Inflation Dynamics During the Financial Crisis

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Abstract

Using confidential product-level price data underlying the U.S. Producer Price Index (PPI), this paper analyzes the effect of changes in firms’ financial conditions on their price-setting behavior during the “Great Recession.” The evidence indicates that during the height of the crisis in late 2008, firms with “weak” balance sheets increased prices significantly, whereas firms with “strong” balance sheets lowered prices, a response consistent with an adverse demand shock. These stark differences in price-setting behavior are consistent with the notion that financial frictions may significantly influence the response of aggregate inflation to macroeconomic shocks. We explore the implications of these empirical findings within the New Keynesian general equilibrium framework that allows for customer markets and departures from the frictionless financial markets. In the model, firms have an incentive to set a low price to invest in market share, though when financial distortions are severe, firms forgo these investment opportunities and maintain high prices in an effort to preserve their balance-sheet capacity. Consistent with our empirical findings, the model with financial distortions—relative to the baseline model without such distortions—implies a substantial attenuation of price dynamics in response to contractionary demand shocks.

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1 Introduction

In spite of substantial and persistent economic slack—as well as a significant tightening of financial conditions—the U.S. economy experienced only a mild disinflation during the “Great Recession” and its aftermath. The absence of significant deflationary pressures during this period is at odds with the canonical New Keynesian framework, which rationalizes the puzzling behavior of inflation by appealing to large unobservable markup shocks. In this paper, by contrast, we analyze inflation dynamics during the 2007–09 financial crisis through the lenses of customer-markets theory, while allowing for departures from the Modigliani–Miller paradigm of frictionless financial markets.

As formalized by Bils (1989), the key idea behind customer markets is that pricing decisions are a form of investment that builds the future customer base. In the presence of financial market frictions, however, firms experiencing a deterioration in the quality of their balance sheets may find it optimal to increase prices—relative to financially healthy firms—and sacrifice future sales in order to boost current cash flows; see, for example, Gottfries (1991); Bucht, Gottfries, and Lundin (2002) and Chevalier and Scharfstein (1996). This suggests that changes in financial conditions may have a direct effect on aggregate inflation dynamics, especially in periods of acute financial distress. Importantly, the interaction of financial factors and the price-setting behavior of firms may limit the deflationary pressures that are often associated with the boom-bust nature of credit-driven cyclical fluctuations, which are typically characterized by a significant deterioration in the quality of borrowers’ balance sheet conditions and large increases in the cost of external finance.

As a first step in our analysis, we construct a novel data set by merging a subset of monthly product-level prices from the U.S. Producer Price Index (PPI), constructed and published by the Bureau of Labor Statistics (BLS), with the respondents’ quarterly income and balance sheet data from Compustat. The micro-level aspect of our data allows us to analyze how changes in financial conditions of these large publicly-traded firms affect their price-setting behavior during the 2005–12 period. Our results indicate that at the peak of the crisis in late 2008—after the collapse of Lehman Brothers—financially vulnerable firms, on average, dramatically increased prices, whereas their financially healthy counterparts lowered prices in response to the ensuing collapse in aggregate demand. During this period, the price increases by firms with “weak” balance sheets generate a differential of 10 percentage points in the monthly producer price inflation relative to firms with “strong” balance sheets, and this differential is highly persistent over time.

Formal regression analysis also indicates that financially weak firms were significantly more likely to increase prices during the height of the financial crisis. At the same time, these firms were less likely to lower prices before the crisis and in its aftermath. These results strongly suggest that firms with weak balance sheets actively manage their prices to maintain cash flows in the face of declining demand and not because they are less well managed and hence less responsive to changes in economic conditions.\(^1\)

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\(^1\)Christiano, Eichenbaum, and Evans (2005), Christiano, Gust, and Roldos (2004), and the empirical work of
To explore the macroeconomic and policy consequences of financial distortions in a customer-markets framework, we build a general equilibrium model, the essential feature of which is that firms face costly price adjustment, while setting prices to actively manage current versus future expected demand. We do so in the context of the “deep habits” framework formulated by Ravn, Schmitt-Grohe, and Uribe (2006), which is augmented with a tractable model of costly external finance. As in Gourio and Rudanko (2011), customer base in our model is a form of investment. The investment literature has long emphasized the notion that financial distortions may create a debt-overhang problem, which leads firms to pass up otherwise positive net present value projects (Myers, 1977). The presence of financial distortions may similarly influence the incentive to invest into customers via price reductions, implying a sensitivity of price-setting decisions to changes in balance sheet conditions—when cash flow is low or external finance is very costly, firms will “disinvest” by maintaining high prices. In this sense, our framework echoes the theoretical insights of Chevalier and Scharfstein (1996) regarding the role of financial market frictions on the cyclical variation in the markup but generalizes their results to a fully dynamic general equilibrium setting. More generally, the framework presented in this paper can be viewed as a special application of liquidity-based asset pricing (Holmstrom and Tirole, 2001) to the New Keynesian pricing theory.

Relative to the baseline model with frictionless financial markets, our model implies a significant attenuation of the response of prices to contractionary demand shocks. Moreover, in a calibration where external finance is extremely costly—as was likely the case at the nadir of the 2007–09 financial crisis—our model implies that inflation rises rather than falls in response to a contractionary demand shock. These theoretical results are consistent with the apparent lack of significant deflationary pressures during the recent recession, and they also suggest that financial factors may help explain sluggish price responses more generally.

One of the defining features of the recent macroeconomic experience is the fact that since the end of 2008, the federal funds rate has been stuck at its effective lower bound. The implications of our model for macroeconomic outcomes at the zero lower bound (ZLB) are striking. In a model with financial distortions, the attenuation of deflationary pressures implies that the ZLB is both harder to achieve, and when achieved, the economy is likely to exit the ZLB environment sooner. In other words, the sharp contractionary nature of a deflationary spiral at the zero lower bound is surprisingly mitigated to a great extent by the presence of financial distortions, a result suggesting that in a customer-markets model, financial frictions may paradoxically improve overall economic outcomes in an environment where the zero lower bound is binding. This echoes recent findings by Barth and Ramey (2002) all emphasize a “cost channel,” whereby firms borrow to finance inputs to production. To the extent that firms with weak balance sheets face higher borrowing costs, which during the crisis increase more sharply than those for firms with strong balance sheet, financially vulnerable firms may pass those cost increases on to their customers in the form of higher prices. Note that both mechanisms imply less price-cutting for financially constrained firms relative to their unconstrained counterparts.

of Denes, Eggertsson, and Gilbukh (2013) and Eggertsson (2011), who argue that certain forms of taxation may improve economic outcomes in situations where short-term interest rates are stuck at their effective lower bound.

We also consider an extension of our benchmark model that allows for heterogeneity in fixed operating costs across firms. The resulting differences in operating efficiency translate directly into differences in financial conditions: A firm with low operating efficiency (i.e., high fixed operating costs) is in a much more precarious financial position because, on average, it will have a more difficult time meeting its liquidity needs using internally-generated funds and, therefore, will face a higher external finance premium. In such context, an adverse financial shock causes financially healthy firms to aggressively cut prices in an effort to gain a market share, whereas their financially constrained counterparts find it optimal to increase prices to avoid relying on costly external finance. The resulting “price war” induces countercyclical dispersion in firm-level inflation rates—as well as in output and employment—a pattern consistent with that reported by Vavra (2011). However, it is important to emphasize that in our framework the countercyclical dispersion in inflation rates (and other variables) arises endogenously in response to differences in financial conditions across firms, whereas Vavra (2011) generates the countercyclical dispersion in inflation rates using an exogenous countercyclical second-moment shock.

The theoretical mechanism that we study has broader implications for the conduct of monetary policy. The standard New Keynesian framework analyzes optimal policy from the perspective of the welfare losses induced by output fluctuations and price dispersion owing to nominal rigidities. In the standard framework, demand shocks imply the so-called divine coincidence, the fact that countercyclical monetary policy achieves the joint stabilization of output and inflation. The trade-off between inflation and output stabilization only arises in circumstances where the “cost-push” shocks move inflation and output in opposite directions. In our model, demand shocks may also lead to opposing movements in inflation and output. However, when firms face varying costs of external finance, demand shocks also lead to increased dispersion in prices, even in the absence of large swings in aggregate inflation. These results suggest that customer-markets models with financial distortions may have starkly different implications for the inflation-output tradeoff—and therefore for the conduct of monetary policy—especially at times of significant distortions in financial markets.

2 Data Sources and Methods

To understand the interaction between price-setting behavior of firms and the condition of their balance sheets, we construct a new firm-level data set using two sources: (1) product-level price data underlying the Producer Price Index published by the Bureau of Labor Statistics; and (2) firm-level income and balance sheet data from Compustat.
2.1 Producer Price Data

The confidential PPI micro-level price data from the BLS form the cornerstone of our analysis for two reasons. First, they allow us to construct firm-level inflation rates, thereby overcoming the limitations of working with aggregate price indexes. And second, they allows us to analyze firm-level price dynamics directly in conjunction with the respondents’ corresponding financial data. Both of these features represent an important advance over any analysis that employs aggregate price series—even if narrowly defined—because price dynamics at the product and firm levels are subject to large idiosyncratic shocks (cf. Nakamura and Steinsson (2008); Bils and Klenow (2004); and Gopinath and Itskhoki (2011)). Moreover, prices at the level of individual products contain potentially important information to understand the economics of price adjustment at the firm level.

From a practical perspective, we focus on the PPI data because they yield a much broader match with the data set of publicly-traded Compustat firms. Economic considerations also point to studying producer prices as they most directly reflect the response of production unit to changes in the underlying economic fundamentals. The CPI data, in contrast, reflect the pricing behavior of non-producing retailers—the so-called outlets—which are subject to price responses by the entire distribution network and therefore may exhibit quite different price-setting behavior. Moreover, the PPI data exclude prices of imported goods, which are an important part of the CPI, but for which data on financial conditions of the underlying production units are not readily available. All told, our sample contains about 100,000 monthly producer price quotes collected by the BLS from 28,300 production units. The time-series range of our data runs from January 2005 through September 2012 and thus fully includes the 2007-09 financial crisis and its aftermath.

Our measure of firm-level inflation—denoted by \( \pi_{j,k,t} \)—is given by the weighted monthly average price changes of goods produced by each firm, after filtering out monthly industry-level (2-digit NAICS) inflation rates (denoted by \( \pi_{k,t} \)). Formally,

\[
\pi_{j,k,t} = \frac{1}{n_j} \sum_{i=1}^{n_j} w_{i,j,k,t} (\Delta p_{i,j,k,t} - \pi_{k,t}),
\]

where \( \Delta p_{i,j,k,t} \) denotes monthly log price changes for each good \( i \) produced by firm \( j \) that operates

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3 The PPI data are described at length for example in Nakamura and Steinsson (2008), Bhattarai and Schoenle (2010) and Goldberg and Hellerstein (2009). The data are representative of the entire U.S. production structure and have the important quality that they are carefully and consistently sampled. In particular, goods within a firm are uniquely identified according to several consistent criteria: their “price-determining” characteristics such as the type of buyer, the type of market transaction, the method of shipment, the size and units of shipment, the freight type, and the day of the month of the transaction. Once a good is identified, prices are consistently collected each month for that very same good and the same customer. Such consistent sampling avoids the problem of having to compute unit prices as is given with many micro data sets. All prices are transaction prices, not list prices as critiqued by Stigler and Kindahl (1970).
in an industry $k$, and $n_j$ the number of goods produced by that firm. Importantly, we use quality-adjusted prices when constructing these inflation rates. For each item, the quality-adjusted price in month $t$ is defined as the ratio of the recorded price $p_{i,j,k,t}^*$ to the base price $p_{i,j,k,t}^b$, where the latter takes into account changes in the item’s quality over time: $p_{i,j,k,t} = \frac{p_{i,j,k,t}^*}{p_{i,j,k,t}^b}$. The fraction of firm-level price changes is constructed analogously using a quarterly price change indicator variable instead of the monthly log price difference but without filtering.

We construct weights very carefully based on the relative importance weights used by the BLS and firm-level value of shipments data recorded by the BLS for computation of the aggregate PPI. We define the within-firm good-level weight $w_{i,j,k,t}$ as follows:

$$w_{i,j,k,t} = \bar{w}_{i,j,k,t} \times \omega_{j',t},$$

where $\bar{w}_{i,j,k,t}$ denotes the relative weight for good $i$ in the production structure of firm $j$, according to the BLS definition. The second term in the expression is an adjustment factor that takes into account the fact that in our merge with Compustat data more than one BLS respondent may fall within the Compustat firm definition $j$. The adjustment factor is therefore defined as the relative value of shipments weight of one BLS firm with respect to all other BLS firms within the same Compustat firm unit.

