Reading Assignments

As usual, here we tell you which pages of each section you must read and which you can skip. We also give comments to help you with the reading and hints to help you with the homework.

I. Section 2.1. We will do everything in Section 2.1.

IMPORTANT: You have probably studied functions before. However, it’s possible that you never got a very clear idea of what a function actually is. You may have thought a function was simply a formula that you could plug numbers into or that you could graph. But the basic idea behind a function is much deeper. Before you can study functions seriously you need to understand this basic idea: a function is a relationship between two quantities, where one quantity depends on the other.

For example, you know from experience that the cost of mailing a first-class letter depends on its weight. Here we have two quantities: cost and weight. One quantity depends on the other: the cost depends on the weight. Therefore the cost is a function of the weight.

Here’s another example. Suppose that a population of bacteria is growing in a test tube and that initially there are 1000 bacteria in the tube. Suppose that you know that the population doubles every 5 hours. Again we have two quantities: the size of the population and the time that has passed since the experiment began, and the size of the population depends on how much time has passed. So the size of the population is a function of the time that has passed since the experiment began. We’ll see later that the formula for this function is

\[ f(t) = 1000 \cdot 2^t. \]

Here’s one more example. Suppose that you have a circular oil spill. (In real life, it’s unlikely that an oil spill would be exactly circular, but often scientists studying such phenomena model it with a simplified situation and then progress to more complicated and realistic situations.) Clearly the area of the oil spill depends on the radius of the circle. So the area is a function of the radius. In this case, it’s easy to write down a formula for this function. If we call the radius of the oil spill \( r \) and we call the area \( a \), then the formula is

\[ a = \pi r^2, \]

which we can also write as \( f(r) = \pi r^2 \) or \( a(r) = \pi r^2 \).

As you read through Chapter 2, keep in mind this basic idea of a function as a relationship between two quantities.

Here are some comments on the reading and hints for the homework:
• Make sure that you know and understand the definition of a function that’s given in the middle of page 143. You will need to use it. We may also ask you to state it on quizzes or exams.

• Make sure that you know all the vocabulary for functions given on page 143.

• Since this is a math course, most of the functions we’ll work with can be expressed as mathematical formulae. Examples 1–5 on pages 144–6 in Section 2.1 are examples of such functions. Notice that in each example you are asked to evaluate the function at a value of \( x \); this just means finding the \( f(x) \)-value that corresponds to that particular \( x \).

For example, consider the function \( f(x) = x^2 + 1 \). If we evaluate the function at \( x = 4 \), we get \( f(4) = (4)^2 + 1 = 17 \). So the function matches the \( x \)-value 4 with the \( f(x) \)-value 17. Similarly, if we evaluate the same function at \( x = \pi \), we get \( f(\pi) = (\pi)^2 + 1 = \pi^2 + 1 \). So the function matches the \( x \)-value \( \pi \) with the \( f(x) \)-value \( \pi^2 + 1 \).

Finally, remember that the \( f(x) \)-value is usually called the \( y \) value. It is also called the output of the function. The \( x \)-value is called the input of the function.

• Example 3 on page 145 gives an example of a piecewise defined function. Such functions will be important in this course, so study this example carefully.

• Make sure you understand how to evaluate expressions like \( f(2 + h) \) or \( f(a + h) \). (See Example 4c on page 145). If you have trouble doing this, then start by replacing the \( x \) in the formula for the function with the word “input”. For example, \( f(x) = x^2 + 1 \) can be rewritten as

\[
f(\text{input}) = (\text{input})^2 + 1.
\]

Now suppose that you need to find \( f(2+h) \). The input here is \( 2+h \). Since \( f(\text{input}) = (\text{input})^2 + 1 \), we get

\[
f(2 + h) = (2 + h)^2 + 1 = 5 + 4h + h^2.
\]

