Goal-Setting, Social Comparison, and Self-Control*

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Abstract

This paper addresses the role of non-binding goals to attenuate time inconsistency. Agents have linear reference-dependent preferences and endogenously set a goal that is the reference point. They face an infinite horizon, optimal stopping problem in continuous time, where there exists an option value of waiting due to uncertainty. Goal-setting attenuates the time-inconsistent agent's tendency to stop too early, and may even lead an agent to wait longer than the first best. In particular, reference dependence is strictly worse for a time-consistent agent. Notably, none of the effects of goal-setting require any form of loss aversion. The model extends to social comparisons, in which each agent measures his outcome against his expectation of his peers outcome. In a heterogeneous group where agents have differing degrees of self-control, comparison to increasingly patient peers induces increasingly patient behavior. Nonetheless, every agent prefers to compare himself to a peer with the lowest degree of self-control possible, regardless of the severity of his own self-control problem.

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1 Introduction

Personal development is a burgeoning, multi-billion dollar industry that focuses on self-improvement on a variety of levels, from career aspirations to lifestyle choices to spiritual well-being. In response to consumer demand, this market offers books, motivational speakers, and personal coaching, as well as innumerable weight-loss programs. Institutional demand has resulted in consulting and employee development programs whose purpose is to raise productivity in the workplace. A central tenet of this industry is that goal-setting is a vital instrument for improving any aspect of life. For example, the "**S**(pecific) **M**(easurable) **A**(ttainable) **R**(elevant) **T**(imely) Goals" mnemonic is ubiquitous in project management, education, and self-help programs.[1] This prescription is supported by extensive empirical evidence that *non-binding* goals tend to increase effort, attention, and persistence (Klein 1991, Latham and Locke 1991, Locke and Latham 2002), and that satisfaction is tied to achievement relative to such a goal, not just final outcomes (Mento, Locke and Klein 1992).

Although much attention has been paid to the problem of present-biased preferences and intrapersonal conflict by individual decision makers, the role of self-set goals in attenuating the self-control problem has been relatively unexplored by economists thus far.[2] In standard economic models, only binding goals, involving external enforcement of punishment and rewards or pre-commitment, can affect motivation and behavior. For example, an employer can motivate a worker's effort choice by prescribing goals if the worker is evaluated and compensated accordingly. Present-biased agents can also enforce personal motivation by using binding precommitments or externally enforced contracts.[3]

However, the prevalence of non-binding goal-setting, and the success of personal development, strongly suggests that less drastic measures may be successful regulatory mechanisms. For example, dieters often set weight targets, and writers set self-imposed page targets and deadlines.[4] Individuals may also set goals for themselves in situations where there exists an option value of waiting due to uncertainty. A student may thus continue his education to achieve a target level of starting salary upon graduation. A person trying to save may set a target level of accumulated wealth upon retirement. In the marriage market, an individual may specify a standard of quality for a prospective spouse to avoid settling too soon.

[1]There exist many variations on this theme, which is generally attributed to Drucker (1954).

[2]Bénabou and Tirole (2004) develop a theory of self-enforcing personal rules, operating through self-reputation. Because agents have imperfect information about their own willpower and imperfect recall, they can achieve internal commitment out of fear of creating precedents and losing faith in themselves.

[3]Classic examples include alcoholics who take Antabuse to preclude drinking and gamblers who place themselves on casino "do not enter" lists.

[4]Ariely and Wertenbroch (2002) conduct an experiment in which students set binding deadlines for themselves, since the students were penalized when they were not met.

In this paper, I propose that a goal provides internal motivation by acting as the reference point for an agent who possesses reference-dependent preferences. I consider agents who derive utility from both standard consumption, arising from outcomes, and comparison utility, derived from the comparison of these outcomes to a self-set goal. At each point in time, the agent sets a goal based on his expectations about the outcome of future behavior, which his next "self" will inherit as a reference point in his utility function. In this way, the agent provides a degree of internal motivation that is desirable if he exhibits time inconsistency due to present-biased time preferences. I consider the polar cases of sophistication and naivete about one's self-control problem to study the impact of differing expectations on behavior and welfare.

The economic setting that I consider is an infinite-horizon, optimal stopping problem in continuous time, where there exists an option value of waiting due to uncertainty. Present-biasedness leads the agent to stop too early because he undervalues this option relative to his time-consistent counterpart. In the preceding examples, this means accumulating too little human capital, retiring with too little wealth, or settling for a mediocre spouse. I show that if the agent has reference dependent preferences, even of the simplest, *linear* kind, he can induce more patient behavior by setting a goal to be evaluated upon stopping, regardless of whether he is sophisticated or naive. Formally, I solve for the unique stationary Markov equilibrium of the intrapersonal game and show that greater reference dependence leads to later exercise of the option. The presence of a goal increases the agent's incentive to wait because he wishes to avoid incurring comparative disutility from falling short of it. For any degree of present-biasedness, there exists a level of reference dependence such that a sophisticated agent can achieve the first best from an ex ante perspective. On the other hand, too much reference dependence causes him to wait *too long*. In particular, reference dependence always decreases the welfare of a time-consistent agent because it causes him to wait longer than the first best, by distorting his incentives at the margin. Thus, goal setting *per se* can itself be a source of intrapersonal conflict. Another key result, which stands in contrast to previous literature and conjectures, is that goal-setting does not require any form of loss aversion or curvature in the comparison utility function to regulate behavior.

Next, I study the optimal goal when it represents an individual's aspiration, rather than an expectation about what he will attain. When the goal is divorced from expectations, there exists a trade-off between consumption and comparison utility. I find that the agent must be sufficiently present-biased and exhibit diminishing sensitivity to gains to set a non-degenerate aspiration. In this case, more present-biased agents set higher aspirational goals.

Finally, I extend the model to social comparisons. In addition to, or instead of, engaging in individual goal-setting, an agent may also look to a peer or role model as a source of comparison. It has long been recognized that people derive utility from comparing their own outcomes, such as wealth, education, and consumption, against those of their peers (Veblen 1953, Duesenberry 1949, Frank 1985). Furthermore, Bandura and Jourden (1991) find that social comparisons affect both individual goal-setting and the interpretation of personal outcomes. I assume that agents are fully aware of their own personal characteristics, but care about "keeping up with the Joneses." Since they are rational and fully informed about one another's characteristics, they must again hold correct expectations regarding everyone's outcomes. I show that comparing oneself with *any* peer improves patience over having no goal at all, even if the peer has an even more severe self-control problem. An agent's patience increases with his peer's degree of self-control, since it effectively sets a higher standard to be met, generating a stronger incentive to wait for a higher stopping value; there also exists a feedback effect between two peers engaged in mutual social comparison. Nonetheless, every agent prefers to compare himself to a peer with the lowest degree of self-control possible, regardless of the severity of his own self-control problem.

The paper proceeds as follows. Section 2 links this paper to related lines of research. Section 3 describes the model. Section 4 derives and characterizes the stationary Markov equilibrium and discusses the welfare implications of goal-setting when the agent is sophisticated. Section 5 derives the equilibrium when the agent is naive and discusses the welfare implications in comparison to the sophisticate. Section 6 considers the optimal choice of aspirational goals. Section 7 extends the model to include social comparisons. Section 8 concludes. Proofs are gathered in the Appendix.

2 Related Literature

This paper lies at the intersection of several lines of research. It links the work on reference dependence with that on self-control, which have each been studied quite separately, by considering the role of reference dependence preferences as an instrument to countervail a self-control problem. It also relates to work on peer effects and social comparisons. Finally, it analyzes behavior and welfare in the context of optimal stopping under uncertainty.

Goal-setting theory posits that goals serve as a reference standard in a cognitive comparison process of self-evaluation and satisfaction (Latham and Locke 1991, Locke and Latham 2002, Bandura 1989). Heath, Larrick and Wu (1999) explicitly argue that a goal acts as the reference point in the prospect theory (Kahneman and Tversky 1979) value function, while Loewenstein (2007) discusses goal-setting in the context of mountaineering, but neither formally endogenizes goals. In parallel and independent work, Suvorov and van de Ven (2008) and Koch and Nafziger (2008, 2009) also propose models of goals as reference points to remedy self-control. While sharing a similar concept, the papers are quite complementary. Both Suvorov and van de Ven (2008) and Koch and Nafziger (2008, 2009) consider a three-period problem where costly effort on a task results in a delayed benefit and a sophisticated, quasi-hyperbolic agent sets a goal regarding both effort and the task benefit. There, loss aversion is required to affect behavior and outcome uncertainty plays a key role. In contrast to related work on reference-dependent preferences, including Koch and Nafziger (2008, 2009), Suvorov and van de Ven (2008), and Kőszegi and Rabin (2006, 2009), none of my results require either loss aversion or uncertainty over final payoffs.

Kőszegi and Rabin (2006) endogenize reference point formation through rational expectations. This framework has been extended to a dynamic setting where agents derive utility from the revision of beliefs (Kőszegi and Rabin 2009, Matthey and Dwenger 2007, Matthey 2008). Abeler, Falk, Goette and Huffman (2009) and Ericson and Fuster (2010) find experimental support for expectations as endogenous reference points. There is also empirical evidence, though debated, that people use reference points in the form of income targets (Camerer, Babcock, Loewenstein and Thaler 1997, Farber 2005, Farber 2008, Crawford and Meng 2008, Fehr and Goette 2007, Goette and Huffman 2005).

The concept of intrapersonal conflict due to intertemporal differences in preferences within the self was first studied by Strotz (1956) and Schelling (1984), and more broadly developed by Ainslie (1992) and Laibson (1997). Self-control can be improved through self-imposed, binding commitments (Brocas and Carrillo 2005, Carrillo 2005, Gul and Pesendorfer 2001, Bisin and Hyndman 2009). Bénabou and Tirole (2004) develop a theory of internal regulation through self-enforcing personal rules based on a mechanism of self-reputation. The intrapersonal problem with time inconsistent preferences shares clear parallels with a principal-agent problem with moral hazard, where optimal compensation schemes have been studied (Ou-Yang 2003, Kadan and Swinkels 2008). However, belief constraints and welfare interpretations can differ markedly between the two settings.

There exists an extensive literature on social comparison and peer effects. When individuals care about status, this concern can lead to conformity (Bernheim 1994) or conspicuous consumption (Bagwell and Bernheim 1996). DeMarzo, Kaniel and Kremer (2008) consider the role of relative wealth concerns in the formation of financial bubbles. Austen-Smith and Fryer (2005) study the influence of cultural norms on school performance. Battaglini, Bénabou and Tirole (2005) endogenize peer group effects when individuals have imperfect information about their self-control problem, but can learn from observing others. Falk and Knell (2004) consider endogenous reference standards in a static, reduced-form social comparison model. Rayo and Becker (2007) propose an evolutionary model of reference dependence and social comparison as optimal mechanisms to maximize fitness.

The real options approach to investment under uncertainty was pioneered by Brennan and Schwartz (1985) and McDonald and Siegel (1986), and has been built upon extensively (Dixit 1993, Dixit and Pindyck 1994). Grenadier and Wang (2007) extend this framework to model the investment decisions of hyperbolic entrepreneurs, while Miao (2008) studies agents with Gul and Pesendorfer's (2001) temptation utility.

3 The Model

I first describe the economic environment, followed by the agent's preferences, which may include both hyperbolic discounting and reference dependence. I focus on an optimal stopping problem, where the self-control problem arises purely from the tension between waiting and stopping today. This framework applies to many economic situations, such as those described above - the student pursuing his education, the person saving a nest egg for retirement, and the person searching for a spouse.[5]

3.1 Optimal Stopping

I consider the standard, continuous-time optimal stopping problem, in which an infinitely lived agent has a non-tradeable option to invest in a project.[6] The problem can also be interpreted as a project termination decision - the agent currently holds a project that has a fixed cost of disinvesting and an uncertain payoff or resale value.

