Due Dates for Reading Assignments
Note: There will be a very short online reading quiz on each reading assignment due one hour before class on its due date.

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Reading Assignments
Here we tell you which pages of each section in the text you must read and which you can skip. We also give comments to help you with the reading, as well as hints to help you with the homework.

I. Section 4.1. We will cover everything in this section except for compound interest (page 306–end of the section). Here are some comments on the reading and hints for the homework:

- In the exponential function \( f(x) = a^x \), the base \( a \) is \( > 0 \). (See the blue box at the bottom of page 302.) Moreover, \( a \neq 1 \) since the function \( f(x) = 1^x \) is a constant function rather than an exponential one.

- The blue box in the middle of page 304 shows the graphs and key features of the exponential functions \( f(x) = a^x \) and \( g(x) = a^{-x} \). Here is a more thorough list of the key features of these functions:

  1. The domain of \( f(x) = a^x \) is \((-\infty, +\infty)\). The domain of \( g(x) = a^{-x} \) is also \((-\infty, +\infty)\).
  2. The range of \( f(x) = a^x \) is \((0, +\infty)\). The range of \( g(x) = a^{-x} \) is also \((0, +\infty)\).
  3. The \( y \)-intercept of \( f(x) = a^x \) is the point \((0, 1)\) since \( a^0 = 1 \). The same is true for \( g(x) = a^{-x} \).
  4. \( f(x) = a^x \) has no \( x \)-intercepts, since \( a^x > 0 \) for all values of \( x \). For the same reason, \( g(x) = a^{-x} \) has no intercepts.
  5. \( f(x) = a^x \) is increasing on \((-\infty, \infty)\), whereas \( g(x) = a^{-x} \) is decreasing on \((-\infty, \infty)\).

- Property #4 in the above list is particularly important since it shows that the exponential equation

\[
 a^x = 0
\]

has no solutions. Similarly, the equation \( a^{-x} = 0 \) has no solutions. For example, equations like \( 3^x = 0 \) and \( 2^{-x} = 0 \) have no solutions. This will be relevant when we solve more complicated equations like the following:

\[
 3^x \cdot x^2 - 3^x \cdot 16 = 0.
\]

To solve this equation, start by factoring out \( 3^x \), getting \( 3^x(x^2 - 16) = 0 \). From this it follows that \( 3^x = 0 \) or \( x^2 - 16 = 0 \). But \( 3^x = 0 \) has no solutions, so the only solutions to the original equation are \( x = \pm 4 \).
II. Section 4.2. We will cover everything in this section except for continuously compounded interest (page 312). We will rarely use a calculator to evaluate exponential functions (see example 1). But there will be some homework problems where you will need a scientific calculator.

Here are some comments on the reading and hints for the homework:

- The natural exponential function \( f(x) = e^x \), which is the subject of this section, is the most important of all the exponential functions, especially in calculus.

**A comment on \( e \):** the number \( e \) is an extremely important number in mathematics, as well as in the physical and natural sciences, but it can be somewhat confusing at first. The best way to think of \( e \) for now is this: (1) it’s a special irrational number (just like \( \pi \)); (2) its value is about 2.718; (3) for reasons that are beyond the scope of this discussion, it’s the ideal base to use for an exponential function.

- Make sure that you can sketch an accurate graph of the function \( f(x) = e^x \) (see Figure 2 on page 311). The simplest way to do this is the following:
  1. Plot the points \((1, e)\), \((0, 1)\) and \((-1, \frac{1}{e})\). Remember that \( e \approx 2.7, \) so \( \frac{1}{e} \approx \frac{1}{3} \).
  2. Connect these points with the usual exponential curve.

- Note that \( f(x) = e^x \) is an exponential function whose base is greater than 1. So the following familiar properties hold:
  1. The domain of \( f(x) = e^x \) is \( (-\infty, \infty) \).
  2. The range of \( f(x) = e^x \) is \( (0, \infty) \).
  3. The \( y \)-intercept of the graph of \( f(x) = e^x \) is \((0, 1)\) since \( e^0 = 1 \).
  4. The graph of \( f(x) \) has no \( x \)-intercepts since \( e^x \) is > 0 for all \( x \). So the equation \( e^x = 0 \) has no solutions.
  5. The graph of \( f(x) = e^x \) is increasing on \( (-\infty, \infty) \).