• Example 4d on page 145 computes and simplifies the expression \( \frac{f(a + h) - f(a)}{h} \). We will emphasize this type of computation, since it is crucial in calculus. Note that \( \frac{f(a + h) - f(a)}{h} \) is not the same as \( \frac{f(a) + f(h) - f(a)}{h} \). Here’s an example that shows why not:

**Example.** Let \( f(x) = x^2 \). Compute \( \frac{f(a + h) - f(a)}{h} \).

**Solution:**

\[
\frac{f(a + h) - f(a)}{h} = \frac{(a + h)^2 - (a)^2}{h} = \frac{a^2 + 2ah + h^2 - a^2}{h} = \frac{2ah + h^2}{h} = 2a + h.
\]

**Important:** This is NOT the same as

\[
\frac{f(a) + f(h) - f(a)}{h} = \frac{(a)^2 + (h)^2 - (a)^2}{h} = \frac{h^2}{h} = h \quad \leftarrow \text{THIS IS WRONG!}
\]
• A function like \( f(x) = 7 \) is called a \textbf{constant} function. This means that for any input \( x \), the output is the constant 7. For example, \( f(3) = 7 \), \( f(0) = 7 \), \( f(-14) = 7 \), \( f(100) = 7 \), \( f(\frac{1}{3}) = 7 \), \( f(a) = 7 \), \( f(a + 1) = 7 \), and so on. In other words, \( f(\text{anything}) = 7 \).

• Hint for homework problems #21 and 23: In problem #21, you must find \( g(a - 1) \). Remember that the entire expression \( a - 1 \) is the input. Similarly, in problem #23, you must find \( f(x + 1) \); in this case the entire expression \( x + 1 \) is the input.

• Hint for homework problem #31: You are asked to find two different expressions: \( f(x + 2) \) and \( f(x) + 2 \). They are not the same!

• Hint for homework problems #36–38: these problems ask you (among other things) to find and simplify the expression \( \frac{f(a + h) - f(a)}{h} \). See Example 4d on page 145 for an example. Remember that \( \frac{f(a + h) - f(a)}{h} \neq \frac{f(a) + f(h) - f(a)}{h} \).

In problem #37, note that \( f(x) = 5 \) is a constant function, so \( f(\text{anything}) = 5 \).

• Hint for homework problem #56: you must solve a quadratic inequality.

II. Section 2.2. We will do everything in Section 2.2 except Example 2 (using a graphing calculator), Example 3 (families of power functions) and Example 6 (the greatest integer function).

Here are some comments on the reading and hints for the homework:

• If the definition of the graph of a function (top of page 153) and the picture next to it confuse you, then just think of a familiar example like \( f(x) = x^2 \). See Figure 3a on page 154, where the usual points are labeled, namely \((0,0)\), \((1,1)\), \((-1,1)\), etc. We could also label a “general” point as \((x,x^2)\). Since \( x^2 \) is just another name for the output \( f(x) \), so we could also label this “general” point as \((x,f(x))\). Keeping this in mind, reread the definition on page 153.

• You must know how to accurately graph all the following functions:

\[
\begin{align*}
  f(x) &= x^2; \\
  f(x) &= x^3; \\
  f(x) &= |x|; \\
  f(x) &= \sqrt{x}; \\
  f(x) &= \frac{1}{x}; \\
  f(x) &= \frac{1}{x^2}.
\end{align*}
\]

These functions are all shown in the text, as well as in a handout called called \textbf{The Library of Basic Functions}, which is posted on LATTE. The best way to graph these functions accurately is to remember the general shape of each, along with a few key “reference points”. For example, to graph \( f(x) = x^3 \) you need to remember the general S-like shape of the curve and to remember some reference points like \((0,0)\), \((1,1)\), \((2,8)\), \((-1,-1)\) and \((-2,-8)\).

• You also need to know how to graph two functions whose graphs we saw in Section 1.10: \( f(x) = c \) (constant function), and \( f(x) = mx + b \) (linear function). You do \textbf{not} need to know how to graph \( g(x) = x^4 \), \( f(x) = x^5 \), \( f(x) = \sqrt{x} \), or \( f(x) = \sqrt[3]{x} \).