At any time t, the agent knows the current value of the project's payoff $x_t \in [0, \infty)$ and decides whether to stop or to wait. In the latter case, the project's payoff evolves as a geometric Brownian motion:

$$dx_t = \mu x_t dt + \sigma x_t dz, \tag{1}$$

where z is a standard Wiener process, μ the average growth rate of x_t, and σ its standard deviation per unit time. At the stopping time $\bar{t}$, the project yields the lump-sum terminal payoff $x_{\bar{t}}$. The cost of stopping at any time is $I > 0$, and is incurred only at the stopping time.[7] Without loss of generality, there is no interim flow payoff, nor any direct cost incurred prior to stopping. Due to the stochastic nature of the payoff process, there exists an option value of waiting, in the hope that a higher project value will be realized at a later date.

[5] For example, a student who is deciding how long to remain in school will incur job search costs when he stops, and will generally obtain a better job outcome from staying in school longer.

[6] Thus, it has the same structure as an American stock option, where an investor holds an option that does not expire and decides when to strike.

[7] Thus, there is no intertemporal separation of costs and benefit. Separating the costs and benefits of stopping certainly exacerbates the self-control problem, but is not necessary to produce intrapersonal conflict.

A more realistic setting might include an observable constant or stochastic flow payoff. In the previous examples, a student might incur some small positive or negative flow payoff from going to school; saving might induce negative flows due to decreased consumption before retirement; and a person searching for a spouse might incur negative flow utility until he finds a match. Such an inclusion has no qualitative effect on the results[8], so I exclude flow payoffs henceforth.

3.2 Time Preferences

The agent may have present-biased preferences, creating a self-control problem. I model this present-biasedness by following Harris and Laibson (2004), who formulate a continuous-time version of quasi-hyperbolic preferences. At any time s, an agent's preferences are divided into a "present," which lasts from time s to time $s+\tau_s$, and a "future," which arrives at time $s+\tau_s$ and persists forever. The length of the present, τ_s, is stochastic and distributed exponentially with parameter $\lambda \in [0,\infty)$. While the agent knows the distribution governing the length of the present, he is unaware of when the future will arrive ex ante.[9] When self s's future arrives at time $s+\tau_s$, he is replaced by a new self who takes control of decision-making. Likewise, the preferences of this self $s+\tau_s$ are divided into a "present" of length $\tau_{s+\tau_s}$ and a "future" that arrives at time $(s+\tau_s)+\tau_{s+\tau_s}$. Hence, when each self's "future" arrives, it "dies" and is replaced by a new self.

Each self s has a stochastic discount function $D_s(t)$:

$$D_s(t) = \begin{cases} e^{-\rho(t-s)} & \text{if} \quad t \in [s, s+\tau_s) \\ \beta e^{-\rho(t-s)} & \text{if} \quad t \in [s+\tau_s, \infty). \end{cases} \tag{2}$$

where $\beta \in [0,1]$ and $\rho > 0$.[10] To ensure that the agent never finds it optimal to wait forever in the optimal stopping problem, let $\rho > \mu$. The function $D_s(t)$ decays exponentially at the rate ρ throughout, but drops discontinuously at time $s+\tau_s$ to a fraction β of its prior level. Here, there are two parameters that determine the extent of the self-control problem. First, the parameter β retains the same role it plays in the discrete-time version, measuring

[8]More precisely, I can include a constant flow payoff $y \in (\underline{y}, \infty)$, where $\underline{y} < 0$ is the minimum flow payoff such that the agent stops immediately for any $x_t \geq 0$. Likewise, incorporating an observable stochastic flow payoff that follows a known process leads to the same qualitative results.

[9]The assumption of a stochastic arrival time of the future allows me to obtain a stationary solution to the stopping problem, but is not necessary to obtain qualitative results. One can obtain stationarity by imposing a deterministic arrival time instead, though comparative statics with respect to differing time preferences would be less general.

[10]The stochastic discount function $D_s(t)$ described by (2) is analogous to the discrete-time, quasi-hyperbolic version, as seen by substituting $\delta = e^{-\rho}$.

how much the future is valued relative to the present. Second, the parameter λ determines the arrival rate of the future, and thus how often preferences change. In particular, when $\lambda \to \infty$ and $\beta < 1$, the agent prefers "instantaneous gratification" (Harris and Laibson 2004), discretely discounting all moments beyond the current instant. When $\beta = 1$ or $\lambda = 0$, the preferences described by Equation (2) are equivalent to those of an exponential discounter with discount rate ρ.

3.3 Goals

The agent's preferences are assumed to be reference-dependent: his utility is composed of both consumption utility, which is based on absolute levels, and of comparison utility, which is concerned with gains and losses relative to a reference point, which here corresponds to his goal. There is much evidence that people react to goals in ways consistent with prospect theory, such as reporting less or more satisfaction with the same outcome due to differing goal levels or prior expectations (Heath et al. 1999, Medvec, Madey and Gilovich 1995, McGraw, Mellers and Tetlock 2005). Equivalently, an agent may incur non-monetary costs and benefits from performance relative to his goal, such as embarrassment and pride.

In optimal stopping with zero flow payoffs, the agent's consumption utility upon stopping at time $\bar{t}$ is simply his net terminal payoff, $x_{\bar{t}} - I$. As in Kőszegi and Rabin (2006, 2009), the agent's comparison utility is closely related to his consumption utility, and is derived by comparing his net terminal payoff at time $\bar{t}$ against his goal at that time, $r_{\bar{t}}$.[11] In the language of personal development, the goal is thus "Specific," i.e. it is well-defined, and "Measurable," i.e. it can be clearly evaluated. His comparison utility increases with the degree to which he exceeds his goal and decreases with the degree to which he falls short of it, a property consistent with empirical evidence (Mento et al. 1992). A key difference is that comparison utility here is simply a linear function, given by

$$\eta(x_{\bar{t}} - I - r_{\bar{t}}), \tag{3}$$

where $\eta \geq 0$, implying that the agent exhibits neither loss aversion nor diminishing sensitivity to gains and losses. Although the absence of these two features deviates from Kahneman and Tversky's (1979) value function, it will demonstrate that *neither is necessary for goals to affect behavior in a meaningful way.* Indeed, one of the main points of the paper is that the effects of goal-setting can be understood completely independently of loss aversion.

[11] Alternatively, he could compare his gross terminal payoff $x_{\bar{t}}$ against his goal for the gross terminal payoff at that time, or separately compare the terminal payoff and cost using this comparison utility function, and the results would be unchanged.

The parameter η can be interpreted as the degree to which the agent cares about, or pays attention to, the difference between his outcome and his goal. It can also be seen as a measure of salience or "goal commitment," which is broadly defined in psychology as the degree to which a person is determined to achieve a goal. A central concept of goal-setting theory is that goal difficulty has little effect on behavior if commitment is not present (Locke and Latham 2002).[12] The absence of goal commitment ($\eta = 0$) corresponds to the absence of reference dependent preferences, when the existence of a goal has no effect on utility, and consequently, his behavior. We can also interpret $\eta = 0$ as the case in which the agent has no goal or an ill-defined goal. Without a well-defined basis against which to make a comparison, it seems natural to believe that an agent cannot incur comparison utility in this case. Such an interpretation accords with results regarding the ineffectiveness of vague goals on motivation and effort (Latham and Locke 1991, Mento et al. 1992). Here, I treat η as a fixed parameter and analyze the agent's subsequent behavior and welfare. The demand for personal development services and products can be interpreted as an attempt by individuals to improve self-regulation and welfare by changing η.[13] This interpretation is not inconsistent with this assumption, insofar as an agent must determine how he would fare for any given η to determine the value of attempting to change his initial η. In this vein, I later consider the agent's preferences over η from an ex-ante perspective.

I assume that the agent only incurs comparison utility when he stops and receives the net terminal payoff. Although he is aware that he will incur comparison utility upon stopping, he does *not* incur any while waiting. This assumption accords with the notion from mental accounting that individuals do not necessarily "feel" gains and losses until they have been realized (Thaler 1999).[14]

For simplicity, overall utility is taken to be additively separable in its two components. The agent's total utility at the stopping time is thus

$$x_{\bar{t}} - I + \eta(x_{\bar{t}} - I - r_{\bar{t}}). \tag{4}$$

At any time s, the goal r_s is taken as given by self s and cannot be changed during his entire "lifetime," having been set by his previous self. Similarly, the goal that self $s + \tau_s$ inherits,

[12] Similarly, McClelland (1962) defines those high in "achievement motivation," or n Ach, as those derive satisfaction from setting achievable and measurable goals for themselves.

[13] For example, the suggestively titled "The Magic Lamp: Goal Setting for People Who Hate Setting Goals" (Ellis 1998) purports to help individuals improve their goal commitment. The inside flap declares, "**The Magic Lamp** . . . [combines] the methods of goal setting with the magic of making your wishes come true." Despite the extravagance of the claim, it suggests that goal commitment is necessary and that individuals may be able to change their existing goal commitment with some effort.

[14] The disposition effect, where stockholders are reluctant to sell losing stocks, and hence realize losses relative to their original buying prices, is consistent with this idea (Odean 1998, Barberis and Xiong 2008).

denoted $r_{s+\tau_s}$, is set by self s, where τ_s, the lifespan of self s, is stochastically determined and a priori unknown to self s.

The assumption that the agent cannot change an inherited goal implies that it can provide a degree of internal motivation to his (present-biased) future selves. Such "goal stickiness" is necessary: if the agent could simultaneously make the stopping decision and set a goal for himself for the *current* period, this goal would have no effect on his current behavior, since his present bias implies that he has no desire to behave more patiently today. He would also have no means by which to impose ex-ante preferences over time-consistent behavior.[15]

3.4 Expectations

In setting the goal, each self forms an expectation of his immediate "descendant"'s payoff if he does not stop himself. His descendant inherits this expectation and compares his own final payoff against this inherited goal if he stops.[16] The goal is clearly "Relevant" his stopping problem. I consider the two polar cases of sophistication and naivete, in the manner of O'Donoghue and Rabin (1999), to study the impact of differing expectations on behavior and welfare. In contrast to the case without reference dependent preferences, holding incorrect expectations directly affects the agent's comparison utility through his goal choice. However, I assume that his goals must be consistent with the outcome he expects to achieve. Because he observes the current project payoff perfectly, he holds no uncertainty over the anticipated outcome upon stopping ex ante, though such beliefs may or may not be correct. If the agent is sophisticated and correctly anticipates his actions, each self must have rational expectations about goal achievement. That is, the agent cannot consistently fool himself about what he can or cannot achieve - he must set goals that are realistic.[17] Likewise, if he is naive, so his beliefs regarding future behavior are incorrect, he must set objectives that he *perceives* to be realistic. That is, the agent's goal must be "Attainable." Note, however, that these assumptions do *not* necessarily imply that each self must have the

[15] Alternatively, we could assume that the agent can change an inherited goal at some cost. Thus imposing zero or infinite costs corresponds to no or maximal "goal stickiness," respectively. While relaxing "goal stickiness" would certainly weaken the effects of a goal, clearly the qualitative findings and comparative statics would still hold as long as it exists, without offering additional insights.

[16] This formulation is consistent with Bandura's (1989) theory that goals serve as both targets to strive for and standards by which outcomes are evaluated, as well as empirical evidence that the degree of self-satisfaction varies depending on goal level. That is, two individuals who attain the same outcome will be unsatisfied or satisfied depending on whether their goals were higher or lower than that outcome, respectively (Mento et al. 1992).

[17] Based on lab and field experiments, Latham and Locke (1991) conclude that goal choice is an integration of what one wants and what one believes is possible, suggesting that goals are realistic. Carrillo and Dewatripont (2008) also discuss the tension between foresight and the credibility of promises in intrapersonal games, arguing that agents cannot simultaneously anticipate future behavior and fool future selves.

same goal. Each self cannot change the goal that he inherits, but is free to choose a different one for his future self if he so desires, as long as that he perceives, whether accurately or not, that it is realistic.

Because each self inherits his goal from his predecessor, it is necessary to specify the source of the agent's goal when he is first able to stop the project. I assume that there exists a "self 0," an ex-ante self, who learns that the stopping opportunity will present itself and forms an expectation of how he will behave once the option becomes available for exercise.

Given such preferences, at any time s the agent chooses the stopping rule that determines a (random) stopping time $\bar{t}$ to maximize the expected present value of his overall utility:

$$\max_{\bar{t}} E_s\{D_s(\bar{t})[x_{\bar{t}} - I + \eta(x_{\bar{t}} - I - r_{\bar{t}})]\}, \tag{5}$$

where E_s denotes the conditional expectation at time s and $D_s(\bar{t})$ is given by Equation (2). When the agent has hyperbolic time preferences, he is prone to stopping too early because he undervalues the future, and thus the option value of waiting (Grenadier and Wang 2007). The question of interest is to what extent setting goals can attenuate this problem.