### 2.2 Indicators of Financial and Product Market Frictions

We use quarterly Compustat data to characterize the firms’ financial conditions and product market characteristics. Our primary measure of financial conditions is the liquid-asset ratio, defined as

$$LIQUIDITY_{j,t} = \frac{\text{Cash and other Liquid Assets}_{j,t}}{\text{Total Assets}_{j,t}},$$

where cash (and other liquid assets) and total assets are measured at the end of month $t$ corresponding to the firm’s fiscal quarter, which is properly aligned with the calendar month and thus each monthly inflation rate.

As a robustness check, we also consider two alternative financial

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4 Note that taking out “industry inflation” does not take into account the weight assigned to each industry for a firm if the firm operates across multiple industries. This is not a problem for our data. The reason is the BLS defines firms as price-setting units in one production unit, which is usually unique to a NAICS code, especially at 2-digit level of filtering. We drop the handful of cases where a firm operates across multiple industries.

5 Other widely used indicators to measure the degree of financial frictions faced by firms include the dividend payout status (Carpenter, Fazzari, and Petersen (1994)); firm size (Gertler and Gilchrist (1994) and Carpenter, Fazzari, and Petersen (1998)); the reliance of trade credit (Nilson (2002)); the presence (or absence) of an external credit rating (Gilchrist and Himmelberg (1995)); the length and/or number of banking relationships Petersen and Rajan (1994); and industrial effects arising from factor intensity differentials (Rajan and Zingales (1998)).
indicators: cash-flow ratio and interest coverage. We define the cash-flow ratio as:

\[ CASH-FLOW_{j,t} = \frac{Operating\ Income_{j,t}}{Total\ Assets_{j,t-1}}; \]  \quad (4)

and the interest coverage ratio as:

\[ INTEREST-COVERAGE_{j,t} = \frac{Interest\ Expense_{j,t}}{Total\ Sales_{j,t}}. \]  \quad (5)

To measure product-market characteristics motivated by the customer-markets theory, which emphasizes the idea that price setting is a form of investment that builds the future customer base, we include into our analysis sales and general administrative expenses (SG&A), a commonly-used indicator of frictions in product markets. We normalize SG&A expenditures by sales:

\[ SGAXR_{j,t} = \frac{SGAX_{t}}{Total\ Sales_{j,t}}. \]  \quad (6)

It is important to emphasize that the intensity of SG&A spending can have opposite implications for pricing decisions. On the one hand, for a firm with a relatively high SG&A ratio—an indication that the firm is likely operating in a customer market environment (e.g., Gourio and Rudanko (2011))—one would expect a stronger incentive to lower prices today in order to expand market share tomorrow. On the other hand, due to the quasi-fixed nature of SG&A expenses, a high ratio of SG&A to sales may be associated with low operating efficiency. In turn, this would force firms to set higher prices today to boost current cash flow, a dynamic that would be exacerbated during periods of financial stress when external financing is very costly. In the empirical analysis, we let the data speak for themselves to see which force dominates the pricing dynamics during the financial crisis.

2.3 Matched Compustat-PPI Sample

To link the BLS micro-level research database with the firm-level Compustat, we use the matching algorithm developed by Schoenle (2010). The algorithm works by running a fuzzy match of the names of firms in the PPI and Compustat databases. After sorting all nonperfect matches in decreasing order of similarity, we then manually select “good” matches in addition to perfect matches.

After applying the algorithm to the two data sets over the 2005-12 period, we successfully
matched 780 Compustat firms, on average, per quarter. Given that we have information on almost 5,000 Compustat firms in an average quarter, this implies a matching rate of 16 percent. In terms of basic characteristics, the firms in the matched sample tend to be larger, a result that is not at all surprising because large firms are more likely to be sampled by the BLS. In terms of their price-setting behavior, we find that there are no significant differences between average monthly inflation rates in the full PPI data set and the matched subsample. At the same time, the frequency of price changes in the matched sample is somewhat higher than in the full sample (see Table 1).

Figure 1 plots the aggregate inflation rates for the full and matched sample of firms. The two series are highly correlated, an indication that our matched sample is broadly representative of the economy as a whole. And lastly, as indicated in Table 2, our matched sample of firms exhibits somewhat lower liquidity, SG&A and interest expense ratios, on average, compared with the sample of all U.S. publicly-traded nonfinancial corporations.

### 3 Inflation Dynamics and Financial Conditions

To analyze the role of financial distortions in determining inflation dynamics, we first compute industry-adjusted firm-level inflation rates as described above. Then for each month \( t \), we sort firms into financially “weak” and “strong” categories based on whether a specified financial indicator in month \( t - 1 \) (i.e., liquid asset ratio, cash flow ratio, or interest coverage ratio) is below/above the median of its distribution in that period. To minimize the switching of firms between the two categories, we use a rolling 12-month backward moving average of financial ratios when sorting firms into financially weak and strong categories. We use the same method to identify firms with a high (low) intensity of SG&A spending. Finally, we compute average weighted monthly inflation for each category using firms sales as weights.

The top panel of Figure 2 shows the industry-adjusted inflation rates for firms with weak balance sheets, while the bottom panel depicts the the same information for firms in a strong financial position. Regardless of the financial indicator used to sort firm into financially weak and strong categories, a strikingly similar picture emerges: At the peak of the crisis in the fourth quarter of 2008, firms with weak balance sheets significantly increased—relative to their industry trend—prices, whereas their financially healthy counterparts substantially lowered prices, a response consistent with the sharp drop in demand that was occurring at that time. Note that such differences in price-setting behavior in response to a contractionary demand shock will lead to a persistent and long-lasting dispersion in prices.

Our next empirical exercise focuses on firms with different product-market characteristics. The top panel of Figure 3 shows the industry-adjusted inflation rates for firms with different intensities of SG&A spending. The bottom panel focuses on a subset of SG&A expenses, namely advertising expense, a narrower indicator of whether a firm is operating in a customer-markets environment. According to both panels, firms that likely operate in the customer-markets environment signifi-
cantly increased prices in the latter part of 2008. In contrast, firms that likely operate in competitive markets lowered their prices significantly, a response consistent with the concomitant drop in demand. These results suggest that a substantial part of the SG&A expenditures reflects overhead costs and that this ratio may not be indicative of whether a firm is operating in a customer-markets environment.

The results in Figures 2 and 3 are difficult to reconcile with the standard price-adjustment mechanism emphasized by the New Keynesian literature, a paradigm where firms’ financial conditions play no role in determining their price-setting behavior. In general, we would expect that firms hit by an adverse demand shock—the kind the U.S. economy experienced in the latter part of 2008—should induce firms to lower prices. Moreover, if the proxies used to measure the strength of firms’ balance sheets were also indicative of the weakness in demand, we would expect financially vulnerable firms to lower prices even more relative to financially strong firms. However, we observe exactly the opposite reaction in the data.

As emphasized by Bils, Klenow, and Malin (2012), disinflationary pressures during the “Great Recession” were most pronounced in nondurable goods industries. The top panel of Figure 4 shows inflation rates of financially strong and weak firms within the durable and nondurable goods sectors, while the bottom panel contains inflation rates for firms with varying intensity of SG&A spending, again within the durable and nondurable goods sectors. According to both panels, the deflationary pressures during the recent financial crisis were concentrated primarily in the nondurable goods sector, a result consisted with that reported by Bils, Klenow, and Malin (2012).

However, within the nondurable goods sector, price deflation reflects solely the massive price cut by financially healthy firms or firms that are unlikely to operate within the customer-markets environment. In contrast, nondurable goods producers with weak balance sheets or those with a high SG&A expense ratio significantly increased prices during the height of the crisis. The fact that inflation dynamics in nondurable goods industries during the financial crisis appear to be shaped significantly by financial conditions and SG&A expenditures is consistent with the notion that nondurable goods are typically frequently-purchased items, subject to habits and past experience, factors at the center of the customer-markets theory. In contrast, we expect customers to be less loyal to past purchase habits when buying large-item durable goods that are purchased relatively infrequently.

3.1 Extensive Margin of Price Adjustment

In this section, we provide new evidence on the importance of the strength of firm balance sheets and the intensity of SG&A spending for price adjustment at the extensive margin—that is, the frequency of price adjustment. As Bhattarai and Schoenle (2010), we estimate a multinomial logit of the form:

$$Pr(Y_{i,j,t+1} = \{-1, 0, 1\}|X_{i,j,t} = x) = \Lambda(\beta'X_{i,j,t}),$$

(7)
where $Y_{i,j,t+1}$ is an indicator variable for price changes at time $t + 1$ of good $i$ produced by firm $j$: $-1 =$ price decrease; $0 =$ no change (base category); and $1 =$ price increase. The set of firm-level explanatory variables $X_{i,j,t}$ includes the liquidity ratio, the SG&A-to-sales ratio, and sales growth at time $t$. In addition, the specification includes time and (3-digit NAICS) industry fixed effects.

To allow for time-series variation in the response coefficients $\beta_t$, we estimate the multinomial logit specification using a four-quarter rolling window. The resulting (time-varying) estimates are used to compute the elasticity of the response variable—that is, the percent change in the probability of a price adjustment for a given percent change in a variable of interest, evaluated at the sample mean of the explanatory variables.

The top panel of Figure 5 depicts the time-varying elasticity estimates of price adjustment with respect to financial conditions, as measured by the liquid-asset ratio; the left panel depicts the elasticity of downward price adjustment, while the right panel depicts the elasticity of upward price adjustment. According to these estimates, firms with weak balance sheets became significantly more likely to increase prices during the financial crisis, as evidenced by the fact that the estimated elasticity of upward price adjustment with respect to liquidity ratio (right panel) dropped noticeably into the negative territory in the latter half of 2008; conversely, financial healthy firms were less likely to increase prices during that period.

In economic terms, an increase in the liquidity ratio of one percentage points in the second half of 2008 is estimated to lower—ceteris paribus—the probability of upward price adjustment 16 basis points relative to no price change. At the same time, the results in the left panel indicate that a high liquidity ratio is consistently associated with a greater likelihood of downward price adjustment. In sum, these results suggest that financially strong firms face lower downward price rigidity—at least at the extensive margin—compared with their weaker counterparts.

The bottom panel of Figure 5 presents the elasticities of price adjustment with respect to the SG&A ratio. The fact that both directional elasticities are consistently negative indicates that firms with a high intensity of SG&A spending are less likely to change their prices—either up or down—relative to no price change. Moreover, there is little evidence to suggest that this pattern has changed appreciably during the financial crisis; at the same time, there is a sizable drop—at least in economic terms—in the elasticity of downward price adjustment during the crisis, an indication that firms with higher SG&A-to-sales ratio were less likely to lower their prices during that period.

4 Model

In this section, we construct a general equilibrium model aimed at explaining two key empirical findings:

1. Firms with weak balance sheets significantly increased prices at the height of the 2007–09 financial crisis, while the financially strong firms lowered prices.
2. Firms with a high SG&A-to-sales ratio—a likely indication that they operate in a customer-markets environment—increased prices significantly during the nadir of the recent financial crisis, while firms with a low SG&A ratio lowered their prices.

The main idea of our model involves monopolistically-competitive firms that set prices in a customer-markets environment, whereby their current pricing decisions influence future market shares. To motivate the competition for market shares implied by the customer-markets hypothesis, we specify preferences that allow for the formation of a customer base—that is, “low” prices today are a form of investment in a future market share (Rotemberg and Woodford (1991)). Specifically, we adopt the good-specific habit model developed by Ravn, Schmitt-Grohe, and Uribe (2006), which we augment with nominal rigidities in the form of quadratic costs to changing prices. Because our empirical analysis indicates that financial factors primarily affect the price-setting behavior of firms in nondurable goods industries, we consider only perishable goods in our model formulation. To explore the influence of financial distortion on price-setting behavior, we augment the framework with a stylized but tractable model of costly external finance.

