Set theory: motivation and first concepts.

A complex example.

- We need to search through a giant library of Russian texts to find all sentences that begin with an accusative objects, to determine if sentence-initial position carries special significance.

  (1) Staruhu ubil Raskol'nikov.
  Old.woman.ACC killed.MASC Raskolnikov
  'Raskolnikov killed the old lady.'

  How do we tell a computer to search for this?
  - One way is to look for a period or a question mark or an exclamation point or a new paragraph, possibly followed by a space or two, and then by a (capitalized) word with an accusative ending.
  - There is a way to do it, called “regular expressions”, which we will ignore, since we're not actually doing the search.
  - But how can we select words with accusative endings?

Simplifying the facts a bit (ignoring neuter nouns etc.).

- Russian has three major declension classes, which have different accusative endings.
  - Class I ending is -u for nouns (which is actually identical to the dative case endings for Class II nouns), and -uju for agreeing feminine adjectives
    (2) ruku – hand.ACC beluju – white.fem.ACC
  - Class II endings are a bit more complex – they are different for animate and inanimate nouns. For animate nouns, the ending is identical to the genitive Class II ending, and to nominative Class I ending -a, and -ogo for agreeing masculine adjectives. For inanimate nouns, the ending is identical to nominative Class II ending, which is a consonant for nouns, and -yj for agreeing masculine adjectives.
    (3) Slona – elephant.ACC belogo – white.masc.anim.ACC
    (4) stol – table.ACC belyj – white.masc.inan.ACC
  - Class III combines these properties – the accusative ending for nouns is the same as for nominative Class III, where these nouns end in a “soft sign” (palatilization mark on a consonant), while agreeing feminine adjectives end in -uju.
    (5) Myshj – mouse.ACC, beluju – white.fem.ACC

  To complicate matters, first-person verbs can also end in -u and -uju, and there is a discourse particle “nu”, which often begins a sentence.

  QUESTION 1. How can we describe the exact bunch of words that we want to find?

Broader and (perhaps) simpler examples.

  o The meaning of a sentence is the set of situations in which it is true.
  o Entailment: sentence X entails sentence Y if the set of situations in which Y is true contains the set of situations in which X is true.

  (6) a. Fido is a dog. b. Fido is a poodle.
  (7) a. John has one good leg. b. John has two good legs.

  QUESTION 2: Based on the definition of entailment above, is there an entailment between (a) and (b)? Between (b) and (a)?
Basic concepts of set theory.

- A SET is a collection of objects.
- Some thing is an ELEMENT OF a set A if that thing is a member of the collection A. Notation: “∈” reads as “is an element of” or “belongs to”.

There is no repetition, so \{annie, boris, annie, chris\} = \{annie, boris, chris\}
The order does not matter at all, so \{annie, boris, chris\} = \{chris, annie, boris\}
Two sets are equal when they have all the same elements.

- A set A is a SUBSET OF a set B if all the elements of A are also in B.
  Notation: “⊆” reads as “is a subset of”.
  Note that a set can have elements that are themselves sets, and subsets that are sets of sets:
  \{a, \{b,c\}, d\} has \{b,c\} as its element, and therefore the single-element set \{\{b,c\}\} as its subset.

QUESTION 3. In the definition of entailment above, does the phrase “is contained within” correspond to “is an element of” or to “is a subset of”?

We can treat meanings of some nouns, verbs, and adjectives as sets (LING 130: Semantics discusses whether this is quite right):
- Nouns: denotations of nouns may vary from situation to situation. A noun denotes a set.
- Adjectives: their denotations also vary. An adjective like red or wooden or blond is a set.
  - Red is the set of all the red things in the situation, apple is the set of all the apples in the situation

QUESTION 4. In the example (1) above, what is the relationship between the meanings of the words dog and poodle? What about the meanings of the words dog and Fido?

A set with only one element in it is called SINGLETON. The unique set containing no elements at all is called “the null set” or “the empty set”, and is written as {} or Ø.

QUESTION 5. Suppose that no dragons or unicorns exist in the actual world. What is wrong with supposing that the meaning of a noun is just the set of objects it denotes in the actual world?

- The INTERSECTION of two sets A and B (A∩B) is the set containing all and only the objects that are elements of both A and B.

- The UNION of two sets A and B (A∪B) is the set containing all and only the objects that are elements of A, of B, or of both A and B.

- The COMPLEMENT of a set A (A’ ) is the set containing all the individuals in the discourse except for the elements of A.  
  -- Note 1: Complement is relative to Universe of Discourse 
  -- Note 2: many notations, including A*, A¹, A

We can combine meanings of some set-denoting words by intersecting the sets they denote. For instance, if in a particular situation Ann and Connor are blond while Betty is a brunette, and at the same time, Betty and Connor are smokers, while Ann does not smoke.

In this situation, what would be the meaning of the expression “blond smoker”?
Some natural language expressions seem to have meanings that can be captured by the operations of intersection, union, and complement. Think of examples below to determine what they are:

(8) a. Fido is a poodle. b. Fido is not a poodle.
(9) a. John is blond. b. John is handsome. c. John is blond and handsome.
(10) a. Leo is here. b. Leo is there. c. Leo is here or there.

• The **Power Set** of a set A (written as \(\mathcal{P}(A)\)) is the set whose members are all the subsets of A.
• A set-theoretic difference (also called the relative complement) \(A - B\) is the set of elements of A that are not in B.

**QUESTION 6:** Given the sets below and assuming that the universe of the discourse is \(\cup\{A, B, C, D, E, F, G\}\), list the members of the following sets:

\[
\begin{align*}
A &= \{1, 2, 3, 4\} & E &= \{\{1\}, 2, \{a, 1\}\} \\
B &= \{a, b, c, d, e, f\} & F &= \{1, c, d\} \\
C &= \{1, 2\} & G &= \{d, e, 2, 3\} \\
D &= \{1, 3, 4, a, b\}
\end{align*}
\]

a. \(C - D = \)

b. \(A \cap F = \)

c. \(A \cap B = \)

d. \(C' \cap F' = \)

e. \(E \cap C = \)

f. \((C \cup D) - (C \cup D) = \)

g. \(F \cup C = \)

h. \(G' \cap C = \)

i. \(A \cap E = \)

j. \((E \cup B) \cup D = \)