- Make sure that you can also graph the function \( g(x) = e^{-x} \). It has all the properties listed above, expect that the graph of \( g(x) = e^{-x} \) is decreasing on \( (-\infty, \infty) \). Note that the equation \( e^{-x} = 0 \) has no solutions.

- We will ask you to solve equations like \( x^2 e^x - 4xe^x + 3e^x = 0 \) and \( x^2 e^{-x} - 16xe^{-x} = 0 \). The key to solving these equations is to start by factoring out \( e^x \) or \( e^{-x} \). Then solve the resulting factored equation, bearing in mind that neither the equation \( e^x = 0 \) nor the equation \( e^{-x} = 0 \) has any solutions.

- We’ll also ask you to solve inequalities involving \( e^x \) and \( e^{-x} \). These are not as intimidating as they look. For example, suppose that you are asked to solve something like the following:
  \[
x^2 e^x + 5xe^x > 0.
  \]
First find the zeros of \( x^2 e^x + 5xe^x = 0 \): \( x^2 e^x + 5xe^x = 0 \Rightarrow e^x(x^2 + 5x) = 0 \Rightarrow e^x(x+5) = 0 \Rightarrow e^x = 0, x = 0 \) or \( x + 5 = 0 \). The equation \( e^x = 0 \) has no solutions, so the only zeros are \( x = 0 \) and \( x = -5 \). Mark these zeros on a number line, then do a sign analysis as we learned in Section 1.7:
So the solution set to the inequality \( x^2e^x + 5xe^x > 0 \) is \((-\infty, -5) \cup (0, +\infty)\).

- Hint for the equations and inequalities in part B of the homework: if you have trouble, look at the examples that are worked out here in these reading notes.

III. Section 4.3: This section introduces logarithms and logarithmic functions. We will do everything in this section.

Here are some comments on the reading and hints for the homework:

- If you haven’t studied logarithms before, it may take you a while to get comfortable with the topic. For one thing, the notation can be difficult: a statement like \( \log_3 9 = 2 \) is less transparent than a statement like \( 3^2 = 9 \).

The best way to get comfortable with logarithms is to do lots of examples right away. The book does eight examples in Examples 1 and 2, but you may want to do more. Here are some examples (the solutions are given underneath).

**Examples:**

\[(a) \log_{10} 100 \quad (b) \log_{10} 10,000 \quad (c) \log_5 25 \quad (d) \log_2 16 \]

\[(e) \log_3 27 \quad (f) \log_3 \frac{1}{3} \quad (g) \log_3 \frac{1}{9} \quad (h) \log_{\frac{1}{2}} \frac{1}{8} \]

\[(i) \log_{\frac{1}{2}} 4 \quad (j) \log_{36} 6 \quad (k) \log_8 2 \quad (l) \log_{10} 1 \]

\[(m) \log_5 1 \quad (n) \log_{\frac{1}{2}} 1 \quad (o) \log_{10} 0 \quad (p) \log_{10}(-1) \]

**Solutions:**

\[(a) 2 \quad (b) 4 \quad (c) 2 \quad (d) 4 \]

\[(e) 3 \quad (f) -1 \quad (g) -2 \quad (h) 3 \]

\[(i) -2 \quad (j) \frac{1}{2} \quad (k) \frac{1}{3} \quad (l) 0 \]

\[(m) 0 \quad (n) 0 \quad (o) \text{doesn’t exist} \quad (p) \text{doesn’t exist} \]

- The properties of logarithms listed in the blue box on page 316 are extremely important. Make sure you know them and that you understand why they are true—especially Properties 3 and 4, which we will use repeatedly throughout the rest of the course. If you find you’re having trouble understanding Properties 3 and 4, construct some concrete examples, like the following.

1. **Example of Property 3:** \( \log_5(5^2) = 2 \). This makes sense, since you know that \( \log_5 25 = 2 \) for precisely the reason that \( 5^2 = 25 \).