4.1 Preferences and Technology

The model contains a continuum of households indexed by \( j \in [0, 1] \). Each household consumes a variety of consumption goods indexed by \( i \in [0, 1] \). The preferences of households are defined over a habit-adjusted consumption bundle \( x^j_t \) and labor \( h^j_t \) as follows:

\[
E_t \sum_{s=0}^{\infty} \beta^s U(x^j_{t+s} - \delta_{t+s}, h^j_{t+s}).
\]

The consumption/habit aggregator is defined as

\[
x^j_t \equiv \int_0^1 \left( \frac{c^j_{it}}{s^i_{it-1}} \right)^{1-1/\eta} di^{1/(1-1/\eta)},
\]

while the demand shock \( \delta_t \) in equation (8) alters the marginal utility of consumption today and hence the final demand. The good-specific habit stock is assumed to be external and thus taken as given by consumers. We assume that the habit evolves according to

\[
s_{it} = \rho s_{it-1} + (1 - \rho) c_{it},
\]

\( \text{Switching cost models of the type considered by Klemperer (1987) would serve the same purpose. We chose the good-specific habit model because of its tractability in a dynamic general equilibrium setting.} \)
where $1 - \rho$ denotes the depreciation rate for the current stock of habit. The dual problem of cost minimization gives rise to a good-specific demand,

$$c_{it}^j = \left(\frac{p_{it}}{p_t}\right)^{-\eta} s_{it-1}^{\theta(1-\eta)} x_t^j,$$

(11)

where $p_{it} \equiv P_{it}/P_t$ is the relative price of variety $i$ in terms of $P_t \equiv \left[\int_0^1 P_{it}^{1-\eta} di\right]^{1/(1-\eta)}$, and the externality-adjusted composite price index $\tilde{p}_t$ is given by

$$\tilde{p}_t \equiv \left[\int_0^1 (p_{it}s_{it-1})^{1-\eta} di\right]^{1/(1-\eta)}.$$

(12)

We assume that there exists a continuum of monopolistically-competitive firms, producing a differentiated variety of goods indexed by $i \in [0, 1]$. The production technology is given by

$$y_{it} = \left(\frac{A_t}{a_{it}} h_{it}\right)^\alpha - \phi, \quad 0 < \alpha \leq 1$$

(13)

where $A_t$ is an aggregate productivity shock that follows an AR(1) process, and $a_{it}$ is an idiosyncratic (i.i.d.) productivity shock distributed as $\log a_{it} \sim N(-0.5\sigma^2, \sigma^2)$. As indicated by equation (13), we allow the production technology to exhibit either decreasing or constant returns-to-scale. In addition, we assume that production is subject to fixed operating costs—denoted by $\phi$—which makes it possible for firms to incur negative income, thereby creating a liquidity squeeze if external financing is costly.

### 4.2 Pricing Frictions and Financial Distortions

To allow for nominal rigidities, we assume that firms face a quadratic cost to adjusting nominal prices, specified as $\gamma/2(P_{it}/P_{it-1} - \bar{\pi})^2 c_t = \gamma/2(\pi_t \cdot p_{it}/p_{it-1} - \bar{\pi})^2 c_t$ (see Rotemberg (1982)). It is worthwhile to note that staggered pricing models such as those of Calvo (1983) would not change the main conclusions of our paper. Rather, convex adjustment costs are adopted for the sake of mathematical tractability.

In the model, firms make pricing and production decisions to maximize the present value of discounted dividends. To introduce financial distortions in a tractable manner, we focus on equity as the only source of external finance. We assume that firms must commit to pricing decisions—and hence production—based on all aggregate information available within the period, but prior to the realization of their idiosyncratic productivity shock. Based on this aggregate information, firms post prices, take orders from customers, and plan production based on expected marginal costs. Firms then realize actual marginal cost and hire labor to meet the demand. Labor must be paid

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*Implicitly, fixed operating costs $\phi$ can include “long-term debt payments,” a coupon payment to perpetual bond holders.*
within the period, and in the presence of fixed operating costs, ex post profits may be too low to cover the total cost of production, in which case, firms must raise external funds.

External funds are raised by issuing new equity. We assume that equity finance involves a constant per-unit dilution cost per dollar of equity issued. We also abstract from firm savings decisions by assuming that all dividends are paid out within the period. Specifically, we assume that ex post equity finance involves a constant per-unit dilution cost $\phi \in (0, 1)$. This tractable formulation of costly external finance allows us to highlight the basic mechanism within a framework that deviates only slightly from the standard good-specific habit model. In particular, in our framework, all firms are identical ex ante, so that only firms with an idiosyncratic productivity shock below a threshold incur negative profits and issue new equity.

4.3 Profit Maximization Problem

The firm’s problem is to maximize the present value of dividend flows, $\mathbb{E}_t[\sum_{s=0}^{\infty} m_{t,t+s} d_{t+t+s}]$, where $d_{it}$ denotes (real) dividend payouts when positive and equity issuance when negative. The formulation of the dilution costs implies that when a firm issues a notional amount of equity $d_{it}(< 0)$, actual cash inflow from the issuance is reduced to $-(1 - \phi)d_{it}$. The firm’s problem is subject to the flow-of-funds constraint:

$$0 = p_{it} c_{it} - w_{it} h_{it} - \frac{\gamma}{2} \left( \frac{\pi_t p_{it}}{p_{it-1}} - \bar{\pi} \right)^2 c_t - d_{it} + \phi \min\{0, d_{it}\};$$

and given the monopolistically competitive product market, to the demand constraint:

$$\left( \frac{A_t}{a_{it}} h_{it} \right)^\alpha - \phi \geq c_{it}. \quad (15)$$

Formally, the Lagrangian associated with the firms’ problem is given by

$$\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} m_{0,t} \left\{ \begin{array}{l}
d_{it} + \kappa_{it} \left[ \left( \frac{A_t}{a_{it}} h_{it} \right)^\alpha - \phi - c_{it} \right]
+ \xi_{it} \left[ p_{it} c_{it} - w_{it} h_{it} - \frac{\gamma}{2} \left( \frac{\pi_t p_{it}}{p_{it-1}} - \bar{\pi} \right)^2 c_t - d_{it} + \phi \min\{0, d_{it}\} \right]
+ \nu_{it} \left[ \left( \frac{p_{it}}{\tilde{p}_t} \right)^{-\eta} s_{it} \right] \left[ x_t - c_{it} \right]
+ \lambda_{it} \left[ \rho s_{it-1} + (1 - \rho) c_{it} - s_{it} \right]\end{array} \right\},$$

where $\kappa_{it}, \xi_{it}, \nu_{it},$ and $\lambda_{it}$ are the Lagrangian multipliers associated with equations (14), (13), (11), (12).

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9 As shown by Gomes (2001) and Stein (2003), other forms of costly external financing can be replicated by properly parametrized equity dilution costs.

10 An interesting extension would involve incorporating precautionary demand for liquid assets in the model. Allowing for costly equity financing and liquidity hoarding, however, would make the distribution of firms’ liquid asset holdings a state variable of the model. We leave this nontrivial extension for future research.
and (10), respectively.

The efficiency conditions are summarized by the following first-order conditions:

1. $d_{it}$:
   
   $$d_{it} : \xi_{it} = \begin{cases} 1 & \text{if } d_{it} \geq 0 \\ 1/(1 - \varphi) & \text{if } d_{it} < 0 \end{cases}$$

2. $h_{it}$:
   
   $$h_{it} : \kappa_{it} = \xi_{it} a_{it} \frac{w_{it}}{\alpha A_{it}} (c_{it} + \phi)^{1 - \alpha}$$

3. $c_{it}$:
   
   $$c_{it} : \mathbb{E}_t^a[\nu_{it}] = \mathbb{E}_t^a[\xi_{it} p_{it}] - \mathbb{E}_t^a[\kappa_{it}] + (1 - \rho) \lambda_{it}$$

4. $s_{it}$:
   
   $$s_{it} : \lambda_{it} = \rho \mathbb{E}_t \{m_{t,t+1} \lambda_{it+1}\} + \theta (1 - \eta) \mathbb{E}_t \left\{m_{t,t+1} \mathbb{E}_{t+1}^{a} \left[\nu_{it+1} \frac{c_{it+1}}{s_{it}}\right]\right\}$$

5. $p_{it}$:
   
   $$p_{it} : 0 = \mathbb{E}_t^a[\xi_{it}] c_{it} - \eta \mathbb{E}_t^a[\nu_{it}] c_{it} - \gamma \left(\frac{\pi_{t} p_{it}}{p_{it-1}} - \bar{\pi}\right) c_{t} + \gamma \mathbb{E}_t \left[m_{t,t+1} \mathbb{E}_{t+1}^{a} \xi_{it+1} \frac{p_{it+1}}{p_{it}} \left(\frac{p_{it+1}}{p_{it}} - \bar{\pi}\right) c_{t+1}\right].$$

Implicit in the last three conditions is the assumption that pricing and production decisions are made prior to the realization of the idiosyncratic productivity shock. Accordingly, these first-order conditions involve the expected value of internal funds $\mathbb{E}_t^a[\xi_{it}]$. Where the expectations are formed using all aggregate information up to time $t$, except, of course, the realization of the idiosyncratic shock. In contrast, the realized values $\xi_{it}$ and $a_{it}$ enter the efficiency conditions (17) and (18) without the expectation operator because equity issuance and labor hiring decisions are made after the realization of the idiosyncratic productivity shock.

Under risk-neutrality and with i.i.d. idiosyncratic productivity shocks, the timing convention adopted above implies that firms are identical ex ante. Hence, we focus on a symmetric equilibrium, whereby all monopolistically-competitive firms choose identical relative price ($p_{it} = 1$), production scale ($c_{it} = c_t$), and habit stock ($s_{it} = s_t$). However, the distributions of labor inputs, dividend payouts, and equity issuance are non-degenerate and depend on the realization of the idiosyncratic shock.

### 4.4 Value of Internal Funds and Value of Customer Base

To analyze how distortions in financial markets interact with pricing (markup) decisions, we now consider the value of internal funds. Define the equity issuance trigger $a_t^E$ as the level of idiosyncratic productivity shocks.

---

1. **Note**: In equation (18), we replace $h_{it}$ by the conditional labor demand $h_{it} = (c_{it} + \phi)^{1 - \alpha} a_{it} / A_{it}$ after we derive the associated first-order condition.

2. **A similar timing convention has been used by Kiley and Sim (2012)** in the context of financial intermediation.
productivity that satisfies the flow-of-funds constraint, when dividends are exactly zero:

\[ a_t^E = \frac{c_t}{(c_t + \phi)^{1/\alpha}} \frac{A_t}{w_t} \left[ 1 - \frac{\gamma}{2} (\pi_t - \bar{\pi})^2 \right]. \]

Using this trigger, we can rewrite the first-order conditions for dividends as

\[ \xi(a_{it}) = \begin{cases} 
1 & \text{if } a_{it} \leq a_t^E \\
1/(1 - \varphi) & \text{if } a_{it} > a_t^E 
\end{cases} \tag{22} \]

This condition simply states that the realized shadow value of internal funds jumps from 1 to \(1/(1 - \varphi) > 1\) because of costly external financing, when the realization of the idiosyncratic productivity shock is greater than the threshold value. Let \(z_t^E\) denote the standardized value of \(a_t^E\); that is, \(z_t^E = \sigma^{-1}(\log a_t^E + 0.5\sigma^2)\). Using equation (22), the expected shadow value of internal funds can be expressed as

\[ \mathbb{E}_t^q[\xi_{it}] = \int_0^{a_t^E} 1dF(a) + \int_{a_t^E}^{\infty} \frac{1}{1 - \varphi}dF(a) = 1 + \frac{\varphi}{1 - \varphi} \left[1 - \Phi(z_t^E)\right] \geq 1, \]

where \(\Phi(\cdot)\) denotes the CDF of the standard normal distribution.

The expected shadow value is strictly greater than unity as long as equity issuance is costly \((\varphi > 0)\) and the firm faces idiosyncratic liquidity risk \((\sigma > 0)\). This makes the firm de facto risk averse when making its pricing decision. Setting the price too low and taking an imprudently large number of orders exposes the firm to the risk of incurring negative operating income, which must be financed through costly equity issuance. On the other hand, by raising the expected markup, \(c_t / (w_t h_t) = c_t A_t / [w_t (c_t + \phi)^{1/\alpha}]\), the firm can lower the probability of external financing \(1 - \Phi(z_t^E)\), and as a result, the expected shadow value of internal funds. Note that the equity financing trigger \(a_t^E\)—and hence the ex ante external financing cost \(\mathbb{E}_t^q[\xi_{it}]\)—are both directly affected by aggregate conditions. For instance, under reasonable parameter values, and holding wages constant, a contraction in demand that causes a reduction in \(c_t\) implies an increase in the expected cost of external finance because with lower production, fixed costs now account for a greater share of total costs.