• The text discusses the vertical line test on page 157. Make sure you can explain \textit{why} this test works.
III. Section 2.3: We will cover the material on page 163 through the middle of page 165. We will not cover the material on the local maximum and minimum values of a function.

Here are some comments on the reading and hints for the homework.

- The section begins with a discussion of how to read function values from a graph. It goes on to discuss how to find the domain and range of a function from its graph. Study Figure 2 at the bottom of page 163; it will help you understand how to find the domain and range of a function if you are just given the graph of the function.

- The text then defines what it means for a function to be increasing on an interval or decreasing on an interval. This topic is essential in calculus. Here’s an example:

The graph of the function $f(x)$ with domain $(-\infty, +\infty)$ is shown below. Then

- $f(x)$ is increasing on the intervals $(-\infty, -3)$ and $(0, 5)$
- $f(x)$ is decreasing on the intervals $(-3, 0)$ and $(5, +\infty)$

Notice two things:

1. The intervals are intervals on the $x$-axis, not the $y$-axis. We always use intervals on the $x$-axis to describe where a function is increasing or decreasing.

2. The intervals are open intervals. Unfortunately, our textbook uses closed intervals. Calculus textbooks use open intervals, so it is a good idea for you to do the same. However, you may use either open or closed intervals, as long as you are consistent.

IV. Section 2.4. We will cover everything in Section 2.4.

Here are some comments on the reading and hints for the homework.

- The formula for the average rate of change of a function $f(x)$ between $x = a$ and $x = b$ is given in the blue box in the middle of page 173. This formula is quite straightforward. Make sure that you study the graph that accompanies the definition in the blue box. It is of central importance in calculus.

- Note that in Example 2b on page 174 you have to compute $\frac{d(a + h) - d(a)}{h}$. Remember that we did this kind of computation in Section 2.1.

- In Example 3c on pages 175–6 the average rate of change is negative. This is because the temperature is falling. In general, if a function measures a quantity that is decreasing over the interval $[a, b]$, then the average rate of change on that interval will be negative.
• Hint for homework problem #6: find $\frac{f(5) - f(1)}{5 - 1}$ by reading off the appropriate coordinates from the graph that’s given. The same idea works for problem #8.

• Hint for homework problem #19: you’re finding $\frac{f(a + h) - f(a)}{h}$ for $f(t) = \frac{2}{t}$. This is similar to work you did in Section 2.1. (See Example 2b on page 174.)

V. Section 2.5. We will do everything in Section 2.5 except horizontal shrinking/stretching of graphs (middle of page 184–middle of page 185) and even and odd functions (middle of page 185–the end of the section).

Here are some comments on the reading and hints for the homework:

• This section is quite straightforward. The main thing to understand about transformations is the following:

1. **Vertical shifting.** When you are graphing, for example, a function of the form $y = f(x) + 3$, you are actually adding 3 to the $y$-coordinate of each point on the graph of $f(x)$. The result is to shift the graph of $f(x)$ up by 3 units.

   On the other hand, the function $y = f(x) - 3$ shifts the graph of $f(x)$ down by 3 units.

2. **Horizontal shifting.** When you graph a function of the form $y = f(x + 4)$, you are actually subtracting 4 from the $x$-coordinate of each point on the graph of $f(x)$. (This is counter-intuitive.) The result is to shift the graph of $f(x)$ to the left by 4 units.

   On the other hand, the function $y = f(x - 4)$ shifts the graph of $f(x)$ to the right by 4 units.

3. **Vertical stretching/shrinking.** When you graph a function of the form $y = 2f(x)$, you are actually multiplying the $y$-coordinate of each point on the graph of $f(x)$ by 2. The result is to stretch the graph of $f(x)$ vertically by a factor of 2.