4 Sophistication

When the agent is quasi-hyperbolic and sophisticated, in that he is fully aware of his present-biasedness, the problem takes on the nature of a dynamic game between successive selves, and there could exist equilibria in which each self chooses a different stopping strategy. Here, I focus on the most natural equilibrium, namely a stationary Markov equilibrium in which each self employs the same threshold strategy. In the Appendix, I verify that the stationary stopping rule is in fact optimal.

Since the geometric Brownian motion is continuous, the project value cannot jump discontinuously from one moment in time to the next. This implies that the sophisticated agent has no uncertainty over the net terminal payoff from stopping: if every self uses the same threshold $\overline{x}$, then $x_{\bar{t}} = \overline{x}$, so the expected net terminal payoff is $\overline{x} - I$. The source of ex ante uncertainty is the *timing* of this stopping - that is, when he chooses to stop. Furthermore, if a self-set goal must be realistic, every self will inherit, set, and meet the same goal in a stationary equilibrium, so that $r_{\bar{t}} = \overline{x} - I$.

To construct the equilibrium, I solve the intrapersonal game backwards.[18] Each self anticipates that his descendants will employ thresholds that maximize their own current

[18] I derive the solution in the same manner as Grenadier and Wang (2007), but with the inclusion of a final goal-dependent payoff.

benefit of waiting, so they will face a problem identical to his own. A stationary solution is thus a fixed point such that current and future selves employ a common threshold.

Because each self controls the stopping decision in the present, and cares about - but cannot directly control - those of the future, two value functions are required to describe the intrapersonal problem. Let all descendants inherit the goal $\hat{r}$. Then the Bellman equation for the continuation value function $v(x, \hat{r})$ is

$$v(x, \hat{r}) = \max\{x - I + \eta(x - I - \hat{r}), e^{-\rho dt} E[v(x + dx, \hat{r})]\}. \tag{6}$$

This continuation value function describes each self s's consideration of future selves, following the random arrival of the future at time τ_s. Beyond time τ_s, he discounts future utility flows exponentially. Hence, the continuation value function also describes how the current self *prefers* future selves to evaluate payoff streams - by discounting exponentially. Thus, v describes the agent's preferences over the future from an ex ante perspective, including those of self 0. If the agent were time consistent, then all selves' preferences would coincide and he would choose the optimal strategy to maximize v. Given the current project value x and his inherited goal $\hat{r}$, he would thus choose the maximum of the observed total utility from stopping and the expected (exponentially) discounted value of waiting for a higher realization of x.

However, if the agent is present-biased ($\beta < 1$), he maximizes a different value function that overweights the present relative to the future. Denoting the goal inherited by the current self by $\overline{r}$, the Bellman equation that describes this current value function $w(x, r)$ is

$$w(x, \overline{r}) = \max\{x - I + \eta(x - I - \overline{r}), (1 - e^{-\lambda dt}) e^{-\rho dt} \beta E[v(x + dx, \hat{r})] + (e^{-\lambda dt}) e^{-\rho dt} E[w(x + dx, \overline{r})]\}. \tag{7}$$

Given the current x and an inherited goal $\overline{r}$, and anticipating that his future selves will inherit $\hat{r}$ (with the knowledge that he sets $\hat{r}$ for his immediate descendant), the current self chooses the maximum of the current total utility from stopping and the expected discounted value of waiting for a higher realization of x, where this discounting discontinuously drops by the factor β upon the random arrival of the future. A future self arrives in the next instant dt with probability $1 - e^{-\lambda dt}$, while the current self remains in control with probability $e^{-\lambda dt}$.[19]

[19] Alternatively, consider a parent who derives utility from the payoff stream of his descendants. The continuation value v describes his evaluation of the stream of his descendants' utilities - he discounts them exponentially. Thus he prefers that every descendant evaluate the payoff streams from his entire family line, including his own, exponentially. When his child becomes the decision-maker, he discounts his own descendants' utilities exponentially, just as his parent did. But he also underweights the stream of his descendants' utilities relative to his own by the factor β, in disagreement with his parent's wishes. Thus, w describes the child's evaluation of the payoff stream from his entire family line, including his own.

4.1 Ex Ante Preferences

To construct the continuation value function $v(x, \hat{r})$, I first suppose that all future selves inherit goal $\hat{r}$ and employ a threshold $\hat{x}$ such that they wait if $x < \hat{x}$ and stop if $x \geq \hat{x}$. By continuity of the geometric Brownian motion, there is zero probability that the project value x_t can jump discontinuously from the "wait" region ($x < \hat{x}$) to the "stop" region ($x \geq \hat{x}$) from one moment to the next. Therefore, I can construct $v(x, \hat{r})$ by considering its behavior in each region separately, then joining them using appropriate boundary conditions.

Using the threshold strategy implies that the value of Equation (6) in the stop region ($x \geq \hat{x}$) is simply given by $x - I + \eta(x - I - \hat{r})$. In the wait region, standard results imply that $v(x, \hat{r})$ must obey the following linear differential equation:

$$\rho v(x, \hat{r}) = \mu x(\frac{\partial v}{\partial x}) + \frac{1}{2}\sigma^2 x^2(\frac{\partial^2 v}{\partial x^2}) \quad \text{if } x < \hat{x}. \tag{8}$$

By definition of the geometric Brownian x, $x = 0$ is an absorbing barrier. The continuation value function must also be continuous at the threshold $\hat{x}$ between the waiting and stopping regions. Therefore, the relevant boundary conditions for v are:[20]

$$\text{Boundary: } v(0, \hat{r}) = 0, \tag{9}$$

$$\text{Value Matching: } v(\hat{x}, \hat{r}) = \hat{x} - I + \eta(\hat{x} - I - \hat{r}). \tag{10}$$

Because the stopping decision is only made by *current* selves, and never by future selves, no optimality condition applies to $v(x, \hat{r})$ if the agent is present-biased.

Combining Equation (8) with conditions (9) and (10), the continuation value function is

$$v(x, \hat{r}) = \begin{cases} [(\hat{x} - I) + \eta(\hat{x} - I - \hat{r})](\frac{x}{\hat{x}})^{\gamma_1} & \text{if } \ x < \hat{x} \\ x - I + \eta(x - I - \hat{r}) & \text{if } \ x \geq \hat{x}, \end{cases} \tag{11}$$

where $\gamma_1 > 1$ is the positive root[21] of the quadratic equation

$$\frac{1}{2}\sigma^2\gamma_1^2 + (\mu - \frac{1}{2}\sigma^2)\gamma_1 - \rho = 0, \tag{12}$$

reflecting the fact that from an ex ante perspective, the agent discounts the future exponentially at the rate ρ.

[20] The first boundary condition is obtained by noting that given any $\hat{r} \geq 0$, which can be verified in equilibrium, the agent never stops if $x = 0$, since he incurs negative overall utility.

[21] The negative root is ruled out by the boundary condition for $x = 0$. To see that $\gamma_1 > 1$, note that $\sigma^2 > 0$ and the left-hand side of the quadratic is negative when evaluated at $\gamma_1 = 0$ and $\gamma_1 = 1$, implying that the negative root is strictly negative and the positive root is strictly greater than 1 if $\mu < \rho$.

4.1.1 Benchmark: The Standard Case

In the standard optimal-stopping problem, the agent is time-consistent ($\beta = 1$ or $\lambda = 0$) and is not reference dependent ($\eta = 0$). His optimal strategy is to use the fixed stopping threshold x^*: at any time s, he waits if $x_s < x^*$ and stops if $x_s \geq x^*$. Time consistency implies that the agent's preferences are fully described by v, so he chooses x^* to maximize v. Thus, the optimality, or smooth pasting, condition applies to the continuation value function v, so the marginal values of waiting and stopping must be equal at the optimal threshold. Since $\eta = 0$ here, the smooth pasting condition is given by:

$$\text{Smooth Pasting: } \frac{dv}{dx}(x^*) = 1. \tag{13}$$

Solving Equation (8) subject to (13) and barrier absorption and value matching conditions (9) and (10) when $\eta = 0$ allows us to determine the optimal threshold x^*:

$$x^* = (\frac{\gamma_1}{\gamma_1 - 1})I, \tag{14}$$

where $\gamma_1 > 1$ is again described by Equation (12) because the continuation value is derived by exponentially discounting the future at the rate ρ in both cases. The standard agent always waits for a project value that exceeds its direct cost ($x^* > I$). Due to the forgone possibility of higher realizations of x in the future, there exists an opportunity cost of stopping today. In equilibrium, his value of the option to stop, denoted $v^*(x)$, is given by

$$v^*(x) = \begin{cases} (x^* - I)(\frac{x}{x^*})^{\gamma_1} & \text{if} \quad x < x^* \\ x - I & \text{if} \quad x \geq x^*. \end{cases} \tag{15}$$

4.2 Present-Biased Preferences

In contrast to the time-consistent case, the present-biased agent maximizes the current value function w, rather than the continuation value v. Proceeding with the derivation of $w(x, \overline{r})$ analogously, I first suppose that all current selves inherit goal $\overline{r}$ and employ a threshold $\overline{x}$ such that they wait if $x < \overline{x}$ and stop if $x \geq \overline{x}$. Again, I can construct $w(x, \overline{r})$ by characterizing it in the "wait" and "stop" regions separately, then joining the two regions through appropriate boundary conditions.

Using the threshold strategy implies that the value of w in the stop region ($x \geq \overline{x}$) is simply given by $x - I + \eta(x - I - \overline{r})$. In the wait region, standard results imply that $w(x, \overline{r})$

must obey the following linear differential equation:

$$\rho w(x,\overline{r}) = \lambda(\beta v(x,\hat{r}) - w(x,\overline{r})) + \mu x(\frac{\partial w}{\partial x}) + \frac{1}{2}\sigma^2 x^2(\frac{\partial^2 w}{\partial x^2}) \quad \text{if } x < \overline{x}. \tag{16}$$

Comparing Equation (16) to Equation (8), the additional term $\lambda(\beta v(x,\hat{r}) - w(x,\overline{r}))$ is the expected value of the change in the current value w that occurs through the stochastic arrival of a transition from the present to the future.

As with v, $x = 0$ is an absorbing barrier and w must clearly be continuous at the threshold $\overline{x}$ between the wait and stop regions. Finally, the smooth pasting condition must apply to w with respect to x, because the optimal threshold is chosen by the current self to maximize his current value function w. At $\overline{x}$, the marginal value of waiting must equal that of stopping so that the current self is unwilling to deviate from stopping. Thus, the relevant boundary conditions for w are

$$\text{Boundary: } w(0,\overline{r}) = 0, \tag{17}$$

$$\text{Value Matching: } w(\overline{x},\overline{r}) = \overline{x} - I + \eta(\overline{x} - I - \overline{r}), \tag{18}$$

$$\text{Smooth Pasting: } \frac{\partial w}{\partial x}(\overline{x},\overline{r}) = 1 + \eta. \tag{19}$$

Because the current self fully anticipates that his future selves will use threshold $\hat{x}$ given goal $\hat{r}$, we can substitute the continuation value function, given by Equation (11) into the differential Equation (16). Under the assumption that $x \leq \hat{x}$, which is satisfied in a stationary equilibrium, it is the value of v in its wait region that applies to (16). Combining Equation (16) with (11) and conditions (17), (18), and (19) yields the solution to the optimal threshold $\overline{x}$ as a function of current goal $\overline{r}$ and the conjectured future goals $\hat{r}$ and thresholds $\hat{x}$.

4.3 Stationary Equilibrium

In a stationary equilibrium, the sophisticated agent knows that all current and future selves employ the same threshold, which implies that $\overline{x} = \hat{x}$. To make clear the effect of goal-setting on the optimal stopping rule, I first consider the sophisticate's stopping threshold given a fixed goal level (i.e. $r = \overline{r} = \hat{r}$), denoted $\overline{x}^{SE}$, provided that this goal is set ex ante and does not change during the stopping decision. Thus, it describes a sophisticate's response to any externally set goal, which can differ from the agent's actual final payoff. Next, I derive the sophisticate's stopping rule when his goals are set internally, denoted $\overline{x}^{SI}$, and therefore are required to be met in equilibrium.