We now consider how the value of internal funds affects the value of marginal sales in the presence of financial market frictions. According to the first-order condition for \(c_{it}\) (equation (19)), the value of marginal sales consists of two terms: current profits and the value of the customer base. From equation (19), after imposing the symmetric equilibrium condition, we have

\[ \frac{\mathbb{E}_t^q[\nu_{it}]}{\mathbb{E}_t^q[\xi_{it}]} = \frac{1 - \mathbb{E}_t^q[\kappa_{it}]}{\mathbb{E}_t^q[\xi_{it}]} + (1 - \rho) \frac{\lambda_t}{\mathbb{E}_t^q[\xi_{it}]} \]  

\[ \text{value of current profit} \quad \text{value of market share} \]  

14
To see that the first term corresponds to the value of marginal profits, we can substitute the first-order condition (18) in the second term of the above expression, which yield

$$1 - \frac{E_t^a[\kappa_{it}]}{E_t^a[\xi_{it}]} = 1 - \frac{E_t^a[\xi_{it}a_{it}]}{E_t^a[\xi_{it}]} \frac{w_t}{\alpha A_t} (c_t + \phi)^{1-\alpha}$$

$$\equiv 1 - \frac{E_t^a[\xi_{it}a_{it}]}{E_t^a[\xi_{it}]} \mu(A_t, c_t, w_t)^{-1},$$

where $\mu(A_t, c_t, w_t) \equiv (A_t/w_t)(c_t + \phi)^{1-\alpha}$ denotes the aggregate (marginal) gross markup. With frictionless financial markets, the value of marginal profit is $1 - 1/\mu(A_t, c_t, w_t)$. The presence of financial distortions importantly tilts the way the firm assess the marginal revenue and cost through the terms $E_t^a[\xi_{it}]$ and $E_t^a[\xi_{it}a_{it}]$.

Using the property of a log-normal distribution (Johnson, Kotz, and Balakrishnan (1994)), the interaction term can be expressed as

$$E_t^a[\xi_{it}a_{it}] = \int_0^{a^E} adF(a) + \int_{a^E}^{\infty} \frac{dF(a)}{1 - \phi} = 1 + \frac{\phi}{1 - \phi} [1 - \Phi(z_t^E - \sigma)] \geq 1.$$

Note that $E_t^a[\xi_{it}a_{it}] \geq E_t^a[\xi_{it}] \geq 1$ and the inequalities are strict if and only if $\phi, \sigma > 0$; note also that this difference measures the covariance between the shadow value of internal funds and the idiosyncratic productivity shock because $E_t^a[\xi_{it}a_{it}] = E_t^a[\xi_{it}]E_t^a[a_{it}] + Cov_t^a[\xi_{it}, a_{it}] = E_t^a[\xi_{it}] + Cov_t^a[\xi_{it}, a_{it}]$, where the last equality reflects the fact that $E_t^a[a_{it}] = 1$. Algebraically, this covariance is given by

$$Cov_t^a[\xi_{it}, a_{it}] = \frac{\phi}{1 - \phi} [\Phi(z_t^E) - \Phi(z_t^E - \sigma)] > 0. \quad (24)$$

With financial market frictions ($\phi > 0$), this covariance term is strictly positive because the firm does not want to rely on costly external financing unless it is experiencing financial distress as a result of an unusually bad liquidity shocks (i.e., the realization of $a_{it}$ is very high). Indeed, the positive covariance term can also be gleaned directly from equation (22). The presence of financial distortions simultaneously increases the value of marginal revenue ($E_t^a[\xi_{it}]$) and the value of marginal cost ($E_t^a[\xi_{it}a_{it}]$), though the latter effect dominates. As a result, an adverse productivity shock, or an increase in demand, makes the firm more conservative in its pricing decision in order to avoid potential losses.

To streamline the notation, we define the financially-adjusted markup

$$\tilde{\mu}(A_t, c_t, w_t) \equiv \frac{E_t^a[\xi_{it}]}{E_t^a[\xi_{it}a_{it}]} \mu(A_t, c_t, w_t) \leq \mu(A_t, c_t, w_t), \quad (25)$$

an expression that will be used to derive a closed-form solution for the value of the customer base.\footnote{This also defines $\tilde{\mu}$ in real terms. Because $E_t^a[\xi_{it}] / E_t^a[\xi_{it}a_{it}] < 1$ due to the positive covariance term, the wedge makes the unadjusted markup $\mu$ go up to compensate for the financial wedge.}
First, however, let $g_t \equiv c_t / s_{t-1} = (s_t / s_{t-1} - \rho) / (1 - \rho)$. By iterating equation (20) forward, one can then verify that the marginal value of an increase in the customer base satisfies:

$$\frac{\lambda_t}{E_t^a[\xi_{it}]} = \theta(1 - \eta) E_t \left[ \sum_{s=t}^{\infty} \tilde{\beta}_{t,s+1} \frac{E_t^a[\xi_{is+1}]}{E_t^a[\xi_{it}]} \left( \frac{\mu_{s+1} - 1}{\mu_{s+1}} \right) \right],$$

(26)

where $\tilde{\beta}_{t,s+1} \equiv m_{s,s+1} g_{s+1} \times \prod_{j=s+1}^{\infty} \rho^{s-t} \theta(1 - \eta)(1 - \rho) g_{t+j} m_{t+j-1,t+j}$ denotes the growth-adjusted discount factor. The marginal value of the customer base is the expected present value of future marginal profits. Substituting the expression for the value of customer base into equation (19) and using the expression for the financially-adjusted markup then yields a closed-form solution for the value of marginal sales:

$$\frac{E_t^a[\nu_{it}]}{E_t^a[\xi_{it}]} = \frac{\tilde{\mu}_t - 1}{\tilde{\mu}_t} + \chi E_t \left[ \sum_{s=t+1}^{\infty} \tilde{\beta}_{t,s} \frac{E_t^a[\xi_{is}]}{E_t^a[\xi_{it}]} \left( \frac{\tilde{\mu}_s - 1}{\tilde{\mu}_s} \right) \right],$$

(27)

where $\chi \equiv (1 - \rho) \theta(1 - \eta) > 0$ if $\theta < 0$ and $\eta > 1$.

In this context, the liquidity condition of the firm—as summarized by the sequence of $E_t^a[\xi_{is}]$, $s = t, \ldots, \infty$—determines the weight that the firm places on current versus future profits when determining the expected price trajectory. If today’s liquidity premium outweighs the future liquidity premia, the firm will places a greater weight on current profits. In fact, using extensive simulations we show that when the current liquidity premium—as measured by $E_t^a[\xi_{it}]$—sufficiently exceeds the next period’s premium, the firm will value current profits more than future profits and, as a result, choose a higher price trajectory, resulting in higher inflation.

4.5 The Phillips Curve

In this section, we analyze how the presence of real and financial distortions affect aggregate inflation dynamics. Specifically, we study the local dynamics of inflation around the steady state, an approach that allows for an easy comparison with the standard literature.

Imposing the symmetric equilibrium ($p_{it} = 1$, $c_{it} = c_t$) and dividing equation (21) through by $E_t[\xi_{it}] c_{it}$ yields the following Phillips curve:

$$1 = \gamma \pi_t (\pi_t - \bar{\pi}) - \gamma E_t \left[ m_{t,t+1} \frac{E_t^a[\xi_{it+1}]}{E_t^a[\xi_{it}]} \pi_{t+1} (\pi_{t+1} - \bar{\pi}) \frac{c_{t+1}}{c_t} \right] + \eta \frac{E_t^a[\nu_{it}]}{E_t^a[\xi_{it}]},$$

(28)

The local dynamics of inflation can be assessed by log-linearizing equation (28) as follows:

$$\hat{\pi}_t = \frac{1}{\gamma} (\hat{\xi}_t - \hat{\nu}_t) + \beta E_t[\hat{\pi}_{t+1}],$$

(29)

\footnote{This derivation of the log-linearized Phillips curve exploits a steady-state relationship $E_t^a[\xi_{i}] / E_t^a[\nu_{it}] = 1/\eta$; see the model appendix for details.}
where \( \hat{\xi}_t \) and \( \hat{\nu}_t \) denote the log-deviations of \( E_a^a[\xi_{it}] \) and \( E_a^a[\nu_{it}] \), respectively. Equation (29) clearly shows that given inflation expectations, the current inflation rate depends crucially on the relative importance of the value of internal funds (\( E_a^a[\xi_{it}] \)) versus the value of marginal sales (\( E_a^a[\nu_{it}] \)).

To highlight the relationship between the model’s structural parameters and inflation dynamics, we can log-linearize equation (27) and substitute the result in equation (29). These steps yield the following Phillips curve

\[
\hat{\pi}_t = -\frac{\omega(\eta - 1)}{\gamma} \left[ \hat{\mu}_t + \mathbb{E}_t \sum_{s=t}^{\infty} \chi \hat{\delta}^{s-t+1} \hat{\mu}_{s+1} + \beta \mathbb{E}_t[\hat{\pi}_{t+1}] \right] + \frac{1}{\gamma} [\eta - \omega(\eta - 1)] \mathbb{E}_t \sum_{s=t}^{\infty} \chi \hat{\delta}^{s-t+1} \left[ (\hat{\xi}_t - \hat{\xi}_{s+1}) - \hat{\beta}_{t,s+1} \right],
\]

where \( \omega \equiv 1 - \beta \theta/(1 - \rho)/(1 - \rho \beta) \), \( \hat{\beta}_{t,s+1} \) is the log-deviation of \( \tilde{\beta}_{t,s+1} \). Note that this formulation of the Phillips curve encompasses a number of special cases:

1. **New Keynesian** (\( \theta = 0, \varphi = 0 \)): No internal habit (\( \theta = 0 \)) implies \( \omega = 1 \) and \( \chi = 0 \). Furthermore, the absence of financial distortions (\( \varphi = 0 \)) implies \( E_a^a[\xi_{it}] = E_a^a[\xi_{it} a_{it}] = 1 \) and hence \( -\hat{\mu}_t = (\hat{w}_t - \hat{A}_t) \). In this case, equation (30) reduces to the traditional New Keynesian Phillips curve.

2. **Internal Habit Only** (\( \theta < 0, \varphi = 0 \)): Without financial distortions, all terms involving \( \hat{\xi}_t - \hat{\xi}_{s+1} \) in the second line of equation (30) disappear. However, because of the sticky customer base, the terms involving \( \hat{\beta}_{t,s+1} \) remain. Recall that \( \hat{\beta}_{t,s+1} \) captures the capitalized growth rate of customer base and thus measures the present value of the marginal benefit from expanding customer base today. According to equation (30), when the firm expects a greater benefit from the future customer base, it is more willing to lower its current price in order to build its customer base.

3. **Financial Distortions Only** (\( \theta = 0, \varphi > 0 \)): Without the sticky customer base, all present-value terms in equation (30) disappear. Reflecting the presence of financial friction, however, \( -\hat{\mu}_t = \tilde{\xi}_t a_{it} - \hat{\xi}_t + \hat{w}_t - \hat{A}_t \), where \( \tilde{\xi}_t a_{it} \) is the log-deviation of \( E_a^a[\xi_{it} a_{it}] \). As discussed above, \( E_a^a[\xi_{it} a_{it}] - E_a^a[\xi_{it}] \) measures the covariance between the shadow value of internal funds and the idiosyncratic productivity shock. Hence, any disturbance that leads to an increase in that covariance may increase the inflation rate above the level that would have prevailed in the Modigliani–Miller paradigm of frictionless financial markets.

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15 In deriving these special cases, we assume a constant returns-to-scale production technology.

16 An interesting empirical implication of this model is that an econometrician who failed to include a proxy for financial frictions in the empirical specification of the Phillips curve may erroneously attribute positive forecast errors to markup shocks.
4. **Internal Habit and Financial Distortions** ($\theta < 0$, $\varphi > 0$): With both a sticky customer base and financial market frictions, firms face a fundamental tradeoff between current profit maximization and the long-run maximization of their market share, which is reflected in the term $(\xi_{t} - \xi_{s+1}) - \beta_{t,s+1}$. The maximization of their market share requires firms to lower current prices. However, firms may be forced to deviate from this dynamic optimization strategy, provided that their current liquidity position $(\xi_{t})$ is sufficiently weak relative to future liquidity position $(\xi_{s+1})$. In that case, firms may raise their current prices to avoid costly external financing, a policy that resembles a myopic optimization of current profits.

The fundamental tradeoff between the maximization of current cash flows and long-run maximization of market shares in our model hinges crucially upon a parameter restriction that $\eta - \omega(\eta - 1) > 0$. Otherwise, a seemingly pathological phenomenon may occur: Firms in strong financial condition may increase their current prices in order to increase their long-run market shares. This condition, however, is not particularly restrictive in our context because, as shown in the model appendix, $\eta - \omega(\eta - 1)$ is determined by the steady-state marginal profit:

$$
\eta - \omega(\eta - 1) = \frac{E_{t}^{q}[\xi_{i}]}{E_{t}^{q}[\nu_{i}]} - \frac{E_{t}^{q}[\kappa_{i}]}{E_{t}^{q}[\nu_{i}]} = \left[1 - \frac{E_{t}^{q}[\kappa_{i}]}{E_{t}^{q}[\xi_{i}]}\right] > 0
$$

In other words, as long as $\theta$, $\rho$ and $\eta$ are chosen such that the steady-state marginal profit is positive, we can exclude such pathological cases.