2. **Example of Property 4** (the harder property): \( 3^{\log_3 9} = 9 \). Note that \( \log_3 9 = 2 \) since 2 is the the power you must raise 3 to in order to get 9. So when you raise 3 to that power, you do indeed get 9. So \( 3^{\log_3 9} = 3^2 = 9 \).
• At the top of page 317, the text uses the fact that $f(x) = \log_a x$ and $f^{-1}(x) = a^x$ are inverse functions to compare the graphs of $f(x) = \log_2 x$ and $f^{-1}(x) = 2^x$. Notice that $f(x) = \log_2 x$ has the following properties:

1. The domain of $f(x) = \log_2 x$ is $(0, +\infty)$ and the range is $(-\infty, \infty)$. (Recall that the domain of $y = 2^x$ is $(-\infty, \infty)$ and the range is $(0, \infty)$).
2. The graph of $f(x) = \log_2 x$ has $x$-intercept 1 and no $y$-intercept. (Recall that the graph of $y = 2^x$ has $y$-intercept 1 and no $x$-intercept.)
3. The graph of $f(x) = \log_2 x$ is increasing on $(0, \infty)$.

These properties are common to all functions of the form $f(x) = \log_a x$, where $a > 1$.

• After discussing the common logarithm (base 10), the text introduces the natural logarithm. This is the logarithm with base $e$, and it is the most important logarithmic function, just as the natural exponential function is the most important exponential function.

If the natural logarithm is new to you, it may take you a while to get used to it. Part of the difficulty is the notation: instead of writing $\log_e x$, which would be consistent with the other logs we’ve studied, we write $\ln x$. In other words,

$$\ln x = \log_e x.$$ 

So, for example, instead of writing $\log_e e^2$, which is clearly equal to 2, we write $\ln e^2$. You may always rewrite $\ln x$ as $\log_e x$.

• Here are some natural logarithms to evaluate:

<table>
<thead>
<tr>
<th>Examples</th>
<th>Solutions</th>
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</thead>
<tbody>
<tr>
<td>(a) $\ln e$</td>
<td>(a) 1</td>
</tr>
<tr>
<td>(b) $\ln e^2$</td>
<td>(b) 2</td>
</tr>
<tr>
<td>(c) $\ln e^4$</td>
<td>(c) 4</td>
</tr>
<tr>
<td>(d) $\ln e^{-3}$</td>
<td>(d) $-3$</td>
</tr>
<tr>
<td>(e) $\ln \frac{1}{e}$</td>
<td>(e) $-1$</td>
</tr>
<tr>
<td>(f) $\ln 1$</td>
<td>(f) 0</td>
</tr>
<tr>
<td>(g) $\ln \sqrt{e}$</td>
<td>(g) $\frac{1}{3}$</td>
</tr>
<tr>
<td>(h) $\ln \frac{1}{\sqrt{e}}$</td>
<td>(h) $-\frac{1}{2}$</td>
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• Note that Properties 3 and 4 in the blue box at the top of page 321 are crucial. Note that they can be written as follows:

$$\log_e e^x = x \quad \text{and} \quad e^{\log_e x} = x.$$ 

A couple of concrete examples may help:

$$\ln e^3 = 3 \quad \text{and} \quad e^{\ln 5} = 5.$$ 

• Hint for homework problems #27bc and others like them: don’t forget that you can always rewrite an expression like $\ln e^4$ as $\log_e (e^4)$.

• Hint for homework problem #46: think about how you did problem #21 in the homework for Section 4.1.
• Hint for homework problems #63 and 65: remember that the domain of \( f(x) = \log_a x \) is \( \{x : x > 0\} \). In other words, if you have a function of the form \( \log_a(\text{input}) \), then \( \text{input} > 0 \).

IV. Section 4.4: We will cover everything in this section except the change of base formula (top of page 328 to the end of the section).

Here are some comments on the reading:

• On page 325 the text states and proves the laws of logarithms. You will probably be able to follow the proofs, but you might find it more illuminating to give an example to illustrate each law. For example, to illustrate the first law, \( \log_a(AB) = \log_a A + \log_a B \), consider following:

\[
\log_2 16 = \log_2(2 \cdot 8) = \log_2 2 + \log_2 8 = 1 + 3, \text{ since } 2^4 = 2^1 \cdot 2^3 = 2^{1+3}.
\]

• When we ask you to do problems using the laws of logarithms, we will—as the text does—use a variety of bases. However, our primary focus will always be on the natural logarithm: \( \ln x \).

• The book points out some false “laws” of logarithms in the middle of page 327. Look at these very carefully, since using them is a common error in Math 5a.

• When the text, in one of its examples, gets a final answer that contains a logarithm (like \( \ln 3 \) or \( \log_2 5 \)), it often uses a calculator to approximate the value of the logarithm. **Don’t do this unless we explicitly ask you to.** For instance, in Example 1c on pages 325–6, you would leave the final answer as \( \log(\frac{1}{2}) \).