4.3.1 Exogenous Goals

Letting $r = \overline{r} = \hat{r}$ and imposing the fixed point condition that $\overline{x} = \hat{x} \equiv \overline{x}^{SE}$, the optimal threshold in response to an externally set goal $r \geq 0$, denoted $\overline{x}^{SE}$, is

$$\overline{x}^{SE} = (\frac{\overline{\gamma}}{\overline{\gamma}-1})I + r(\frac{\eta}{1+\eta})(\frac{\overline{\gamma}}{\overline{\gamma}-1}), \quad \text{with } \overline{\gamma} \equiv \beta\gamma_1 + (1-\beta)\gamma_2, \tag{20}$$

where γ_2 is the positive root[22] of the quadratic equation

$$\frac{1}{2}\sigma^2\gamma_2^2 + (\mu - \frac{1}{2}\sigma^2)\gamma_2 - (\rho + \lambda) = 0. \tag{21}$$

The only difference between the quadratic equation for γ_2 and that of γ_1 is the presence of the parameter λ, the hazard rate for the arrival of the future. It is apparent that $\gamma_2 \geq \overline{\gamma} \geq \gamma_1$, with equality only if the future never arrives ($\lambda = 0$), i.e. preferences never change. The parameter γ_2 reflects the fact that each self's expected "lifetime" shortens with λ, while the degree to which this feature affects the stopping decision is determined by the degree of present-biasedness, measured by $1 - \beta$. If the agent is time-consistent ($\overline{\gamma} = \gamma_1$) and not reference-dependent ($\eta = 0$), $\overline{x}^{SE}$ equals x^*, the solution to the standard stopping problem.

Equation (20) makes clear the effect of goal-setting on the optimal stopping rule. The first term is the agent's stopping threshold in the absence of reference dependence or a goal: if $\eta = 0$, then $\overline{x}^{SE} = (\frac{\overline{\gamma}}{\overline{\gamma}-1})I < x^*$.[23] A hyperbolic discounter without reference dependent preferences stops earlier than the standard agent, and this impatience is exacerbated as the present is more overweighted, i.e. as β decreases, and as preferences change more frequently, i.e. as λ increases. Recall that it is the combination of parameters β and λ that determines the degree of the self-control problem. In fact, (20) reveals that $\overline{\gamma}$ is a sufficient statistic for measuring the sophisticated agent's degree of impulsiveness.[24] The second term in (20) is the effect of the goal on the stopping threshold. Because an agent is motivated to avoid settling for a lower project value only if there exists a potential comparative penalty from falling short, the goal r only induces more patient behavior as long as $r > 0$, even in the case of instantaneous gratification. No potential penalty is imposed when $r = 0$, so behavior is unchanged by this goal and it is equivalent to having no goal.[25]

Proposition 1 *In a stationary equilibrium with exogenous goal r, the sophisticate's stopping*

[22] As before, the negative root is ruled out by the boundary condition for $x = 0$.

[23] If the agent is not reference dependent ($\eta = 0$), then Equation (20) is identical to the sophisticate's threshold obtained by Grenadier and Wang (2007).

[24] Clearly, $\overline{\gamma}$ is decreasing in β and increasing in λ, so comparative statics on each parameter separately yield the same qualitative results.

[25] This feature makes clear that η in itself does not simply act as a subsidy to reaching a higher net x.

threshold exhibits the following properties:

1. *The threshold decreases with the degree of impulsiveness:* $\frac{\partial \overline{x}^{SE}}{\partial \overline{\gamma}} < 0$.
2. *The threshold increases with the level of the goal:* $\frac{\partial \overline{x}^{SE}}{\partial r} > 0$.
3. *Responsiveness to a goal increases with the degree of goal commitment:* $\frac{\partial^2 \overline{x}^{SE}}{\partial r \partial \eta} > 0$.
4. *Responsiveness to a goal decreases with the degree of impulsiveness:* $\frac{\partial^2 \overline{x}^{SE}}{\partial r \partial \overline{\gamma}} < 0$.

It is intuitive that the agent's threshold should decrease with his degree of impulsiveness. First, he undervalues the future himself. Second, being sophisticated, he anticipates that future selves will undervalue their own futures as well, decreasing the value of waiting even further. A second intuitive result is that, regardless of his degree of impulsiveness (including if he is not), the agent's threshold increases with the level of the goal if he is reference dependent. Raising r increases the potential cost of settling for a lower project value. Thus, a goal induces more patient behavior by providing an additional incentive to wait for a higher realization of the project value. This result is consistent with experimental evidence that task performance increases with the goal difficulty, whether externally- or self-set.[26]

Consistent with empirical evidence on goal commitment (Klein, Wesson, Hollenbeck and Alge 1999), responsiveness to a goal is increasing in the agent's degree of reference dependence. An agent who cares more about falling short of his goal is more motivated to avoid such an outcome than one who cares less about this comparison. Finally, an agent's responsiveness to a goal is decreasing in his degree of impulsiveness, illustrating the interaction between present-biasedness and reference dependence. A more impulsive agent values the future less, so he has a weaker incentive to avoid incurring a comparative penalty assessed in the future. Thus, he not only exhibits more impatience in the absence of a goal, but is also less responsive to a given goal.

4.3.2 Endogenous Goals

When the goal is internally set, rational expectations implies that each self correctly anticipates his descendant's threshold strategy, so that $\overline{r} = \overline{x} - I$ and $\hat{r} = \hat{x} - I$. But since $\overline{x} = \hat{x}$, this implies that $r = \overline{r} = \hat{r} = \overline{x} - I$. Each self correctly expects his descendants to use the same stopping rule as he does, so every self inherits and meets the same goal in equilibrium. That is, whether goals are externally or internally set, sophistication implies that $\overline{x} = \hat{x}$ and $\overline{r} = \hat{r}$. Thus, the optimal threshold with internally set goals, denoted $\overline{x}^{SI}$, must satisfy

[26] Locke and Latham (2002) even find a positive linear relationship between goal difficulty and performance.

$r = \overline{x}^{SI} - I$ and can be derived by imposing this condition on Equation (20), yielding

$$\overline{x}^{SI} = (\frac{\overline{\gamma}}{\overline{\gamma} - 1 - \eta})I, \qquad \begin{array}{ll} \text{with} & \overline{\gamma} \equiv \beta\gamma_1 + (1-\beta)\gamma_2 \\ \text{and} & \eta < \overline{\gamma} - 1, \end{array} \tag{22}$$

and the equilibrium value functions w^{SI} and v^{SI}:

$$w^{SI}(x, r = \overline{x}^{SI} - I) = \begin{cases} \beta(\overline{x}^{SI} - I)(\frac{x}{\overline{x}^{SI}})^{\gamma_1} + (1-\beta)(\overline{x}^{SI} - I)(\frac{x}{\overline{x}^{SI}})^{\gamma_2} & \text{if} \quad x < \overline{x}^{SI} \\ x - I + \eta(x - \overline{x}^{SI}) & \text{if} \quad x \geq \overline{x}^{SI}, \end{cases} \tag{23}$$

$$v^{SI}(x, r = \overline{x}^{SI} - I) = \begin{cases} (\overline{x}^{SI} - I)(\frac{x}{\overline{x}^{SI}})^{\gamma_1} & \text{if} \quad x < \overline{x}^{SI} \\ x - I + \eta(x - \overline{x}^{SI}) & \text{if} \quad x \geq \overline{x}^{SI}. \end{cases} \tag{24}$$

The value of Equation (23) in its wait region is the project's expected present value to the current self, given the current value of the project's payoff, $x < \overline{x}^{SI}$, and the optimal threshold $\overline{x}^{SI}$. This is essentially the weighted average of two time-consistent option values, where the first, weighted by β, uses the discount rate ρ, which is reflected in γ_1. The second, weighted by $1-\beta$, uses the discount rate $\rho+\lambda$, which is reflected in γ_2. The value of Equation (24) in its wait region is the expected present value of the option to stop, using only the discount rate ρ, reflected in γ_1. Thus, v^{SI} also represents the option value that each current self would prefer his future selves to use from an ex ante perspective (or would like to commit them to), though he is resigned to the knowledge that they will maximize w rather than v.

Figure 1 depicts the equilibrium value functions when goals are self-set. The presence of reference dependence increases the marginal value of waiting, increasing the slope of both the continuation and current value functions upon stopping, and thus the incentive to wait. The fact that v lies above its respective w reflects the fact that an exponential discounter values the option more than a present-biased agent. Independently of η, the slope of v is flatter than that of w at the equilibrium stopping threshold, reflecting the fact that the exponential discounter prefers to wait longer than the present-biased agent, and thus, that the present-biased agent prefers his future selves to wait longer than they actually do.

Proposition 2 *By inducing more patient behavior, reference dependence attenuates impulsiveness in a stationary equilibrium with endogenous goals:* $\frac{\partial \overline{x}^{SI}}{\partial \eta} > 0$.

From Equation (22), it is clear that the equilibrium threshold increases with the degree of goal commitment for any given degree of impulsiveness if λ is finite. An agent with a high degree of reference dependence has a stronger incentive to meet his goal, since he puts more weight on the comparative disutility from falling short. It is only in the instantaneous gratification case that the agent, with infinite impatience and finite goal commitment, is

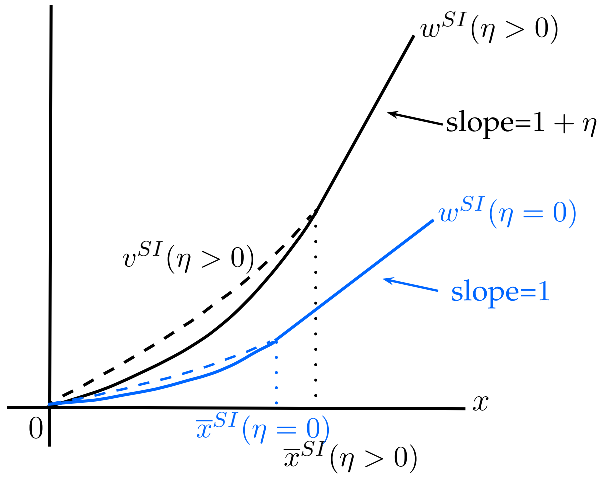

Figure 1: Option values with endogenous goals when $\beta < 1$ for varying degrees of η. The black lines represent the value functions when $\eta > 0$. The blue lines represent the value functions when $\eta = 0$. The solid lines depict the current value functions w, which are smooth and continuous everywhere. The dashed lines depict the continuation values v, which are kinked at the equilibrium thresholds $\overline{x}^{SI}$ and coincide with their respective current value functions w thereafter.

unaffected by goal-setting. As noted in Equation (20), the agent *is* responsive to a goal even in the instantaneous gratification case, as long as $r > 0$. But when the goal is self-set, his anticipation of extreme impatience makes him unable to set a realistic penalty to improve his patience, so his behavior is unchanged by reference dependence.[27]

4.4 Welfare

From an ex-ante perspective, the agent, no matter how severe his degree of present-biasedness or reference dependence, prefers that his future selves behave according to a time-consistent strategy. Therefore, I use the preferences of self 0, which determine the ex ante optimum, to evaluate the agent's welfare. Since all selves possess reference-dependent preferences, self 0 must take both consumption and comparison utility into account when evaluating welfare. Such an analysis allows us to evaluate the welfare consequences for a heterogeneous population of individuals, with varying degrees of present-biasedness and goal commitment. It also allows self 0 to determine the value of adjusting his degree of goal commitment η to improve future behavior, since this choice may be costly.