### 4.6 The Rest of the Model

The household faces a budget constraint

$$
b_{t+1}^{j} + \int_{0}^{1} p_{it} c_{it}^{j} di + \int_{0}^{1} p_{it} s_{F, it}^{j} di = w_{t} h_{t}^{j} + (1 + r_{t-1}) b_{t}^{j} + \int_{0}^{1} [\max\{d_{it}, 0\} + p_{it}^{S}] s_{F, it}^{j} di,
$$

where $b_{t}^{j}$ is the amount of government debt held by the household $j$, $s_{F, it}^{j}$ is the share of firm $i$ directly owned by the household $j$, $p_{it}^{S}$ is the time-$t$ value of shares outstanding at time $t - 1$, and $p_{it}^{S}$ is the ex-dividend value of equity at time $t$. The last two terms are related via the accounting identity $p_{it}^{S} = p_{it-1}^{S} + e_{it}$, where $e_{it}$ is the value of new shares issued at time $t$.

The costly equity finance assumption implies that $e_{it} = -(1 - \varphi) \times \min\{d_{it}, 0\}$. Using the equity accounting identity and the fact that $\int_{0}^{1} p_{it} c_{it}^{j} di = \tilde{p}_{t} x_{t}^{j}$ (see equation (11)), we can rewrite the budget constraint as

$$
b_{t+1}^{j} + \tilde{p}_{t} x_{t}^{j} + \int_{0}^{1} p_{it} s_{F, it+1}^{j} di = w_{t} h_{t}^{j} + (1 + r_{t-1}) b_{t}^{j} + \int_{0}^{1} [\max\{d_{it}, 0\} + (1 - \varphi) \min\{d_{it}, 0\} + p_{it}^{S}] s_{F, it}^{j} di. \quad (31)
$$
The above expression then makes it clear that in general equilibrium, costly equity finance takes the form of sales of new shares at a discount. Because the owners of old and new shares are the same entity, there are no direct wealth effects associated with costly external finance—the losses borne by the old shareholders are offset exactly by the gains that accrue to the new shareholders.

Denoting the multiplier for the budget constraint by $\Lambda_t$ and maximizing equation (8) subject to equation (31) yields the following first-order conditions:

$$
x_t^j : \Lambda_t \tilde{p}_t = U_x(x_t^j - \delta_t, h_t^j) \quad (32)
$$

$$
h_t^j : \Lambda_t w_t = -U_h(x_t^j - \delta_t, h_t^j) \quad (33)
$$

$$
b_t^{j+1} : \Lambda_t = \beta E_t[\Lambda_{t+1}(1 + r_t)] \quad (34)
$$

$$
s_{F,t}^{j+1} : \Lambda_t = \beta E_t \left[ \Lambda_{t+1} \left( \frac{\tilde{d}_{t+1} + \tilde{p}_{t+1}^S}{\tilde{p}_t^S} \right) \right] \quad (35)
$$

where $\tilde{d}_{t+1} \equiv E_{t+1}^a[\max\{d_{it+1}, 0\}] + (1 - \varphi) E_{t+1}^a[\min\{d_{it+1}, 0\}]$, and we used the fact that $p_t^S = \tilde{p}_t^S$ in the symmetric equilibrium. From equation (32), it follows that $\Lambda_t = U_x(x_t^j, h_t^j) / \tilde{p}_t$. Because in the symmetric equilibrium, $\Lambda_t = \Lambda_t$, $p_{it} = 1$, and $s_{it-1} = s_{t-1}$, it follows that $\tilde{p}_t = s_{t-1}^0$. Hence $\Lambda_t = U_x(x_t, h_t) / s_{t-1}^0$ and

$$
m_{t,t+1} = \beta \frac{U_x(x_{t+1} - \delta_{t+1}, h_{t+1})}{U_x(x_t - \delta_t, h_t)} \frac{s_{t-1}^0}{s_t^0}.
$$

Equations (32) and (33) together imply the following efficiency condition:

$$
\frac{w_t}{\tilde{p}_t} = \frac{w_t}{\tilde{p}_t} = \frac{U_h(x_t - \delta_t, h_t)}{U_x(x_t - \delta_t, h_t)}.
$$

Lastly, we assume that the monetary authority sets the nominal interest rate using a Taylor-type rule that responds to the inflation and output gaps:

$$
r_t = \max \left\{ 0, (1 + r_{t-1})^{\rho_r} \left[ (1 + \bar{r}) \left( \frac{\pi_t}{\pi^*} \right)^{\rho_r} \left( \frac{y_t}{y_t^*} \right)^{\rho_y} \right]^{1 - \rho_r} - 1 \right\}. \quad (36)
$$

The rule also allows for policy inertia, as reflected in letting $\rho_r \in (0, 1)$. In our baseline specification, we set $\rho_y = 0$, so that only inflation enters the Taylor rule. We also allow the policy rate to be bounded below by zero, which allows us to explore the role of financial distortions at the zero lower bound, a topic of particular relevance in the current environment.

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17 Note that the case when $d_{it} < 0$ corresponds to an investment made by the representative household—that is, the household purchases new shares issued by the firm. The term $(1 - \varphi)$, therefore, reflects a “discount” to the household.

18 In the numerical implementation of the model, we also assume an adjustment cost for the nominal wage by introducing market power associated with differentiated labor. For expository purposes, we skipped those details; the complete derivation of the dynamic wage Phillips curve is contained in the model appendix.

19 When analyzing the dynamics of the economy when the policy rate is at the zero lower bound, we use the
5 Calibration

There are three sets of parameters in the model: (1) parameters related to preferences and technology; (2) parameters governing the strength of nominal rigidities and monetary policy response; and (3) parameters determining the strength of financial market friction.

We set the time discounting factor equal to 0.99. We set the deep habit parameter $\theta$ equal to -0.8 following Ravn, Schmitt-Grohe, and Uribe (2006). We also choose a fairly persistent habit formation such that only 5 percent of the habit stock is depreciated in a quarter in order to highlight the firms’ incentive to compete for market share. However, this is only marginally more persistent than in Ravn, Schmitt-Grohe, and Uribe (2006). The CRRA parameter is then set equal to 1, given that the deep habit specification provides a strong motive to smooth consumption. We set the elasticity of labor supply equal to 5. For the aggregate technology shock process, we assume $\rho_A = 0.90$, a somewhat lower value than that employed in the real business cycle literature; this choice reflects the fact our model has a number of elements that generate persistent dynamics of the endogenous quantities.

The elasticity of substitution is a key parameter in the customer-markets model—the greater the firm’s market power, the greater is its incentive to invest in customer base. Broda and Weinstein (2006) provide a set of point estimates for the elasticity of substitution for the U.S. economy. Their estimates lie in the range between 2.1 and 4.8, depending on the characteristics of products (commodities vs. differentiated goods) and subsamples (before 1990 vs. after 1990). Using the post-1990 data, Broda and Weinstein (2006) estimate the median value of the elasticity of substitution for differentiated goods at 2.1, a product category that is most relevant for the deep habits model. Accordingly, we set $\eta = 2$; Ravn, Schmitt-Grohe, Uribe, and Uuskula (2010) also provides a point estimate of 2.48 using their structural estimation method.

Another important parameter is the fixed operating cost, $\phi$. This parameter is jointly determined with the returns to scale parameter $\alpha$. We set $\alpha$ first, then choose $\phi$ so that the dividend payout ratio (relative to income) hits the post-WWII mean value of 2.5 percent. Decreasing returns-to-scale enhance the link between the financial market friction and the pricing decision. We choose $\alpha = 0.8$. While this degree of returns to scale parameter is not unusual in empirical investment literature based on Compustat data (see Hennessy and Whited (2007)), the model’s dynamics are not substantially affected by varying the $\alpha$ between 0.8 and 1; in this sense, this is our “preferred” calibration. With the chosen $\alpha$, $\phi$ and $\eta$, the average markup is equal to 1.19.

To calibrate the degree of financial distortions, we set the equity dilution cost $\varphi = 0.30$, the value deterministic simulation routine of Adjemian, Bastani, Juillard, Mihoubi, Perendia, Ratto, and Villemot (2011) to allow for a fully nonlinear solution, after replacing the max operator in equation (36) with a smooth approximation, namely: $\max\{x, 0\} \approx 0.5 \times (\sqrt{2x^2 + \epsilon^2} + x)$ (in our simulations, we set $\epsilon = 1 e - 4$). The use of a nonlinear solution method is important in this case because the shocks that push the policy rate to the zero lower bound are typically large and thus place the economy in the region where local dynamics in the neighborhood of a nonstochastic steady state may not adequately approximate the nonlinear response of the economy to such shocks.
used by Cooley and Quadrini (2001). When we consider the dilution cost as an exogenous shock process, we set the persistence of the financial shock at 0.90. However, in our crisis experiment—a simulation exercise that imposes an extreme degree of financial distortions—we let $\varphi = 0.50$. The volatility of the idiosyncratic shock is calibrated as 0.05 at a quarterly frequency, which implies a moderate amount of idiosyncratic uncertainty. With the fixed operating cost calibrated as described above, the combination of $\sigma = 0.05$ and $\varphi = 0.30$ yields $E^a[\xi_i] = 1.127$.

For the parameters related to nominal rigidities, we set the adjustment costs of nominal prices $\gamma_p = 10.0$ and wages $\gamma_w = 30.0$. These values are close to the point estimates of 14.5 and 41.0 in Ravn, Schmitt-Grohe, and Uribe (2006), who show that deep habit model substantially enhances the persistence of inflation dynamics without the help of implausibly large amount of adjustment friction in nominal prices. Finally, we set the inertial Taylor coefficient at a conventional level of 0.75, and the coefficient of inflation gap as 1.5, which is in line with the New Keynesian literature.

6 Model Simulations

Our main goal in developing the above model was to gain an insight as to why the substantial and persistent slack in the productive capacity of the U.S. economy during the Great Recession and its aftermath did not result in significant disinflation, let alone an outright deflationary spiral that was of much concern to both academics and policy makers. To study the dynamics of inflation and other endogenous variables in a financial crisis, this section reports results from two experiments. First, we consider an extreme crisis situation. In the model, the crisis corresponds to a calibration, which implies that raising external funds—though not impossible—is tremendously costly, and the firms finance themselves almost entirely through internal funds. In the second experiment, we consider an economy that experiences a temporary bout of financial turmoil, corresponding to a (finite) number of periods in which the cost of external finance increases from its normal level.

6.1 Financial Crisis and Inflation Dynamics

To implement a financial crisis in the model, we set $\varphi = 0.50$, which implies the dilution effect of new equity of 50 percent. Admittedly, such an extreme situation seems plausible only at the nadir of a financial crisis, such as that following the collapse of Lehman Brothers in the early autumn of 2008, a period during which virtually no firm could contemplate raising outside equity to finance its operating cost and investment. With this calibration, we analyze the impact of both financial shocks and conventional supply and demand shocks on the pricing decisions of firms in our model.

Figure 6 compares the impact of an adverse one standard deviation technology shock across the two model economies: an economy with financial distortions (solid blue line) and an economy with frictionless financial markets (dotted red line). In both cases, the negative technology shock leads a sizable drop in economic activity (panel (a)) and a short-lived burst of inflation (panel
responses consistent with the canonical New Keynesian theory. As indicated in panel (a), the presence of financial distortions amplifies somewhat the response of output to a technology shock, a result consistent with the standard financial accelerator mechanism.

Although differences in the response of output across the two models are fairly modest, the initial response of inflation is quite different. In the model with financial frictions, the initial response of inflation is almost double that of the response from the model without financial distortions. The explanation for this difference can be found in panels (e)–(f). The timing assumptions of the model imply that firms are aware of the negative aggregate productivity shock before making their pricing decisions. In the presence of financial market frictions, this reduces the firms’ expected internal cashflows and increases the probability that they will require costly external finance, which causes an increase of 400 basis points in the shadow value of internal funds (panel (f)). To protect themselves against the idiosyncratic tail event in which the ex post cashflows are negative and they must raise costly external finance, firms boost their markups relative to the model without financial distortions (panel (e)).