   On the other hand, the function $y = \frac{1}{2}f(x)$ shrinks the graph of $f(x)$ vertically.

4. **Reflecting graphs.** When you graph a function of the form $y = -f(x)$, you are actually multiplying the $y$-coordinate of each point on the graph of $f(x)$ by $-1$. The result is to reflect the graph across the $x$-axis. Similarly, when you graph $y = f(-x)$, you are multiplying the $x$-coordinate of each point on the graph of $f(x)$ by $-1$. The result is to reflect the graph across the $y$-axis.

• Hint for homework problem #64: Here there is no formula for the function $g(x)$ so you can’t graph the transformations by making use of a formula. However, you can easily find the $x$- and $y$-coordinates of a number of points on the graph of $g(x)$. Then you can apply the appropriate transformation to the $x$- or $y$-coordinate of each of these points. For example, in part (f), you would multiply the $y$-coordinate of each point by 2.
IV. Section 2.6: We will do everything in this section except graphical addition (Example 2 on page 192) and finding the domain of compositions of functions (the domain parts of Example 3a on page 193 and Example 4 on page 194).

Here are some comments on the reading and hints for the homework:

- In general, the material in this section is quite straightforward. Remember not to worry about domains of compositions of functions as you read the examples in the text.

- A comment on the notation for compositions of functions: you may find it easier to rewrite \((f \circ g)(x)\) as \(f(g(x))\) and to rewrite \((g \circ f)(x)\) as \(g(f(x))\). But you do need to know what the symbol \(\circ\) means.

- You will notice that there are a lot of homework problems involving decomposition of functions. This is something that the book doesn’t pay sufficient attention to; it only does one example on the topic (Example 6). But we will emphasize decomposition of functions in class, since it is essential to know for calculus.

- Hint for homework problem #27: Find \(g(2)\) from the graph. Then use the result as the input for \(f(x)\). The same kind of idea will work for #28–32.

VII. Section 2.7: We will do everything in this section. Here are some comments on the reading and hints for the homework:

- The basic idea of a one-to-one function, as shown in Figure 1 on page 199, should make sense to you. The algebraic definition (given in the blue box on page 199) may seem more difficult. Don’t worry: in general we won’t ask you to work with that definition. Instead, we’ll use the horizontal line test (also given on page 199).

  So, for instance, in Example 3 on page 200, we would expect you to recognize that the graph of \(f(x)\) is a non-horizontal line and therefore passes the horizontal line test, proving that \(f(x)\) is one-to-one. You would not have not use the algebraic definition of one-to-one, although of course you are welcome to.

- It may take a little time to absorb the definition of the inverse of a function given at the top of page 201. Read it several times, making sure you go through the end of Example 4.

- It is extremely important to realize that \(f^{-1}(x)\) does NOT mean \(\frac{1}{f(x)}\). (See the comment in the margin of page 227.) Instead, \(f^{-1}(x)\) means the inverse function of \(f(x)\).

- The Inverse Function Property given in the blue box on page 201 is the key property of inverse functions. Whenever we ask you to check that two functions are inverses (unless the functions are given by graphs) you can use this property. See Example 5 on page 202.

- The algorithm for finding the formula for the inverse of a function is given in the middle of page 202. It is clear and straightforward.

- Hint for homework problems #12–14, 20: you can do these problems graphically, using the horizontal line test.
• Hint for homework problem #43: the algebra needed in this kind of problem is a little tricky. Start as follows:

\[ y = \frac{1}{x + 2} \implies y(x + 2) = 1 \implies yx + 2y = 1. \]

Then solve for \( x \).

VIII. Modeling with Functions. The last topic in Chapter 3 is called Modeling with Functions, and it covers applications of functions (word problems). We will cover this topic using a handout instead of the text. So your reading assignment for this section is the handout called Introduction to Modeling with Functions. This handout is posted on LATTE, in both the reading assignments section and the handouts section.