Sophistication implies that the agent always anticipates that he will meet his self-set goal in equilibrium. He is thus aware that once he actually stops at some point in the future, his

[27]As shown in the following sections, this will not be the case when he is naive or when he is placed in a heterogeneous peer group.

comparative utility will equal zero in equilibrium, so his overall utility will consist only of the net terminal payoff, just as in the $\eta = 0$ case. Therefore, the ex ante self wants to maximize the expected present discounted value of his overall utility, which exactly equals the net terminal payoff alone, as if he were time consistent. That is, he wants to choose the stopping threshold that maximizes $v^{SI}(x)$. This problem is identical to that of a standard agent who is dynamically consistent and not reference dependent. Thus, the first-best threshold for any sophisticated agent, no matter his degree of present-biasedness or goal commitment, coincides with the threshold that a standard agent chooses: $x^* = (\frac{\gamma_1}{\gamma_1 - 1})I$. This implies that for any degree of impulsiveness, there exists a degree of countervailing goal commitment that enables the agent to employ the first-best threshold strategy (i.e. $\overline{x}^{SI}(\eta^*) = x^*$). Unsurprisingly, this optimal goal commitment η^* is increasing in $\overline{\gamma}$ - as impulsiveness increases, the degree of goal commitment required to attenuate it increases.

Proposition 3 *For every $\overline{\gamma} \in (1, \infty)$, there exists an η^* such that the reference-dependent agent with $\eta = \eta^*$ achieves the first best, given by*

$$\eta^* = \frac{\overline{\gamma} - \gamma_1}{\gamma_1}.$$

This analysis makes clear the differential effect of increasing the degree of goal commitment η versus increasing the stopping cost I. While both have the same qualitative effect of increasing $\overline{x}^{SI}$, only the former improves welfare by closing the gap between equilibrium and first-best behavior if $\eta < \eta^*$. In contrast, the latter has no effect on the self-control problem, since both the equilibrium *and* first-best thresholds increase with I.

On the other hand, the degree of reference dependence can also be sufficiently high that the agent waits *longer* than is first-best. Consider now an agent who is dynamically consistent ($\overline{\gamma} = \gamma_1$) and reference dependent ($\eta > 0$). Although reference dependence does not distort final overall utility directly, it changes the *marginal* value of stopping. Because the agent has an incentive to avoid incurring comparative disutility, the marginal value of waiting at the first best threshold x^* exceeds the marginal value of stopping, so the agent waits longer. Although he achieves a payoff that exceeds x^*, its ex-ante discounted value is lower than the first best. Hence, reference dependence distorts his behavior *away from the first best* and causes him to be overly patient.[28] Since a time-consistent agent has no self-control problem, reference dependence offers no beneficial value, so $\eta^* = 0$ if $\overline{\gamma} = \gamma_1$.

[28] In contrast, Kőszegi and Rabin (2009) find that in the absence of uncertainty over final payoffs, reference dependence can lead to apparently impatient behavior, even without present-biased preferences. There, the reference point is a vector of plans (beliefs) over time, and the agent derives comparison utility in each period of a consumption-savings problem. If he values contemporaneous comparison utility more than future comparison utility, then he may overconsume relative to the ex-ante optimum.

The same argument can clearly be applied to present-biased agents, if η overcompensates for the conflict in time preferences between current and future selves. Moreover, reference dependence can be so high that even an impulsive agent would be better off in the absence of goal-setting. That is, when $\eta > \overline{\eta}$, he would wait so long under goal-setting that he is actually *better off* if *he cannot set goals for himself* (i.e. $\eta = 0$) and behaves impatiently. Thus, goal-setting can itself be a source of intrapersonal conflict, since it can cause an agent to wait longer than is optimal from an ex ante perspective. The level of reference dependence required to be detrimental to the agent's welfare is increasing in his degree of impulsiveness.

Corollary 1 *For every $\overline{\gamma} \in (1, \infty)$, there exists a range of η such that the sophisticated agent waits* longer *than the first best: $\eta^* < \eta < \overline{\gamma} - 1$. Moreover, he is strictly* worse off *under goal-setting if $\overline{\eta} < \eta < \overline{\gamma} - 1$, where $\overline{\eta} \geq \eta^*$ is increasing in $\overline{\gamma}$ and is defined by the following condition:*

$$(\frac{\overline{\gamma}-1}{\overline{\gamma}-1-\overline{\eta}})^{\gamma_1-1}(\frac{1}{1+\overline{\eta}})-1=0.$$

In particular, a time-consistent agent stops at $\overline{x}^{SI} = (\frac{\gamma_1}{\gamma_1-1-\eta})I$ and is strictly worse off if he is reference-dependent ($\eta > 0$).

Although the sophisticate's self-set goal level is pinned down by rational expectations, he may further regulate his behavior by seeking to adjust his goal salience or commitment, behavior that can be strictly welfare-improving. Beyond providing strategies to determine and set well-defined goals, personal development attempts to aid such endeavors by suggesting concrete ways to increase goal salience, such as writing down and reviewing goals daily and continually visualizing their achievement (Tracy 2003). Prescriptively, welfare improvement can be achieved purely through programs or services such as those offered by personal development, which educate individuals about appropriate goal-setting.

While there is much focus on the need for goal commitment, there is evidence that the converse problem is recognized as well. In psychology, dysfunctional perfectionism is defined as "overdependence of self-evaluation on the determined pursuit of personally demanding, self-imposed, standards in at least one highly salient domain, despite adverse consequences" (Shafran, Cooper and Fairburn 2002). Shu (2008) finds that decision-makers can search too long when they have an ideal focal outcome in mind. Consistent with the idea that η can be detrimentally high, Goldsmith (2008) discusses the prevalence of "goal obsession" among successful executives, many of whom regret sacrificing health or family life in the pursuit of their careers.

5 Naivete

To highlight the impact of expectations on behavior and welfare, I now consider the case of naivete, where the agent mistakenly believes that he will be dynamically consistent.

Because the naif holds incorrect beliefs about his future behavior, it is the *perceived*, rather than actual, behavior of future selves that influences his stopping decision. Here, let $\hat{x}$ be the threshold that the naif perceives future selves will employ, such that they wait if $x < \hat{x}$ and stop otherwise, and let $\hat{r}$ be their goal. Both the sophisticate and naif have identical *evaluations* of their future behavior, discounting it exponentially. Thus, the naif's (perceived) continuation value v is still given by Equation (11). However, the sophisticate and naif differ drastically in their *beliefs* over their future behavior. The key difference is that the naive agent believes that given a goal $\hat{r}$, future selves will choose $\hat{x}$ such that

$$\hat{x} = (\frac{\gamma_1}{\gamma_1 - 1})I + \hat{r}(\frac{\eta}{1+\eta})(\frac{\gamma_1}{\gamma_1 - 1}). \tag{25}$$

The naive agent believes that future selves will be exponential discounters, so he thinks they will choose a stopping rule that maximizes v, rather than w. Thus, the naif derives (25) by combining the smooth pasting condition with respect to x with Equation (11), in addition to the absorbing barrier (9) and value matching (10) conditions.[29]

Let $\overline{x}$ be the threshold used by the current self and $\overline{r}$ be the goal that he inherits. Again, because the naif's preferences are identical to those of the sophisticate, the wait region of his current value function w is still given by Equation (16).

5.1 Equilibrium

Since the naif's beliefs about future selves' behavior are incorrect, current and perceived future selves employ different stopping thresholds, so a fixed point condition does not apply. Rather, $\hat{x}$ is given by Equation (25). As before, I will first consider the naif's response to any fixed goal level, denoted $\overline{x}^{NE}$, before describing his stopping threshold when goals are self-set, denoted $\overline{x}^{NI}$.

5.1.1 Exogenous Goals

Let the naif's goal be set ex ante and unchanged during the stopping decision, so $r = \overline{r} = \hat{r}$. Assuming that $\overline{x}^{NE} \equiv \overline{x} \leq \hat{x}$ (and verifying that this holds in equilibrium), we can combine (16) with the (perceived) continuation value function (15) in its wait region and stopping

[29]Equivalently, Equation (25) can be derived from Equation (20) by evaluating $\overline{x}^{SE}$ when $\beta = 1$ and the goal is some $\hat{r}$.

rule (25), and boundary conditions (17), (18), (19) to obtain an implicit function for the naive agent's threshold $\overline{x}^{NE}$ when the goal is exogenous and fixed:

$$\overline{x}^{NE} = \frac{1}{(\gamma_2 - 1)(1+\eta)}[\beta(\gamma_2 - \gamma_1)(\hat{x}^{NE} - I + \eta(\hat{x}^{NE} - I - r))(\frac{\overline{x}^{NE}}{\hat{x}^{NE}})^{\gamma_1} + \gamma_2(1+\eta)I + \gamma_2\eta r], \quad (26)$$

where $\hat{x}^{NE} = (\frac{\gamma_1}{\gamma_1 - 1})I + r(\frac{\eta}{1+\eta})(\frac{\gamma_1}{\gamma_1 - 1})$. Unsurprisingly, the comparative statics with respect to $\overline{x}^{NE}$ are analogous to those described in Proposition (1).[30]

5.1.2 Endogenous Goals

Since each self, including self 0, persistently misperceives future preferences, the current self inherits the same goal that he will pass on to his descendant (i.e. $\overline{r} = \hat{r}$). Thus, whether goals are externally or internally set, the condition that $\overline{r} = \hat{r}$ must hold. Moreover, a self-set goal is perceived to be realistic, $\hat{r} = \hat{x} - I$. Imposing both conditions on Equation (25) yields the perceived threshold employed by future selves when the goal is endogenous, denoted $\hat{x}^{NI}$:

$$\hat{x}^{NI} = (\frac{\gamma_1}{\gamma_1 - 1 - \eta})I. \quad (27)$$

Thus, $\hat{x}^{NI}$ is precisely the threshold employed by a time-consistent agent whose goals are endogenously set.[31] Likewise, the naif's equilibrium threshold when the goal is endogenous, denoted $\overline{x}^{NI}$, is derived by imposing $r = \hat{x}^{NI} - I$ on Equation (26):

$$\overline{x}^{NI} = \frac{1}{(\gamma_2 - 1)(1+\eta)}[\beta(\gamma_2 - \gamma_1)(\hat{x}^{NI} - I)(\frac{\overline{x}^{NI}}{\hat{x}^{NI}})^{\gamma_1} + \gamma_2(\eta\hat{x}^{NI} + I)]. \quad (28)$$

His current value function w^{NI} is given by

$$w^{NI}(x, \overline{r} = \hat{x}^{NI} - I) = \begin{cases} \beta(\hat{x}^{NI} - I)(\frac{x}{\hat{x}^{NI}})^{\gamma_1} + (1+\eta)(\frac{\gamma_1 - 1}{\gamma_2 - \gamma_1})(\hat{x}^{NI} - \overline{x}^{NI})(\frac{x}{\overline{x}^{NI}})^{\gamma_2} & \text{if} \quad x < \overline{x}^{NI} \\ x - I + \eta(x - \hat{x}^{NI}) & \text{if} \quad x \geq \overline{x}^{NI}. \end{cases}$$

Proposition 4 *The naive agent stops after the sophisticated agent, but falls short of his goal*[32]*:* $\overline{x}^{SI} < \overline{x}^{NI} < \hat{x}^{NI}$.

The naif incorrectly believes that he will be more patient in the future, so he sets higher goals accordingly. But when faced with the stopping decision in the present, he undervalues

[30] Since the intuition is virtually identical, the details are provided in the Appendix.

[31] Note that it also corresponds to (22) when $\beta = 1$ or $\lambda = 0$.

[32] As shown in the Appendix, this result also holds for exogenous $r \geq 0$: $\overline{x}^{SE} < \overline{x}^{NE} < \hat{x}$, where $\hat{x}$ is given by (25). The intuition is essentially identical. When $r = \hat{x} - I$, the naif's over-optimism leads him to set his goal higher than the sophisticate's, making the gap between their stopping thresholds even larger.

the future more than he had expected, and stops earlier, falling short of his overly ambitious goal. The fact that he stops after his sophisticated counterpart can be attributed to two factors. First, irrespective of reference dependence, both the sophisticate and the naif overweight the present. But because the sophisticate correctly foresees that he will also undervalue the option to wait in the future, he undervalues the present option even further. In contrast, the naif is more optimistic about future behavior, so he values the present option relatively more.[33] The second effect is caused by the interaction of reference dependence and expectations. The naif's optimism leads him to set higher goals for himself than the sophisticate, who realistically tempers his expectations. Since he incurs a higher potential penalty upon stopping, the naif has a larger incentive to wait for a higher project value.