As discussed above, the presence of severe financial distortion causes the value of internal cashflows and the value of marginal sales to move in tandem (panels (f) and (g)) because financial distortions create a direct link between the two valuations, which does not exist in an economy with frictionless financial markets. In fact, the value of marginal sales in a model without financial frictions moves in the opposite direction compared with model that allows for financial distortions. Because an increase in today’s prices erodes both the current and future customer base, firms in a perfect capital markets environment are more concerned about the future customer base relative to current profits, and the marginal value of sales falls in response to an adverse productivity shock. In the model with financial frictions, by contrast, the substantial increase in the value of internal funds causes the marginal value of sales to rise in response to such an aggregate shock.

Finally, as shown by the closed-form solution for the marginal value of sales (see equation (27)), the benefits of future customer base are heavily discounted when liquidity conditions today are expected to be much worse than in the future. As a result, the firms’ pricing decisions are dominated by the concern about current—relative to future—profits, a form of myopia that is optimal from the firm’s perspective, but one that leads to a significant increase in prices, which is suboptimal from the perspective of a purely monopolistic firm that internalizes the effect of all pricing decisions on demand.

Figure 7 shows the case of an adverse aggregate demand shock, which directly affects the marginal utility of consumption (see equation (8)), thereby replicating the so-called autonomous spending cut. The contractionary demand shock implies very similar output paths across the two models but again markedly divergent inflation trajectories. In the absence of financial distortions, prices decline substantially, leading to a persistent deflation. With imperfect capital markets, however, firms boost their prices in an effort to insure against the need to issue costly external
funds to meet their liquidity needs. As a result, inflation increases modestly in response to a contractionary demand shock.

Note that in both models, the negative demand shock leads to a sharp increase in the markup. The countercyclical markups in a demand-driven cycle arise naturally in models with deep habits. Our simulations indicate that financial distortions can substantially amplify the countercyclical behavior of markups in such models. According to our results, the increase in the markup in the model with financial distortions is double that implied by the model without such distortions (panel (e)). Panel (f) and (g) show that the driving force behind the countercyclical nature of markups is the deterioration in the firms’ liquidity positions, which causes firms to increase prices in order to maintain short-term profits in the face of weakening demand.

Figure 8 analyzes how the cyclical behavior of markups—in response to adverse technology and demand shocks—interacts with nominal and financial frictions of the model. As shown in the top left panel, the behavior of the markup in response to a negative technology shock depends crucially on the degree of nominal rigidities. In the model with flexible prices and internal habit, the markup is strongly countercyclical, regardless of the efficiency of the financial system (solid blue and red lines). Introducing financial market imperfections does induce some countercyclicality in the markup: Although the markup declines initially in response to a negative technology shock, the markup is weakly countercyclical, on balance, when nominal rigidities are present (dotted blue and red lines). Note that in this case, financial distortions substantially amplify the countercyclical dynamics of the markup, as the acute financial distress brought about by the adverse technology shock induces firms to increase their prices.

The right panel considers the implications of a contractionary demand shock. In this case, nominal rigidities plays a central role in generating the countercyclical markup. However, even with nominal rigidities, the response of the markup to a contractionary demand shock eventually crosses into negative territory, which suggests limited countercyclical behavior over a somewhat longer horizon. The presence of financial distortions, however, significantly strengthens the countercyclical behavior of the markup over the medium run.

A notable feature of these results is the fact that the response of the markup to both types of shocks in the model with financial frictions and without nominal rigidities is considerably more pronounced compared with the responses implied by the model that contains both forms of distortions. This implies that in a flexible-price environment, firms facing significant frictions in capital markets would like to increase their prices even more in response to both types of shocks, suggesting that nominal rigidities in the presence of financial distortions serve as a stabilizing mechanism in periods of acute financial distress. The bottom right panel shows that this is indeed the case: With a greater increase in the markup, output in the model with financial distortions but without nominal rigidities (blue solid line) drops considerably more in response to a contractionary demand shock, compared with the model that features both nominal and financial distortions (dotted blue
6.2 Financial Shocks and Inflation Dynamics

This section analyzes the macroeconomic implications of financial shocks. That is, rather than considering a crisis situation in which it is virtually impossible to raise external equity, we introduce financial turmoil in the model by considering a shock that temporarily boosts the cost of external funds. To explore this possibility, we assume that the equity issuance cost parameter \( \varphi \) follows an AR(1) process of the form

\[
\varphi_t = \varphi f_t, \quad \log f_t = \rho_f \log f_{t-1} + \epsilon_{t,f},
\]

and consider a financial shock \( \epsilon_{t,f} \) that increases—upon impact—equity dilution costs by 25 percent from their steady-state level; the equity issuance cost parameter then converges back to its normal level, following the autoregressive dynamics specified in equation (37).

Figure 9 shows the responses of the model economy to a one standard deviation demand shock; the solid blue line depicts the case where the contractionary demand shock is accompanied by a simultaneous shock to the cost of external finance, whereas the dotted red line shows the case where the economy is perturbed only by the demand shock. In the figure, therefore, the differences between the solid blue and dotted red lines measures the additional impact of a financial shock. These simulations show that the economy becomes significantly more sensitive to demand shocks when financial distortions become more severe. According to panels (a) and (b), the temporary increase in external financing costs has large additional effects on economic activity: The decline in both output and hours worked in response to a contractionary demand shock more than doubles when the economy concurrently also experiences an adverse financial shock. The response of inflation is also amplified substantially when both shocks hit the economy. In that case, in fact, the behavior of inflation is observationally equivalent to that implied by a frictionless economy that is experiencing an adverse technology shock. In our model, by contrast, a deterioration in the firms’ liquidity positions shrinks the financial capacity of the economy, which then directly shifts the Phillips curve upward.

Panels (e), (f), and (g) show the essential mechanism at work: When the economy is hit by both types of shocks, the markup, the value of internal funds, and the value of marginal sales all increase sharply relative to the case when the economy is perturbed only by a demand shock. Note that a significant portion of the increase in the shadow value of internal funds reflects the economy’s endogenous response to the temporary increase in financial distortions. The additional deterioration in economic outlook brought about by the financial shock increases the probability that firms will require costly external finance, which causes the shadow value of internal funds to rise further, amplifying the effect of the initial shock. This within-period financial multiplier plays

\[\text{Note that under our calibrations, this shock increases the level of equity dilution cost from 0.3 to 0.375, a degree of financial distortions that is significantly below that assumed in the crisis situation.}\]
a key role in enhancing the propagation mechanism of shocks in our model.

It is useful to compare the responses of the economy in the throes of a financial crisis—that is, when the cost of external finance is permanently at 0.5 (the solid blue lines in Figure 7)—with the case where the economy is experiencing a temporary increase in financial market frictions (the solid blue lines in Figure 9). The responses in Figure 7 reflect the macroeconomic consequences of a contractionary demand shock of the economy with significantly more distorted financial markets compared with the responses in Figure 9. However, the same-sized demand shock has a disproportionately greater effect in the case when the economy is experiencing a temporary financial distress. It is precisely in this sense that the prospect of an improvement in financial conditions leads to worse macroeconomic outcomes because it incentivizes the firms to postpone investing in their market shares.

6.3 Role of the Zero Lower Bound

We close the benchmark simulation exercises by analyzing the interaction of financial distortions and pricing behavior in an environment with a binding zero lower bound (ZLB) on nominal interest rates. To create a ZLB situation, we implement the “paradox of thrift,” a scenario in which the agents’ time discounting factor increases exogenously for a certain number of periods before returning back to its normal level. Specifically, we assume the following shock process

$$\beta_t = \tilde{\beta} v_t, \ log v_t = \rho_v \ log v_{t-1} + \epsilon_{t,v},$$

and set $\epsilon_{t,v} = 0.009$ for $t = 1, \ldots, 4$ and $\epsilon_{t,v} = 0.0$ for $t = 5, \ldots, \infty$. This sequence of shocks causes the discount factor $\beta$ to peak at 1.016—an environment of “hyper patience”—and then return gradually to its normal level.

Figure 10 shows the macroeconomic implications of such discount rate shocks under our baseline calibration. In a ZLB environment, firms in the model with financial distortions (the solid blue lines), are much more reluctant—compared with the firms operating in a frictionless economy (the dotted red lines)—to cut their prices in order to support their current cashflows. In fact, firms in the frictionless economy take a very aggressive stance by cutting prices 20 percent (at an annual rate) upon the impact of the shock. In a ZLB environment, this price response translates into a sharp increase in the real interest rate (panel (f)), which, in turn, leads to a significant decline in both output and hours worked. These simulations suggest that once the economy is at the ZLB, the incentive of firms to cut prices in order to maintain their market shares can be a highly destabilizing force, the effects of which, however, are substantially mitigated by the need of firms to maintain high current cashflows owing to the presence of financial distortions.\footnote{We also performed an experiment in which the economy is hit by the same sequence of discount rate shocks, except the degree of nominal rigidities (nominal price and wage adjustment costs) was five times as large as in our baseline calibration. In the model without financial distortions, the greater nominal rigidities make it difficult for}
7 Equilibrium Dispersion of Firm-Specific Inflation Rates

In the above simulations, we exploited the notion of symmetric equilibrium, where all firms in the economy chose the same inflation rate. We now extend the model to generate a non-degenerate distribution of firm-specific inflation rates. This extension illustrates how firms with strong balance sheets try to drive out weaker competitors by undercutting their prices in periods of financial distress.

7.1 Heterogeneous Operating Costs

To introduce heterogeneity in operating costs across firms, we modify the production technology according to

\[ y_{it} = \left( \frac{A_t}{a_{it}} \right)^{a} - \phi_i, \]  

(38)

where \( \phi_i \) denotes fixed operating costs of firm \( i \), which can take on one of \( N \)-values from a set \( \Phi = \{ \phi_1, \ldots, \phi_N \} \) with \( 0 \leq \phi_1 < \ldots < \phi_N \). We denote the measure of firms at each level of operating efficiency by \( \omega_1, \ldots, \omega_N \), with \( \sum_{k=1}^{N} \omega_k = 1 \). By assuming that \( N \) finite, we can also assume that each group of firms faces the same distribution of the idiosyncratic technology shock: \( \log a_{it} \sim N(-0.5\sigma^2, \sigma^2) \).

The introduction of heterogeneous operating costs implies that the external financing trigger for a class of firms with an operating costs \( \phi_k \) now equals

\[ a^{E}_t(\phi_k) = \frac{c_{it}}{(c_{it} + \phi_k)^{1/\alpha}} \frac{A_t}{w_t} \left[ p_{it} - \frac{\gamma}{2} \left( \frac{p_{it}}{p_{it-1}} - 1 \right)^2 \frac{c_{it}}{p_{it}} \right]; \]  

(39)

and the first-order condition for dividends (17) are given by

\[ \xi(a_{it}; \phi_k) = \begin{cases} 1 & \text{if } a_{it} \leq a^{E}_t(\phi_k) \\ \frac{1}{1 - \phi} & \text{if } a_{it} > a^{E}_t(\phi_k) \end{cases} \]  

(40)

From equation (40), the expected shadow value of internal funds is given by

\[ \mathbb{E}^a_t[\xi_t | \phi_k] = \Phi(z^{E}_t(\phi_k)) + \frac{1}{1 - \phi} \left[ 1 - \Phi(z^{E}_t(\phi_k)) \right] = 1 + \frac{\phi}{1 - \phi} \left[ 1 - \Phi(z^{E}_t(\phi_k)) \right] \geq 1. \]

Note that \( a^{E}_t(\phi_k) < 0 \) and hence \( d\mathbb{E}^a_t[\xi_t | \phi_k]/d\phi_k > 0 \). This implies that at a lower level of operating efficiency, the firm is more likely to have difficulties meeting its liquidity needs using only internally generated funds and hence faces a higher expected external finance premium.

