1. Specification of sets

• Listing all elements of the set
Examples: sets specified in previous handout, alphabet, dictionary

• Predicate notation
Example: simple nouns in formal semantics, our Russian-accusatives problem
  ○ Russell's paradox

• Recursive rules – one way to avoid paradoxes
Example: rewrite rules (the set of sentences of a language)

2. Cardinality

\[ |A| \]

• a natural number, when the set is finite

• special other cardinalities for infinite sets
  ○ natural numbers
  ○ real numbers
  ○ ...

Cardinality of power sets

We'll talk extensively about counting elements of various sets in the future.

3. Set-theoretic equalities

• attached

• Proving them using subset arguments, and by contradiction – and using correspondences with “and”, “or” and “not”

Question 1: prove all of the equalities.

Simplifying expressions and proving new equalities – the importance of being pedantic

Question 2: Simplify \( A \cap (B - A) \), using the set-theoretic equalities

Question 3: Show \( ((A \cup C) \cap (B \cup C')) \subseteq (A \cup B) \), using the set-theoretic equalities

4. Cartesian products and pairs (and n-tuples)

Ordered pair can be defined using ordinary sets:
\[ <a,b> = \{\{a\},\{a,b\}\} \]

• now, order matters

• extending further: \( <a,b,c> = <<a,b>,c> \)

A \times B (cartesian product)

Examples: student-faculty questions, \( F = \{\text{Alex, Beatrice, Cyrill}\} \)
S = \{\text{Yael, Zee}\}

faculty end-of-year reports for students, sets as above
tic-tac-toe games among students (ok to play with self)

5. Relations

Any set of ordered pairs is a relation
  • 'relational nouns' = mother, teacher, cousin, neighbour
    ○ projections onto first and second coordinate
    ○ domain, range
    ○ relations in a set, relations from one set to another
    ○ inverses and complements of relations

Partee et al. Exercise 1 on page 36

6. Functions – special relations

A relation R from A to B is a function iff
  1. each element in the domain is paired with only one element in the range
  2. the domain of R is equal to A
(look at the first set, A, to see that it occurs exactly once)

A dynamic perspective: transformations, maps, mappings, correspondences

Partial functions – satisfy (1) but not (2).
  • onto = surjection (vs. just into)
  • one-to-one = injection
  • both = one-to-one-correspondence = bijection

7. Composition of relations and functions

  • defining composition
  • inverses and composition
  • extending this to functions of ordered n-tuples

Partee et al. Exercises 2-4 on page 37.

8. Properties of relations

  • reflexivity
  • symmetry
  • transitivity
  • connectedness

Diagrams of relations

Properties of inverses and complements