5.2 Welfare

In the absence of reference dependence, the naive agent is clearly better off than his sophisticated counterpart from an ex-ante perspective. When $\eta = 0$, there is no comparison utility and the ex ante self wants to maximize the expected present discounted value of his net terminal payoff alone. Therefore, the first best is to employ the threshold $x^* = (\frac{\gamma_1}{\gamma_1 - 1})I$. Since $\overline{x}^{SI}(\eta = 0) < \overline{x}^{NI}(\eta = 0) < x^*$, the naive agent is unambiguously better off due to the "sophistication effect," whereby the sophisticate's realistic pessimism leads to a relatively detrimental outcome (O'Donoghue and Rabin 1999).

However, the naif's welfare is less rosy when he has reference dependent preferences, because he incurs direct comparative disutility from falling short of his optimistic expectations. From an ex ante perspective, the naif's *true* continuation value when goals are self-set is given by $\tilde{v}^{NI}(x)$:

$$\tilde{v}^{NI}(x, r = \hat{x}^{NI} - I) = \begin{cases} [\overline{x}^{NI} - I + \eta(\overline{x}^{NI} - \hat{x}^{NI})](\frac{x}{\overline{x}^{NI}})^{\gamma_1} & \text{if} \quad x < \overline{x}^{NI} \\ x - I + \eta(x - \hat{x}^{NI}) & \text{if} \quad x \geq \overline{x}^{NI}, \end{cases} \tag{29}$$

where $\hat{x}^{NI}$ and $\overline{x}^{NI}$ are defined by Equations (27) and (28), respectively. Since his unanticipated impulsiveness leads him to fall short of his overly optimistic goal, he incurs disutility $\eta(\overline{x}^{NI} - \hat{x}^{NI})$ upon stopping, in addition to receiving the net terminal payoff $\overline{x}^{NI} - I$.

Because the naive agent incurs comparative disutility that his sophisticated counterpart does not, the welfare comparison between naivete and sophistication is not as clear-cut as in the absence of reference-dependence, and there are circumstances in which the naif is unambiguously *worse off* than the sophisticate. For example, consider the case when reference dependence is quite high: let $\eta \geq \eta^*$, so $\overline{x}^{SI} \geq x^*$. Since the naif always waits

[33] This is the same result obtained by Grenadier and Wang (2007), whose model corresponds to $\eta = 0$ here.

longer than his sophisticated counterpart, the ex-ante expected present discounted value of his net terminal payoff is strictly less than that of the sophisticate. Furthermore, the naif incurs comparative disutility from falling short of his overly optimistic goal while the sophisticate incurs no comparative disutility. Thus, the naif is worse off than the sophisticate in both components of overall utility when $\eta \geq \eta^*$.

Proposition 5 *When goals are endogenous, let* $\tilde{v}^{N^*}(x)$ *denote the (true) first-best option value that the naif can attain, and let* $v^{S^*}(x)$ *denote the first-best option value that the sophisticate can attain. When* $\eta > 0$*, the first best that the naif can achieve is strictly lower than that of the sophisticate:* $\tilde{v}^{N^*}(x) < v^{S^*}(x)$.

In the previous section, I have shown that the sophisticate's maximum attainable option value is achieved when $\eta = \eta^*$. In contrast, the naif persistently sets his goal to be $\hat{x}^{NI} - I$, and incurs comparative disutility from falling short. Given that he sets this goal, the threshold that would maximize his overall utility, but which *no value of* η *can attain* due to his over-optimism, is the one that allows him to meet it. If this were possible, he would incur no comparative disutility and the option value would only consist of the discounted value of consumption utility. But since $\hat{x}^{NI} > x^*$, the value of stopping at $\hat{x}^{NI}$ is strictly less than that of stopping at x^* for any $\eta > 0$. Thus, in contrast to the "sophistication effect" that sophisticates are worse off than naifs in the absence of reference-dependence (O'Donoghue and Rabin 1999), the first-best that the naif can achieve is strictly lower than that of the sophisticate when goals are self-set.

Proposition 5 implies, in contrast to the sophisticate's case, that the naif's ignorance regarding his limitations is a handicap that cannot be overcome by changing η alone. Beyond education about appropriate goal commitment, the naif would need to recognize his limitations to correct his expectations, which may be more difficult to achieve. Clinical psychologists have long recognized that individuals may persistently fall short of self-set goals while simultaneously maintaining the belief that they can attain them. Hamacheck (1978) describes neurotic perfectionists as individuals "whose efforts - even their best ones - never seem quite good enough, at least in their own eyes. It always seems to these persons that they could - and should - do better ..." This result also illustrates the pertinence of advice found in personal development on how to calibrate one's expectations properly, to avoid the disappointment that arises from unattainable goals (Tracy 2003).

6 Aspirational Goals

Thus far, I have assumed that the agent's goal must coincide with the outcome he expects to achieve. Because there is no uncertainty over the realized outcome, regardless of whether *beliefs* are correct or not, it is unclear how the agent could simultaneously anticipate a particular outcome with certainty, knowingly set a goal that differs from that expectation, and also derive utility from comparing the outcome to a goal that he knew to be unrealistic.

An alternative interpretation is that the goal represents an individual's aspiration, rather than an expectation about what he will attain. In this case, the goal may arguably be divorced from expectations without logical conflict, so that he can strategically set goals for future selves even if he is aware that they are unrealistic. If an agent were to choose his goals freely ex ante, the question of interest is what aspirational goal optimizes ex-ante welfare. I assume that the feasible set of goals is given by $r \in [-I, \infty)$.[34] Since a goal can only be set for a future self, he seeks to maximize his total utility from an ex-ante perspective. Thus, the optimal aspirational goal r^* is chosen to maximize the continuation value function $v(x, \hat{r})$ described by Equation (11) in its wait region, anticipating future selves' behavior given r^*. His decision involves a trade-off between consumption and comparison utility. Choosing a more ambitious goal increases his material payoff by incentivizing him to wait for a higher project payoff, but reduces his comparison utility.[35] I focus on the case of sophistication, where the agent is aware of and would like to attenuate his self-control problem.[36]

Proposition 6 *When the agent's comparison function is linear, his ex-ante welfare is monotonically decreasing in his goal r. Thus, the aspirational goal r^* that maximizes ex-ante welfare is the lowest possible.*

Consider an agent who has no self-control problem ($\overline{\gamma} = \gamma_1$). Any positive goal induces him to wait longer than is first-best to maximize expected consumption utility. Thus, he is clearly better off setting a goal which is lower than the outcome he will actually achieve. But even in the more interesting case when the agent has a self-control problem ($\overline{\gamma} > \gamma_1$) and can benefit from additional motivation, choosing the lowest possible goal is optimal from an ex ante perspective for *any* degree of present-biasedness, $\overline{\gamma} \in (\gamma_1, \infty)$. For each

[34]Since the project payoff process is bounded below by zero, the lowest net terminal payoff the agent can receive is $-I$, if he stops the process when it is zero.

[35]Brunnermeier and Parker (2005) consider the optimal choice of subjective beliefs when the agent faces a trade-off between material outcomes and belief-based utility. In their setting, there is no incentive to distort beliefs in order to change actions. The only motivation for belief distortion is the benefit for anticipatory utility flows, so any belief distortion decreases material benefits and pessimistic beliefs can only hurt.

[36]Since the naif believes that he has no self-control problem, he sees no instrumental value in a positive goal.

of the two components of v, namely the expected discounted values of consumption and comparison utility, the two forces affecting each are the utility incurred upon stopping and the time value of waiting for it. As the agent's goal decreases, the decrease in consumption utility reduces the value function, but time discounting counteracts this reduction, since achieving a lower terminal payoff does not require waiting as long. On the other hand, as the goal decreases, both the increase in comparison utility and time discounting positively affect the value function, since he realizes higher comparison utility and stops earlier due to the weaker force on self-discipline. Since the comparison utility function is linear, the latter effect dominates the former, so that choosing the lowest possible goal is globally optimal.

Proposition 7 *To set a non-degenerate aspirational goal ($r^* > -I$), the agent must be sufficiently impulsive and his sensitivity to gains must be diminishing sufficiently fast.*

The finding that an agent prefers to set the lowest possible goal no matter how severe his self-control problem is reliant on both the asymmetric effect of time discounting, which is due to the trade-off between consumption and comparison utility, and the linearity of comparison utility, which implies that the marginal benefit from exceeding a given goal is constant. More realistically, the agent may exhibit diminishing sensitivity to gains, so that the marginal benefit from exceeding his goal is decreasing as the gap between low aspirations and realized outcomes increases. Given the preceding intuition, it is not surprising that the agent must have a sufficiently severe self-control problem and exhibit sufficiently diminishing sensitivity to gains in order to prefer setting a non-degenerate goal.

Proposition 8 *More impulsive agents set higher aspirational goals: $\frac{\partial r^*}{\partial \bar{\gamma}} > 0$. Nonetheless, more impulsive agents stop earlier.*

As an agent's impulsiveness becomes more severe, the motivational benefit of setting a more ambitious aspirational goal increases. On the other hand, undervaluing the future implies that his responsiveness to a given goal declines as well, weakening the benefits of setting a higher goal relative to the loss in comparison utility. Nonetheless, the former effect dominates the latter. Thus, when Proposition 7 holds, more impulsive agents set higher aspiration goals for themselves in order to improve their patience, despite the ensuing loss in comparison utility. Although their aspirational goals are higher than those of less impulsive agents, they still stop earlier.

7 Social Comparison

Thus far, I have assumed that an agent's only point of reference is his own expected net terminal payoff. But in addition to, or rather than, engaging in self-comparison, he may look to a role model or peer(s) as the basis of comparison. Many previous theories of social comparison have considered their costly and wasteful effects. For example, Frank (1985) argues that the pursuit of status decreases societal welfare, because individuals engage in costly signaling in a zero-sum game. Here, I explore whether social comparison can serve to attenuate the self-control problem by providing individuals with reference points that allow them to set goals. Previously, Battaglini et al. (2005) have shown how the presence of a peer can ameliorate or exacerbate one's impulsive behavior, since his behavior can act as either "good news" or "bad news" about one's own ability to resist temptation. In their model, because agents have incomplete information about their own and their peers' vulnerability to temptation but are aware that they are correlated, observing a peer's behavior offers information about one's own degree of self-control. On the other hand, I consider the effects of social comparison under full information and instead emphasize how they can be motivated to exercise patience because they derive utility from comparing their own outcomes to others'.

Consider two agents, i and j, who compare themselves against one another. When the agent's goal is no longer a matter of pure self-comparison, the natural extension of sophistication is that the agent's goal must be derived from correct expectations about his own and his peer's outcomes, given full information about one another's characteristics.[37] Suppose that each agent's goal, or reference point, is a convex combination of his own expected net terminal payoff and his peer's[38]:

$$r_i = \alpha_i(\overline{x}_i - I) + (1-\alpha_i)(\overline{x}_j - I), \text{ where } a_i \in [0,1]$$
$$r_j = \alpha_j(\overline{x}_j - I) + (1-\alpha_j)(\overline{x}_i - I), \text{ where } a_j \in [0,1].$$

If $\alpha_i = 1$, agent i evaluates himself exclusively against his own expected net terminal payoff, so this case is equivalent to the preceding analysis of an individual agent. If $\alpha_i = 0$, agent i evaluates himself exclusively against his expectation of his peer's (i.e. agent j's) net terminal payoff.[39] I also allow for differences in agents' degrees of impulsiveness (β_k, λ_k), reference

[37]As in the case of self-comparison, the model could also be extended to the case in which the agent persistently misperceives his peer's "ability," with similar insights.

[38]Due to linearity of the reference point and the comparison utility function, this formulation is equivalent to assuming that an agent separately compares his net terminal payoff against its own expectation and the expectation of his peer's net terminal payoff, and that his comparison utility is formed by taking a convex combination, dictated by his parameter α, of these two comparisons.

[39]Although the expectation of a peer's net terminal payoff and that peer's actual material outcome coincide, I assume that an agent's goal is the expectation, so that his comparison utility is incurred when he himself stops. If the comparison were made once both agents had stopped, then comparison utility could be incurred after one's stopping time. For example, if two identical agents face separate payoff processes with identical parameters, they will use the same thresholds but stop at different times. Since the stopping is irreversible and the expectation of his peer's outcome coincides with its realization, the assumption that the comparison is assessed upon stopping against an expectation seems reasonable.

dependence (η_k), and the characteristics of their projects (μ_k, σ_k), where $k = i, j$.