Firms to aggressively cut their prices, and as a result, the deflationary pressures are much weaker than in the baseline calibration. This is consistent with the results of Christiano, Eichenbaum, and Rebelo (2011), who document that increased price flexibility exacerbates a deflationary spiral. In the model with financial distortions, in contrast, it makes quantitatively very little difference whether firms face mild or severe nominal rigidities.
7.2 Aggregation

To aggregate this model, we must modify the definition of the symmetric equilibrium. Specifically, all firms with the same $\phi_k$ choose the same price level:

$$P_{it}^{1-\eta} = \sum_{k=1}^{N} 1(\phi_i = \phi_k) \cdot P_{kt}^{1-\eta}.$$  \hspace{1cm} (41)

Aggregate inflation dynamics are then determined by a weighted average of $N$-groups. Because $\pi_t \equiv P_t/P_{t-1} = 1/P_{t-1} \left( \int_0^1 P_{it}^{1-\eta} di \right)^{1/(1-\eta)}$, using equation (41), we can express the aggregate inflation rate as

$$\pi_t = \frac{1}{P_{t-1}} \left( \int_0^1 \sum_{k=1}^{N} 1(\phi_i = \phi_k) \cdot P_{kt}^{1-\eta} di \right)^{1/(1-\eta)}$$

$$= \frac{1}{P_{t-1}} \left[ \sum_{k=1}^{N} P_{kt}^{1-\eta} \int_0^1 1(\phi_i = \phi_k) di \right]^{1/(1-\eta)}$$

$$= \left[ \sum_{k=1}^{N} \omega_k \left( \frac{P_{kt}}{P_{t-1}} \right)^{1-\eta} \right]^{1/(1-\eta)}$$

$$= \left[ \sum_{k=1}^{N} \omega_k \left( \frac{P_{kt}}{P_{kt-1}} \right)^{1-\eta} \left( \frac{P_{kt-1}}{P_{t-1}} \right)^{1-\eta} \right]^{1/(1-\eta)}.$$  \hspace{1cm} (42)

Hence, the aggregate inflation rate equals a weighted average of inflation rates of heterogeneous groups,

$$\pi_t = \left[ \sum_{k=1}^{N} \omega_k P_{kt-1}^{1-\eta} \pi_{kt-1} \right]^{1/(1-\eta)},$$  \hspace{1cm} (42)'

where $\pi_{kt} \equiv P_{kt}/P_{kt-1}$, is a type/sector–specific inflation rate and $p_{kt} \equiv P_{kt}/P_t$ is a type/sector–specific relative price. Note that the relative price $p_{kt}$ can no longer be normalized to one in the symmetric equilibrium. In equilibrium, there exists a non-degenerate dispersion of relative prices—that is, the notion of a symmetric equilibrium is restricted to types/sectors. The following Phillips curve describes the inflation dynamics of $k$-type firms:

$$0 = \frac{p_{kt}}{c_t} - \eta \mathbb{E}_t^q [\nu_{it} | \phi_k] c_{kt} - \gamma \pi_{kt} \pi_t \left( \pi_{kt} \pi_t - \bar{\pi} \right)$$

$$+ \gamma \mathbb{E}_t \left[ m_{t,t+1} \frac{\mathbb{E}_t^q [\xi_{it+1} | \phi_k]}{\mathbb{E}_t^q [\xi_{it} | \phi_k]} \pi_{kt+1} \pi_{t+1} \left( \pi_{kt+1} \pi_{t+1} - \bar{\pi} \right) \frac{c_{t+1}}{c_t} \right].$$  \hspace{1cm} (43)
The same modified symmetric equilibrium can be applied to aggregate output:

\[ c_{jt} = \sum_{k=1}^{N} 1(\phi_i = \phi_k) \cdot c_{kt}^j. \]

Because the household sector is still characterized by a symmetric equilibrium, we can delete the \( j \) superscript. Using equation (11), we can express the individual demand for the products produced by firms with efficiency rank \( k \) as

\[ c_{kt} = \left( \frac{p_{kt}}{\bar{p}_t} \right)^{-\eta} s_{kt-1}^{-\eta} x_t, \tag{44} \]

where

\[ \bar{p}_t = \left[ \sum_{k=1}^{N} \omega_k p_{kt}^{1-\eta} s_{kt-1}^{-\eta(1-\eta)} \right]^{1/(1-\eta)} \tag{45} \]

and

\[ x_t = \left[ \sum_{k=1}^{N} \omega_k \left( \frac{c_{kt}}{s_{kt-1}} \right)^{1-1/\eta} \right]^{1/(1-1/\eta)} \tag{46} \]

Note that equations (45) and (46) are derived by applying the same steps used to derive equations (42)–(12) and (9).

Aggregate demand then satisfies

\[ c_t = \left[ \sum_{k=1}^{N} \omega_k \left\{ \exp(0.5\alpha(1 + \alpha)\sigma^2) h_{kt}^\alpha - \phi_k \right\}^{1-1/\eta} \right]^{1/(1-1/\eta)}, \tag{47} \]

where the type-specific conditional labor demand is given by\(^{22}\)

\[ h_{kt} = \left[ \frac{c_{kt} + \phi_k}{\exp(0.5\alpha(1 + \alpha)\sigma^2)} \right]^{1/\alpha}, \tag{48} \]

and

\[ h_t = \sum_{k=1}^{N} h_{kt}. \tag{49} \]

### 7.3 Countercyclical Dispersion of Inflation Rates

For maximum intuition, we consider in the simulations only two types of firms: (1) financially “strong” and (2) financially “weak” firms. Financially strong firms are characterized by having

\(^{22}\)The expression \( \exp(0.5\alpha(1 + \alpha)\sigma^2) \) is the expected value of \( 1/a_{it} \), which is strictly greater than one due to Jensen’s inequality.
\( \phi_1 = 0 \). For financially weak firms, we set \( \phi_2 \) to the value used in the baseline calibration. Lastly, we assume identical measures for the two types of firms—that is, \( \omega_1 = \omega_2 = 0.5 \). The exercise in this section seeks to answer the following question: In periods of financial turmoil, do financially strong firms slash their prices to drive out their weaker competitors? To answer this question, we analyze the impact of a financial shock, which as in Section 6.2, corresponds to a temporary increase in equity dilution costs.

The solid blue line in panel (a) of Figure 11 shows the response of relative prices \( (p_{kt} = P_{kt}/P_t) \) for financially strong firms, whereas the red dotted line depicts the corresponding response of the financially weak counterparts. In response to an adverse financial shock, financially strong firms cut their prices—behavior consistent with the concurrent decline in aggregate demand—while firms with weak balance sheets actually increase their prices in an effort to avoid costly external financing. Panel (b) translates this difference in the price-setting behavior into type-specific inflation rates \( (i.e., \pi_{kt} = P_{kt}/P_{kt-1} \text{ for } k = 1, 2) \). Clearly evident is the countercyclical behavior of the dispersion in inflation rates, a result consistent with that documented by Vavra (2011). What is different here is that the countercyclical dispersion in inflation rates arises endogenously in response to the differences in financial conditions across firms, whereas Vavra (2011) relies on an exogenous second-moment (i.e., uncertainty) shock that is calibrated countercyclically.

Panel (c) shows the differential responses of output across the two types of firms, together with the response of aggregate output (green line). Reflecting their “victory” in the price war, financially strong firms expand output, whereas their financially weaker counterparts slash production. Again, the dispersion in output and employment at the micro level is generated endogenously by the distortions in financial markets.

The dynamics of the relative market shares for the two types of firms are shown in panel (e). Consistent with their aggressive pricing behavior, financially strong firms significantly expand their market share during the downturn. Because of the deep-habit preferences, the customer base of financially strong firms expands only gradually, though the expansion is quite persistent. Moreover, the customers that switched products during the downturn form a loyal group, as a substantial part of them stays with the same products, even after the relative prices of these products return to their normal level. For example, after 20 quarters, the relative prices of the financially strong firms are back to their normal level, but their relative habit shares remain high, which highlights the primary reason why undercutting competitors’ prices can be such a profitable investment.

In Figure 12, we consider a case, which we call the paradox of financial strength. The idea behind this exercise is to see whether firms with very robust balance sheets can slash their prices so aggressively that they will drive out the financially weaker firms to such an extent so as to generate a sizable drop in aggregate output. Such a scenario can be implemented in several different ways. One way is to make the contribution of the habit to the final demand more important and more

\[23\] Unlike the experiment reported in Figure 9, this exercise for simplicity abstracts from the simultaneous impact of the demand shock.
persistent by choosing higher numbers for \( \theta \) and \( \rho \). Alternatively, we can reduce the price elasticity of demand by lowering \( \eta \). In Figure 12 we illustrate the first case by setting \( \theta = -0.850 \) (baseline is \(-0.800\)) and \( \rho = 0.985 \) (baseline is \(0.950\)).

In this experiment, we consider two cases: (1) \( \phi_1 = 0 \) (labeled as case 1); and (2) \( \phi_1 = 0.8\phi_2 \) (labeled as case 2). Note that the latter case has more financially vulnerable firms, but at the same time, it weakens the relative strength of firms with strong balance sheets. The paradox of financial strength can be seen from the fact that the case with more financially vulnerable firms \( (\phi_1 = 0.8\phi_2) \) results in a milder drop in aggregate output in response to a temporary financial shock, compared with the case in which a smaller fraction of firms face liquidity pressures but there is a greater difference in the relative strength of firms’ balance sheets \( (\phi_1 = 0) \). This is because the first group of firms (financially strong firms) in case 2 cannot slash their prices as aggressively as can their counterparts in case 1. According to panel (a), the price cut by financially strong firms in case 1 is double that of the price cut by financially strong firms in case 2. The aggressive pricing strategy of financially strong firms in the first case drives down the output of financially weaker firms disproportionately more, resulting in a more severe recession. These findings suggest that macroeconomic stabilization policies that in periods of acute financial distress focus on providing support to financially vulnerable firms may be able more successful in avoiding catastrophic economic outcomes.

8 Conclusion

In this paper, we have investigated the effect of financial conditions on price-setting behavior during the “Great Recession.” We did through the lenses of customer-market theory, which emphasizes the idea that price-setting is a form of investment that builds the future customer base.

To motivate our analysis, we used confidential, individual producer prices from the BLS and Compustat to compare pricing behavior across firms with weak balance sheets relative to firms with strong balance sheets. We find strong evidence that at the peak of the crisis firms with relatively weak balance sheets increased prices, while firms with strong balance sheets lowered their prices. Similarly, firms that likely have high fixed operating costs—as evidenced by a high intensity of their SG&A spending—increased their prices, while firms with presumably better operating efficiency lowered prices. Regression analysis shows that liquidity positions and operating efficiency significantly influence the firms’ price-setting behavior during the height of the 2007–09 financial crisis.

We explored the implications of these empirical findings within the context of a New Keynesian framework that allows for customer markets and departures from the Modigliani-Miller paradigm of frictionless financial markets. In our model, firms have an incentive to set a low price to invest in market share. When financial distortions are severe, firms forgo these investment opportunities and maintain high prices. The model implies a substantial attenuation of price dynamics relative to
the baseline model without financial distortions in response to contractionary demand shocks. This implies that in the context of the zero lower bound, financial frictions can paradoxically improve overall economic outcomes.

References


Table 1: Summary Statistics, PPI and Matched Sample

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<th>Full PPI</th>
<th>Matched PPI Sample</th>
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<td>Std. Dev.</td>
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<td>0.120%</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.517%</td>
<td>0.564%</td>
</tr>
<tr>
<td>Median</td>
<td>0.158%</td>
<td>0.170%</td>
</tr>
<tr>
<td><strong>Monthly Frequency of Price Changes</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Trimmed Mean</td>
<td>13.80%</td>
<td>17.12%</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>1.11%</td>
<td>1.55%</td>
</tr>
<tr>
<td>Median</td>
<td>13.88%</td>
<td>17.28%</td>
</tr>
<tr>
<td><strong>Monthly Frequency of Price Changes, Weighted</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Trimmed Mean</td>
<td>41.28%</td>
<td>52.39%</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>5.27%</td>
<td>9.00%</td>
</tr>
<tr>
<td>Median</td>
<td>42.48%</td>
<td>52.11%</td>
</tr>
<tr>
<td><strong>Number of Firms</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Trimmed Mean</td>
<td>23628</td>
<td>769</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>1515</td>
<td>109</td>
</tr>
<tr>
<td>Median</td>
<td>23369</td>
<td>780</td>
</tr>
</tbody>
</table>

NOTE: We compute the above statistics using the micro price data underlying the PPI (full sample) and our sample matched to Compustat. The time period is from January 2005 through December 2012. First, we compute monthly inflation rates and the frequency of price changes at the level of the firm as the (weighted) means of log price changes and the price change indicators using within-firm importance weights. Second, we take (sales-weighted) means in each monthly cross section of firms. Finally, we report trimmed means, medians, and standard deviations of these means, as well as of the average monthly number of firms in the data.
Table 2: Summary Statistics, COMPUSTAT and Matched Sample