• Hint for homework problems #13 and 46: you can start by combining any two of the terms and then, once that is done, combining the result with the third term. For example, suppose you had to simplify

\[
\ln 5 - \frac{1}{3} \ln 6 - \ln 2.
\]

One way to do it is this:

\[
\ln 5 - \frac{1}{3} \ln 6 + \ln 2 = (\ln 5 - \frac{1}{3} \ln 6) + \ln 2 = (\ln 5 - \ln 6^{\frac{1}{3}}) + \ln 2 = \ln \left(\frac{5}{6^{\frac{1}{3}}}\right) + \ln 2
\]

\[
= \ln \left(\frac{5}{6^{\frac{1}{3}}} \cdot 2\right) = \ln \left(\frac{10}{6^{\frac{1}{3}}}\right)
\]

Another way to do it is this:

\[
\ln 5 - \frac{1}{3} \ln 6 + \ln 2 = (\ln 5 + \ln 2) - \frac{1}{3} \ln 6 = (\ln 5 + \ln 2) - \ln 6^{\frac{1}{3}} = \ln 10 - \ln 6^{\frac{1}{3}}
\]

\[
= \ln \left(\frac{10}{6^{\frac{1}{3}}}\right)
\]

There are other orders you can use as well.

• Hint for homework problem #68: See Example 5.

• Hint for homework problem #69: See Example 5a for how to solve for \( P \).
V. Section 4.5: We will do everything in this section except solving equations graphically (Solution 2 of Example 3, Solution 2 of Example 8, and Example 9). We will also add some inequalities involving logarithms to the section; these will be introduced in class.

Here are some comments on the reading and hints for the homework:

- In this section, the text frequently uses a calculator to get a decimal approximation of an answer. **Don’t do this unless we explicitly ask you to.**

- In Section 4.2 we solved equations like those in Example 5. But the type of equation solved in Example 4 is new. Notice its resemblance to a quadratic equation; this will help you solve the equation.

- Logarithmic equations are generally harder than exponential ones. The most common error that people make is **failing to use properties of logs before exponentiating an equation.** For example, consider the equation

\[ \ln x + \ln(x - 5) = \ln 6. \]

It is **wrong** to start by exponentiating each term. In other words

\[ e^{\ln x} + e^{\ln(x-5)} \neq e^{\ln 6}. \]

The correct first step is to use properties of logarithms to rewrite the left hand side of \( \ln x + \ln(x - 5) = \ln 6 \) as a **single** logarithm:

\[ \ln(x \cdot (x - 5)) = \ln 6 \Rightarrow \ln(x^2 - 5x) = \ln 6. \]

Now we **can** exponentiate both sides, getting

\[ e^{\ln(x^2-5x)} = e^{\ln 6} \Rightarrow x^2 - 5x = 6, \text{ etc.} \]

- When solving a logarithmic equation, it’s essential that you check that each solution is valid. For example, suppose you’re solving

\[ \ln 3 + 2\ln x = \ln(2 - x). \]

To solve this, first use properties of logarithms, getting

\[ \ln 3 + \ln x^2 = \ln(2 - x) \Rightarrow \ln(3x^2) = \ln(2 - x). \]

Exponentiating both sides gives

\[ 3x^2 = 2 - x \Rightarrow 3x^2 + x - 2 = 0 \Rightarrow (3x - 2)(x + 1) = 0 \Rightarrow x = \frac{2}{3}, \; x = -1. \]

But \( x = -1 \) is not a valid solution, since if we plug it back into the original equation we get \( \ln(-1) \), which is undefined. So the only solution is \( x = \frac{2}{3} \).

- We’ll finish the section by doing some material that is not in the text: solving equations like \( x - 2x \ln x = 0 \) and inequalities like like \( x^2 - 3x^2 \ln x > 0 \). Your instructor will present these in class.

- **Hint for homework problem #26:** this problem is straightforward if you start by multiplying both sides of the equation by the denominator of the left hand side.

- **Hint for homework problem #31:** think about how you would tackle the quadratic-type equation \( x^4 + 4x^2 - 21 = 0 \).
• Hint for homework problems #40 and 46: note that the base in these equations is 10.

• Hints for parts (a) and (b) of problem 2 in Part B: remember that \( \ln x \) is only defined on \((0, +\infty)\). So your number line should only be drawn for \( x > 0 \). Also, the best test points to use are powers of \( e \).