When peers have different degrees of self-control, how does the degree of equilibrium patience vary between them? If there is a difference in their threshold strategies, then both agents incur non-zero comparative utility, in contrast to the individual case. Thus, the choice of a peer who will maximize ex ante welfare includes a trade-off between the expected discounted value of the two components of his overall utility, the material payoff and comparison utility. I focus on the case in which the agents differ in their degrees of impulsiveness, $(\beta_i, \lambda_i) \neq (\beta_j, \lambda_j)$, but are identical in all other characteristics $(\alpha, \eta, \mu, \sigma)$.[40]

For simplicity, let each agent derive comparison utility exclusively from the expected outcome of his peer, $\alpha_i = \alpha_j = 0$, implying that $r_i = \overline{x}_j - I$ and $r_j = \overline{x}_i - I$ in equilibrium.[41] As before, I use $\overline{\gamma}$ as the measure of impulsiveness, where $\overline{\gamma}_k = \beta_k \gamma_1 + (1-\beta_k)\gamma_{2k}$ for $k = i, j$. Without loss of generality, let agent i have more patience than agent j: $\overline{\gamma}_i < \overline{\gamma}_j$. Recall that Equation (20) describes an agent's optimal threshold in a stationary equilibrium with an arbitrary goal level. Therefore, I can immediately derive the optimal thresholds in a peer group by substituting the appropriate goals. Since each agent has complete information about both his own and his peer's preferences, the optimal thresholds must satisfy the following system of equations simultaneously in equilibrium, where we substitute the relevant goals ($r_i = \overline{x}_j - I$, $r_j = \overline{x}_i - I$) to obtain the optimal thresholds:

$$\overline{x}_i = (\frac{\overline{\gamma}_i}{\overline{\gamma}_i - 1})I + (\overline{x}_j - I)(\frac{\eta}{1+\eta})(\frac{\overline{\gamma}_i}{\overline{\gamma}_i - 1}) \tag{30}$$

$$\overline{x}_j = (\frac{\overline{\gamma}_j}{\overline{\gamma}_j - 1})I + (\overline{x}_i - I)(\frac{\eta}{1+\eta})(\frac{\overline{\gamma}_j}{\overline{\gamma}_j - 1}). \tag{31}$$

Thus, the agents' equilibrium thresholds lie at the intersection of the agents' optimal threshold functions, where $\eta < \min\{\overline{\gamma}_i - 1, \overline{\gamma}_j - 1\}$.[42]

For the sophisticated agent, belonging to a homogeneous peer group ($\overline{\gamma}_i = \overline{\gamma}_j$) is equivalent to pure self-comparison.[43] Intuitively, there is no difference between comparing oneself against one's own expected outcome and comparing oneself against the expected outcome of an identical twin. Hence, all of the preceding welfare analysis for the individual case applies for agents in a homogeneous peer group.

Proposition 9 *In a heterogeneous peer group:*

[40] Allowing the other parameters to differ leads to similar comparative statics in the expected directions.

[41] The results can easily be extended the case when $\alpha_i \neq 0$ and $\alpha_j \neq 0$.

[42] This is a technical assumption to ensure that the equilibrium thresholds are bounded. It implies that if each agent were to engage in self-comparison (or were part of a homogeneous peer group), his threshold would be positive and well-defined.

[43] Unsurprisingly, this result also holds if $\alpha_i = \alpha_j > 0$.

1. *Relative to having no goal ($\eta = 0$), having any peer increases an agent's stopping threshold, even if the peer is more impulsive:* $\frac{\partial \overline{x}_i}{\partial \eta} > 0$ *for all* $\overline{\gamma}_j \in (1, \infty)$.

2. *Each agent's stopping threshold is decreasing in his peer's degree of impulsiveness:* $\frac{\partial \overline{x}_i}{\partial \overline{\gamma}_j} < 0$ *and* $\frac{\partial \overline{x}_j}{\partial \overline{\gamma}_i} < 0$.

3. *The more patient agent always has a higher stopping threshold than his more impulsive peer: If* $\overline{\gamma}_i < \overline{\gamma}_j$, *then* $\overline{x}_i > \overline{x}_j$.

In the absence of a goal or peer, an agent k stops too early, choosing the stopping threshold $(\frac{\overline{\gamma}_k}{\overline{\gamma}_k - 1})I$. A peer provides the agent with a goal that increases the potential cost of settling for a lower project value, since he incurs a comparative penalty if he falls short of his peer. The fact that the peer is relatively more impulsive merely implies that the goal will not be as high as it would be with a less impatient peer. Hence, the potential penalty and consequently, the agent's incentive to wait, increase with his peer's degree of self-control. This result is consistent with Bandura and Jourden's (1991) finding that individuals adjust their goals in response to their peers' performance. When their peers' performance is lower than their own, individuals adjust their goals downward and are content with a lower performance level because they are still outperforming their peers. In particular, as long as at least one member of the group exhibits some self-control (e.g., $\overline{\gamma}_i$ is finite), *both* members exhibit more patient behavior than they would in the absence of goal, even if the other has a preference for instantaneous gratification (i.e. $\overline{\gamma}_j \to \infty$). The mitigation of j's preference for instantaneous gratification stands in contrast to the purely individual and homogeneous cases, when goal-setting could not improve his patience. When agent i has some self-control, however limited, he stops at $\overline{x}_i > I$ even in the absence of a goal. This sets a potential penalty that j wishes to avoid, mitigating his extreme impulsiveness; in turn, j's more patient behavior has a positive feedback effect on i's own behavior.[44] It is only when both agents prefer instantaneous gratification that social comparison does not improve patience, just as the individual setting, when j is unable to provide such a penalty for himself. Because the agent with more self-control is able to wait longer even in the absence of a goal and is more responsive to goals, he sets a higher stopping threshold than his more impulsive counterpart.

7.1 Optimal Peers

I now consider the choice of peer that maximizes ex ante welfare, maintaining the assumption that $r_i = \overline{x}_j - I$ and $r_j = \overline{x}_i - I$. This decision involves a trade-off between consumption

[44] This feedback effect between heterogeneous peers certainly does not rely on $\eta_i = \eta_j$, and is proportional to the product $\eta_i \eta_j$.

and comparison utility, since choosing a relatively less impulsive peer increases the material payoff but induces negative comparison utility and increases the expected wait.

Proposition 10 *An agent's ex ante welfare v_i is monotonically increasing in his peer's degree of impulsiveness: $\frac{\partial v_i}{\partial \overline{\gamma}_j} > 0$. Thus, the partner j^* who maximizes agent i's ex ante welfare is the most impulsive possible: $\overline{\gamma}_j^* \to \infty$ for all $\overline{\gamma}_i \in (\gamma_1, \infty)$.*

The finding that any agent, no matter his degree of impulsiveness, prefers to choose the most impulsive peer possible mirrors that of Proposition 6, where every agent with a linear comparison utility function sets the lowest aspirational goal possible.[45] This similarity is unsurprising, since choosing a peer whose expected outcome can differ from one's own is akin to choosing an aspirational goal that can differ from one's expected outcome. In both cases, there is the same trade-off between consumption and comparison utility, and the same forces that affect an aspirational goal-setter, apply to the agent who selects the optimal peer.

As in the case of intrapersonal comparison, there is the question of whether the marginal utility of a social comparison remains the same between any pair of peers. It seems reasonable that two peers with similar characteristics might choose to compare themselves against one another. But at some point, the gap between two agents may be too large to merit credible comparison. For example, it seems unlikely that an honors student derives significant utility from comparing himself to a remedial student. Diminishing sensitivity to comparative gains would be consistent with this phenomenon and would lead to a less extreme choice of the optimal peer, just as in the case of aspirational goal-setting, described in Proposition 7.

8 Conclusion

This paper addresses the role of self-set, non-binding goals as a source of internal motivation in an optimal stopping problem. When agents have linear reference-dependent preferences and endogenously set a goal regarding the expected outcome that serves as the reference point, they can attenuate the self-control problem, and sophisticates can even achieve the first best from an ex ante perspective. Too much reference dependence, on the other hand, leads an agent to wait longer than the first best, and is always detrimental in the absence of present-biasedness. Notably, none of the effects of goal-setting require any form of loss aversion or curvature in the comparison utility function, nor do they rely on ex-ante uncertainty

[45] In contrast, Battaglini et al. (2005) find that the optimal peer is always slightly weaker, $0 < \beta_j < \beta_i$, when agents infer information about their own degree of self-control by observing the behavior of others. There, having a much weaker partner is undesirable because it is "bad news" about one's own willpower, while a stronger partner provides relatively less information.

over outcomes. These findings suggest that the demand for goods and services aimed at educating individuals about goal-setting and goal commitment may be a rational response to impulsiveness, and that the need for external enforcement may be overestimated.

When the goal represents an aspiration and can be divorced from expectations, there exists a trade-off between consumption and comparison utility. I find that the agent must be sufficiently present-biased and exhibit diminishing sensitivity to gains in order to set a non-degenerate aspirational goal. In this case, more impulsive agents set higher aspirational goals to counteract impatience. Otherwise, the incentive to maximize comparison utility dominates and he prefers to set the lowest aspirational goal possible.

I extend the model to social comparisons, where each agent compares his material outcome against the expectation of his peer's. Peer comparison induces more patient behavior relative to the absence of a goal. Nonetheless, every agent prefers to compare himself to a peer with the least self-control possible, regardless of the severity of his impulsiveness.

The model presented here can be readily distinguished from other proposed explanations of goal-setting behavior by observables. That goals are simply reminders of tasks to do is belied by evidence that people report less or more satisfaction with the same outcome when their goals differ (Heath et al. 1999, Medvec et al. 1995, McGraw et al. 2005). While publicly announced goals may also influence behavior through social censure or praise, that individuals respond to goals outside the public domain (Locke and Latham 2002) indicates that social reputation concerns are not the sole driver. My model predicts that a reference dependent agent may be motivated by a self-set goal even if it is privately known; if he compares himself to a peer, his behavior is affected by information about that peer's achievements regardless of whether his relative performance will be publicized. Another alternative is that a goal can act as a self-monitoring tool for an individual with imperfect information about the extent of his self-control problem (Bénabou and Tirole 2004). Regardless of whether the agent is sophisticated or naive, the model presented here can be distinguished from a self-reputation model in several ways. The two models predict very different responses to information. Whether or not his comparison utility function is linear, the reference-dependent agent is motivated by a goal regarding a future outcome, but is unresponsive to information about past behavior. In contrast, knowledge about past behavior leads the imperfectly informed agent to update his beliefs about his willpower and behave accordingly. Thus, a reference-dependent agent might keep a list of future accomplishments to motivate their achievement, while an imperfectly informed agent would prefer to remind himself of past accomplishments. Likewise, while external constraints on previous behavior have a detrimental effect on the future behavior of an imperfectly informed agent because they weaken his reputation capital, they should have no such influence on the reference-dependent one.

The current model limits attention to a single stopping decision. A natural extension is to consider goal-setting when the agent faces a multi-stage project. In this richer setting, the agent may also be able to "break down" or aggregate goals to improve his welfare. This decision corresponds to Read, Loewenstein and Rabin's (1999) discussion of "motivated bracketing," where an agent with a self-control problem chooses to frame a problem narrowly or broadly to accomplish a goal. In Hsiaw (2009), I show that in the presence of uncertainty over outcomes and loss aversion, the decision to frame narrowly or broadly involves a trade-off between greater motivation and more expected comparative disutility due to outcome variance when goal evaluation is frequent. Similarly, the agent could perform the same exercise when facing multiple projects. There, an important consideration is the timing of the comparative evaluation if a goal pertains to multiple outcomes realized at different times.

While this paper considers situations in which goals are evaluated upon the endogenous stopping time, one can imagine situations in which the relevant goals are time-dependent, i.e. "Timely." For example, a dieter might set a target weight to achieve by the end of the year, and feel bad about herself on December 31 if she has not reached it. In social comparisons, attaining a favorable outcome before a peer or rival can provide positive utility as well. In such settings, an agent might explicitly set time-contingent goals or engage in continual evaluation, relative to expectations about himself or to his peer.