<table>
<thead>
<tr>
<th></th>
<th>Full Sample</th>
<th>Matched Sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liquidity Ratio</td>
<td>Truncated Mean</td>
<td>0.217</td>
</tr>
<tr>
<td></td>
<td>Std. Dev.</td>
<td>0.136</td>
</tr>
<tr>
<td></td>
<td>Median</td>
<td>0.134</td>
</tr>
<tr>
<td>Operating Income Ratio</td>
<td>Truncated Mean</td>
<td>0.005</td>
</tr>
<tr>
<td></td>
<td>Std. Dev.</td>
<td>0.032</td>
</tr>
<tr>
<td></td>
<td>Median</td>
<td>0.024</td>
</tr>
<tr>
<td>Interest Expense Ratio</td>
<td>Truncated Mean</td>
<td>0.068</td>
</tr>
<tr>
<td></td>
<td>Std. Dev.</td>
<td>0.021</td>
</tr>
<tr>
<td></td>
<td>Median</td>
<td>0.017</td>
</tr>
<tr>
<td>SG&amp;A Ratio</td>
<td>Truncated Mean</td>
<td>0.495</td>
</tr>
<tr>
<td></td>
<td>Std. Dev.</td>
<td>0.227</td>
</tr>
<tr>
<td></td>
<td>Median</td>
<td>0.281</td>
</tr>
<tr>
<td>Sales Growth</td>
<td>Truncated Mean</td>
<td>1.08%</td>
</tr>
<tr>
<td></td>
<td>Std. Dev.</td>
<td>3.91%</td>
</tr>
<tr>
<td></td>
<td>Median</td>
<td>1.03%</td>
</tr>
<tr>
<td>Sales</td>
<td>Truncated Mean</td>
<td>833.120</td>
</tr>
<tr>
<td></td>
<td>Std. Dev.</td>
<td>92.874</td>
</tr>
<tr>
<td></td>
<td>Median</td>
<td>67.613</td>
</tr>
<tr>
<td>Number of Unique Firms</td>
<td></td>
<td>6326</td>
</tr>
</tbody>
</table>

NOTE: We compute the above statistics using the full Compustat database for 2005 through 2012 and our matched sample. First, we compute at the firm level and quarterly frequency the ratio of cash and other liquid assets to total assets, the ratio of operating income to total assets, the ratio of interest expenses to total assets, and the ratio of sales and administrative expenses to total assets. Second, we compute time-series averages for each firm of these ratios, sales growth, and total sales. Finally, we report trimmed means, medians, and standard deviations of these means, as well as the number of unique firms in the data.
Table 3: Baseline Calibration

<table>
<thead>
<tr>
<th>Description</th>
<th>Calibration</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Preferences and production</strong></td>
<td></td>
</tr>
<tr>
<td>Time discounting factor, $\beta$</td>
<td>0.99</td>
</tr>
<tr>
<td>Constant relative risk aversion, $\gamma_x$</td>
<td>1.00</td>
</tr>
<tr>
<td>Deep habit, $\theta$</td>
<td>$-0.80$</td>
</tr>
<tr>
<td>Persistence of deep habit, $\rho$</td>
<td>0.95</td>
</tr>
<tr>
<td>Elasticity of labor supply, $1/\gamma_h$</td>
<td>5.00</td>
</tr>
<tr>
<td>Elasticity of substitution, $\eta$</td>
<td>2.00</td>
</tr>
<tr>
<td>Persistence of technology shock, $\rho_A$</td>
<td>0.90</td>
</tr>
<tr>
<td>returns to scale, $\alpha$</td>
<td>0.80</td>
</tr>
<tr>
<td>Fixed operation cost, $\phi$</td>
<td>0.26</td>
</tr>
<tr>
<td><strong>Nominal rigidity and monetary policy</strong></td>
<td></td>
</tr>
<tr>
<td>Price adjustment cost, $\gamma_p$</td>
<td>10.0</td>
</tr>
<tr>
<td>Wage adjustment cost, $\gamma_w$</td>
<td>30.0</td>
</tr>
<tr>
<td>Monetary policy inertia, $\rho^r$</td>
<td>0.75</td>
</tr>
<tr>
<td>Taylor rule coefficient for inflation gap, $\rho^\pi$</td>
<td>1.50</td>
</tr>
<tr>
<td><strong>Financial Frictions</strong></td>
<td></td>
</tr>
<tr>
<td>Equity issuance cost, $\varphi$</td>
<td>0.30, 0.50</td>
</tr>
<tr>
<td>Idiosyncratic volatility (a.r.), $\sigma$</td>
<td>0.20</td>
</tr>
<tr>
<td>Persistence of financial shock, $\rho_{\varphi}$</td>
<td>0.90</td>
</tr>
</tbody>
</table>
Figure 1: Producer Price Inflation, Full vs. Matched PPI Sample

Note: The solid line depicts the 3-month moving average of monthly inflation calculated using the full PPI sample, while the dotted line depicts the corresponding inflation calculated using the subsample of Compustat firms.
Figure 2: Producer Price Inflation by Selected Financial Characteristics

Note: The top panel shows the industry-adjusted 3-month moving average of monthly inflation rates for financially weak firms, while the bottom panel shows the corresponding inflation rates for financially strong firms; see text for details.
Figure 3: Producer Price Inflation by Selected Product-Market Characteristics

Note: The top panel shows the industry-adjusted 3-month moving average of monthly inflation rates for firms with different product-market characteristics; see text for details.
Figure 4: Producer Price Inflation by Selected Firm Characteristics and Production Sector

Note: The top panel shows the 3-month moving average of monthly inflation rates for firms in different financial positions within the durable and nondurable goods sector. The bottom panel shows the 3-month moving average of monthly inflation rates for firms with different product-market characteristics within the durable and nondurable goods sectors; see text for details.
Figure 5: Elasticities of Directional Price Changes

Note: The solid line in each panel depicts the estimated time-varying elasticity of the decision to adjust prices upwards or downwards—relative to a base category of no change—based on a multinomial logit specification; see text for details. Dotted lines denote robust 95% confidence intervals.
Figure 6: Impact of a Technology Shock

Note: Blue responses are from the model with financial distortions; red responses are from the model without financial distortions.
Figure 7: Impact of a Demand Shock

Note: Blue responses are from the model with financial distortions; red responses are from the model without financial distortions.
Figure 8: Nominal Rigidities, Financial Distortions, and the Dynamics of Markups

Note: Blue responses are from the model with financial distortions; red responses are from the model without financial distortions.
Figure 9: Impact of a Demand Shock with Higher External Financing Costs

Note: Both models contain financial distortions. Blue responses are from the model with temporarily elevated external financing costs; red responses are from the model without a temporary increase in external financing costs.
Figure 10: Impact of a Discount Rate Shock

Note: Blue responses are from the model with financial distortions; red responses are from the model without financial distortions.
Figure 11: Impact of a Financial Shock with Firm Heterogeneity

Note: Blue responses are those of firms with strong balance sheets; red responses are those of firms with weak balance sheets; and green responses are those of the aggregate economy.
Figure 12: Paradox of Financial Strength

Note: Cases 1 and 2 correspond to $\phi_1 = 0$ and $0.8\phi_2$, respectively.
Appendices

A Log-linearization of Phillips Curve

From (28), we derive the steady state relationship between the value of internal funds and the value of marginal sales as

$$\frac{E^a[\nu_i]}{E^a[\xi_i]} = \eta.$$  \hspace{1cm} (A-1)

(19) in the steady state implies

$$E^a[\nu_i] = E^a[\xi_i] - E^a[\kappa_i] + (1 - \rho)\lambda$$

Dividing this expression through by $E^a[\nu_i]$ yields

$$1 = 1/\eta - \frac{E^a[\kappa_i]}{E^a[\nu_i]} + (1 - \rho)\frac{\lambda}{E^a[\nu_i]}$$  \hspace{1cm} (A-2)

Since the ratio between the marginal value of customer base and the marginal value of sales is determined by (20) as

$$\frac{\lambda}{E^a[\nu_i]} = \frac{\beta \theta (1 - \eta)}{(1 - \rho \beta)},$$  \hspace{1cm} (A-3)

combining (A-2) and (A-3) yields

$$\frac{E^a[\kappa_i]}{E^a[\nu_i]} = 1/\eta - 1 - (\eta - 1)\frac{\beta \theta (1 - \rho)}{(1 - \rho \beta)}$$

 Subtracting the above expression from the inverse of (A-1) yields

$$\frac{E^a[\xi_i]}{E^a[\nu_i]} - \frac{E^a[\kappa_i]}{E^a[\nu_i]} = \frac{E^a[\xi_i]}{E^a[\nu_i]} \cdot \left[1 - \frac{E^a[\kappa_i]}{E^a[\xi_i]}\right]$$

$$= 1 + (\eta - 1)\frac{\beta \theta (1 - \rho)}{(1 - \rho \beta)} = \eta - \omega(\eta - 1)$$

where $\omega \equiv 1 - \frac{\beta \theta (1 - \rho)}{1 - \rho \beta}$. Hence, requiring $\eta - \omega(\eta - 1) > 0$ is equivalent to a strictly positive marginal profit in the steady state.

B Equilibrium Dispersion of Prices in the Steady State

In the steady state, the Phillips curve implies

$$p_k = \eta\frac{E^a[\nu_i|\phi_k]}{E^a[\xi_i|\phi_k]}.$$  \hspace{1cm} (B-1)

From the FOC for $s$, we have

$$\frac{\lambda_k}{E^a[\xi_i|\phi_k]} = \frac{\theta (1 - \eta) \beta E^a[\nu_i|\phi_k]}{1 - \rho \beta E^a[\xi_i|\phi_k]}.$$  \hspace{1cm} (B-2)
Combining the two yields
\[ \lambda_k \frac{\mathbb{E}^a[\xi_i|\phi_k]}{\mathbb{E}^a[\xi_i|\phi_k]} = p_k \frac{\theta(1 - \eta)\beta}{\eta(1 - \rho\beta)}. \] (B-3)

The FOC for \( c \) and \( h \) in the steady state imply
\[ \frac{\mathbb{E}^a[\nu_i|\phi_k]}{\mathbb{E}^a[\xi_i|\phi_k]} = -\frac{\mathbb{E}^a[\xi_i|\phi_k]}{\alpha A} \left( c_k + \phi_k \right)^{\frac{1}{\alpha}} + p_k + (1 - \rho)\frac{\lambda_k}{\mathbb{E}^a[\xi_i|\phi_k]} \] (B-4)

Substituting (B-1) and (B-3) in (B-4) yields
\[ p_k = \frac{\eta(1 - \rho\beta)}{(\eta - 1)(1 - \rho\beta) - \theta\beta(1 - \rho)} \frac{\mathbb{E}^a[\xi_i|\phi_k]}{\alpha A} \left( c_k + \phi_k \right)^{\frac{1}{\alpha}}. \] (B-5)

The external financing triggers in the steady state are given by
\[ a_E^k = \frac{p_k c_k A}{(c_k + \phi_k)^{1/\alpha}w}. \] (B-6)

The consumption aggregators in the steady state imply
\[ c_k = \left( \frac{p_k}{p_l} \right)^{-\frac{\eta}{s_k}} \frac{\theta^{1-\eta}}{s_k^{\theta(1-\eta)}} \] (B-7)

and
\[ x = \left[ \sum_{m=1}^{N} \omega_m \left( c_m^{1-\theta} \right)^{1-1/\eta} \right]^{1/(1-1/\eta)}. \] (B-8)

Equilibrium consistency requires
\[ 1 = \left[ \sum_{m=1}^{N} \omega_m P_m^{1-\eta} \right]^{1/(1-\eta)}, \] (B-9)

which is the steady state version of (12) with \( \pi = \pi_k = 1 \). Finally labor market and goods market clearing conditions imply
\[ \frac{w}{\hat{p}} x^{-\gamma_x} = \zeta h^\gamma_h \] (B-10)

and
\[ c = \left[ \sum_{m=1}^{N} \omega_m \left[ \exp(0.5\alpha(1 + \alpha)\sigma^2)h_m^{\alpha} - \phi_m \right]^{1-1/\eta} \right]^{1/(1-1/\eta)} \] (B-11)

where the type conditional labor demand satisfies
\[ h_k = \left[ \frac{c_k + \phi_k}{\left( \exp(0.5\alpha(1 + \alpha)\sigma^2) \right) h_m^{\alpha} - \phi_m} \right]^{1/\alpha} \] (B-12)

and
\[ h = \sum_{m=1}^{N} h_m \] (B-13)
Deep-habit adjusted price index in the steady state satisfies

\[ \bar{p} = \left[ \sum_{m=1}^{N} \omega_m p_m^{1-\eta} c_m^{\eta(1-\eta)} \right]^{1/(1-\eta)}. \]  \hspace{1cm} (B-14)

which is the steady state version of (45). (B-5) \sim (B-14) can then be solved for \( 4N + 5 \) variables: \( p_k, c_k, a_k^E, h_k \) for \( k = 1, \ldots, N \) and \( x, w, \bar{p}, h \) and \( c \).