $t$, and 2 years later the machine was worth $2000.
   (a) Express $v$ as a function of $t$. (Hint: let the date of purchase count as $t = 0$.)
   (b) At what point does the graph of the function cross the $t$-axis? What interpretation does this point have for the value of the computer workstation?

7. The stopping distance $D$ (at some fixed speed) of regular tires on glare ice is a function of the air temperature $F$, in degrees Fahrenheit. Tests have shown that when the temperature is $10^\circ$ the stopping distance is 135 ft, and when the temperature is $-20^\circ$ the stopping distance is 75 ft.
   (a) Express $D$ as a linear function of $F$.
   (b) Note that the domain of the function $D$ has certain restrictions. The stopping distance of a vehicle can’t be negative. Moreover, ice occurs only when $F \leq 32^\circ$. Find the domain of the function $D$. 