The interaction of hyperbolic discounting and reference-dependent preferences has implications for a number of other contexts. For example, contracting between a principal and agent may be underestimating the effect of external incentive schemes if they are reinforced by goal-setting. Likewise, the existence of peer effects through social comparison suggests that group or team settings may be beneficial to managers or educators who are interested in improving productivity. Further pursuit of these or related lines of inquiry would enrich our understanding of such effects and their interactions with standard mechanisms.

References

Abeler, Johannes, Armin Falk, Lorenz Goette, and David Huffman, "Reference Points and Effort Provision," IZA Discussion Paper Series 3939, IZA January 2009.

Ainslie, George, *Picoeconomics: The Interaction of Successive Motivational States within the Person*, Cambridge University Press, April 1992.

Ariely, Dan and Klaus Wertenbroch, "Procrastination, Deadlines, and Performance: Self-Control by Precommitment," *Psychological Science*, 2002, *13* (3), 219–24.

Austen-Smith, David and Roland G. Fryer, "An Economic Analysis of "Acting White"," *The Quarterly Journal of Economics*, May 2005, *120* (2), 551–583.

Bagwell, Laurie Simon and B Douglas Bernheim, "Veblen Effects in a Theory of Conspicuous Consumption," *American Economic Review*, June 1996, *86* (3), 349–73.

Bandura, Albert, "Self-Regulation of Motivation and Action Through Internal Standards and Goal Systems," in Lawrence A. Pervin, ed., *Goal Concepts in Personality and Social Psychology*, Lawrence Erlbaum Associates, Publishers, 1989, pp. 19–63.

——— **and Forest J. Jourden**, "Self-Regulatory Mechanisms Governing the Impact of Social Comparison on Complex Decision Making," *Journal of Personality and Social Psychology*, June 1991, *60* (6), 941–951.

Barberis, Nicholas C. and Wei Xiong, "Realization Utility," Working Paper 14440, National Bureau of Economic Research October 2008.

Battaglini, Marco, Roland Bénabou, and Jean Tirole, "Self-control in peer groups," *Journal of Economic Theory*, August 2005, *123* (2), 105–134.

Bénabou, Roland and Jean Tirole, "Willpower and Personal Rules," *Journal of Political Economy*, August 2004, *112*, 848–886.

Bernheim, B. Douglas, "A Theory of Conformity," *The Journal of Political Economy*, oct 1994, *102* (5), 841–877.

Bisin, Alberto and Kyle Hyndman, "Procrastination, Self-Imposed Deadlines and Other Commitment Devices," Working Paper March 2009.

Brennan, Michael J. and Eduardo S. Schwartz, "Evaluating Natural Resource Investments," *The Journal of Business*, 1985, *58* (2), 135–157.

Brocas, Isabelle and Juan D. Carrillo, "A theory of haste," *Journal of Economic Behavior & Organization*, January 2005, *56* (1), 1–23.

Brunnermeier, Markus K. and Jonathan A. Parker, "Optimal Expectations," *The American Economic Review*, 2005, *95* (4), 1092–1118.

Camerer, Colin, Linda Babcock, George Loewenstein, and Richard Thaler, "Labor Supply of New York City Cabdrivers: One Day at a Time," *The Quarterly Journal of Economics*, May 1997, *112* (2), 407–441.

Carrillo, Juan D., "To be consumed with moderation," *European Economic Review*, January 2005, *49* (1), 99–111.

___ **and Mathias Dewatripont**, "Promises, Promises, ...," *The Economic Journal*, August 2008, *118*, 1453–1473.

Crawford, Vincent P. and Juanjuan Meng, "New York City Cabdrivers' Labor Supply Revisited: Reference-Dependence Preferences with Rational-Expectations Targets for Hours and Income," UCSD Discussion Paper 2008-03, University of California, San Diego July 2008.

DeMarzo, Peter M., Ron Kaniel, and Ilan Kremer, "Relative Wealth Concerns and Financial Bubbles," *Review of Financial Studies*, 2008, *21* (1), 19–50.

Dixit, Avinash, *Art of Smooth Pasting, The (Fundamentals of Pure and Applied Economics)*, Routledge, May 1993.

Dixit, Avinash K. and Robert S. Pindyck, *Investment under Uncertainty*, Princeton University Press, 1994.

Drucker, Peter F., *The Practice of Management*, Harper & Row, 1954.

Duesenberry, James Stemble, *Income, Saving and the Theory of Consumption Behavior*, Harvard University Press, 1949.

Ellis, Keith, *The Magic Lamp: Goal Setting for People Who Hate Setting Goals*, Three Rivers Press, 1998.

Ericson, Keith M. and Andreas Fuster, "Expectations as Endowments: Evidence on Reference-Dependent Preferences from Exchange and Valuation Experiments," *SSRN eLibrary*, 2010.

Falk, Armin and Markus Knell, "Choosing the Joneses: Endogenous Goals and Reference Standards," *The Scandinavian Journal of Economics*, 2004, *106* (3), 417–435.

Farber, Henry S., "Is Tomorrow Another Day? The Labor Supply of New York City Cabdrivers," *Journal of Political Economy*, 2005, *113* (1), 46–82.

___ , "Reference-Dependent Preferences and Labor Supply: The Case of New York City Taxi Drivers," *American Economic Review*, June 2008, *98* (3), 1069–1082.

Fehr, Ernst and Lorenz Goette, "Do Workers Work More if Wages Are High? Evidence from a Randomized Field Experiment," *American Economic Review*, May 2007, *97* (1), 298–317.

Frank, Robert, *Choosing the Right Pond: Human Behavior and the Quest for Status*, Oxford University Press, 1985.

Goette, Lorenz F. and David Huffman, "Affect as a Source of Motivation in the Workplace: A New Model of Labor Supply, and New Field Evidence on Income Targeting and the Goal Gradient," IZA Discussion Paper Series 1890, IZA December 2005.

Goldsmith, Marshall, "Are You Too Obsessed with Your Goals?," Harvard Business Online March 2008. [online] `http://discussionleader.hbsp.com/goldsmith/2008/03/goal_obsession.html`.

Grenadier, Steven and Neng Wang, "Investment Under Uncertainty and Time-Inconsistent Preferences," *Journal of Financial Economics*, 2007, *84*, 2–39.

Gul, Faruk and Wolfgang Pesendorfer, "Temptation and Self-Control," *Econometrica*, November 2001, *69* (6), 1403–1435.

Hamacheck, Don E., "Psychodynamics of normal and neurotic perfectionism," *Psychology: A Journal of Human Behavior*, February 1978, *15* (1), 27 – 33.

Harris, Christopher and David Laibson, "Instantaneous Gratification," Working paper, Harvard University May 2004.

Heath, Chip, Richard P. Larrick, and George Wu, "Goals as Reference Points," *Cognitive Psychology*, 1999, *38* (1), 79–109.

Hsiaw, Alice, "Goal Bracketing and Self-Control," Working Paper June 2009.

Kadan, Ohad and Jeroen M. Swinkels, "Stocks or Options? Moral Hazard, Firm Viability, and the Design of Compensation Contracts," *Review of Financial Studies*, 2008, *21* (1), 451–482.

Kahneman, Daniel and Amos Tversky, "Prospect Theory: An Analysis of Decision under Risk," *Econometrica*, 1979, *47* (2), 263–291.

Kőszegi, Botond and Matthew Rabin, "A Model of Reference-Dependent Preferences," *Quarterly Journal of Economics*, November 2006, *121* (4), 1133–1165.

——— **and** ———, "Reference-Dependent Consumption Plans," *American Economic Review*, June 2009, *99* (3), 909–936.

Klein, Howard J., "Further evidence on the relationship between goal setting and expectancy theories," *Organizational Behavior and Human Decision Processes*, August 1991, *49* (2), 230–257.

———**, Michael J. Wesson, John R. Hollenbeck, and Bradley J. Alge**, "Goal Commitment and the Goal-Setting Process: Conceptual Clarification and Empirical Synthesis," *Journal of Applied Psychology*, 1999, *84* (6), 885–896.

Koch, Alexander K. and Julia Nafziger, "Self-Regulation through Goal Setting," IZA Discussion Paper Series 3893, IZA December 2008.

Laibson, David, "Golden Eggs and Hyperbolic Discounting," *Quarterly Journal of Economics*, 1997, *112* (2), 443–477.

Latham, Gary P. and Edwin A. Locke, "Self-Regulation through Goal Setting," *Organizational Behavior and Human Decision Processes*, December 1991, *50* (2), 212–247.

Locke, Edwin A. and Gary P. Latham, "Building a practically useful theory of goal setting and task motivation: A 35-year odyssey.," *American Psychologist*, September 2002, *57* (9), 705–717.

Loewenstein, George, "Because It Is There: The Challenge of Mountaineering . . . for Utility Theory," *Kyklos*, 2007, *52* (3), 315–343.

Matthey, Astrid, "Yesterday's Expectation of Tomorrow Determines What You Do Today: The Role of Reference-Dependent Utility from Expectations," Jena Economic Research Papers 2008-003 January 2008.

——— **and Nadja Dwenger**, "Don't Aim Too High: The Potential Costs of High Aspirations," Jena Economic Research Papers 2007-097 December 2007.

McClelland, David C., "Business Drive and National Achievement," *Harvard Business Review*, July/August 1962, *40* (4), 99–112.

McDonald, Robert and Daniel Siegel, "The Value of Waiting to Invest," *The Quarterly Journal of Economics*, 1986, *101* (4), 707–728.

McGraw, A. Peter, Barbara A. Mellers, and Philip E. Tetlock, "Expectations and emotions of Olympic athletes," *Journal of Experimental and Social Psychology*, July 2005, *41* (4), 438–446.

Medvec, Victoria Husted, Scott F. Madey, and Thomas Gilovich, "When less is more: Counterfactual thinking and satisfaction among Olympic medalists," *Journal of Personality and Social Psychology*, October 1995, *69* (4), 603–610.

Mento, Anthony J., Edwin A. Locke, and Howard J. Klein, "Relationship of Goal Level to Valence and Instrumentality," *Journal of Applied Psychology*, 1992, *77* (4), 395–405.

Miao, Jianjun, "Option exercise with temptation," *Economic Theory*, March 2008, *34* (3), 473–501.

Odean, Terrance, "Are Investors Reluctant to Realize Their Losses?," *The Journal of Finance*, 1998, *53* (5), 1775–1798.

O'Donoghue, Ted and Matthew Rabin, "Doing It Now or Later," *The American Economic Review*, 1999, *89* (1), 103–124.

Ou-Yang, Hui, "Optimal Contracts in a Continuous-Time Delegated Portfolio Management Problem," *Review of Financial Studies*, 2003, *16* (1), 173–208.

Rayo, Luis and Gary S. Becker, "Evolutionary Efficiency and Happiness," *Journal of Political Economy*, 2007, *115*, 302–337.

Read, Daniel, George Loewenstein, and Matthew Rabin, "Choice Bracketing," *Journal of Risk and Uncertainty*, December 1999, *19* (1), 171–197.

Schelling, Thomas C, "Self-Command in Practice, in Policy, and in a Theory of Rational Choice," *American Economic Review*, May 1984, *74* (2), 1–11.

Shafran, Roz, Zafra Cooper, and Christopher G. Fairburn, "Clinical perfectionism: a cognitive-behavioural analysis," *Behaviour Research and Therapy*, 2002, *40* (7), 773 – 791.

Shu, Suzanne B., "Future-biased search: the quest for the ideal," *Journal of Behavioral Decision Making*, October 2008, *21* (4), 352–377.

Strotz, R.H., "Myopia and Inconsistency in Dynamic Utility Maximization," *The Review of Economic Studies*, 1956, *23* (3), 165–180.

Suvorov, Anton and Jeroen van de Ven, "Goal Setting as a Self-Regulation Mechanism," Working Papers w0122, Center for Economic and Financial Research (CEFIR) October 2008.

Thaler, Richard H., "Mental accounting matters," *Journal of Behavioral Decision Making*, 1999, *12* (3), 183–206.

Tracy, Brian, *Goals!: How to Get Everything You Want - Faster Than You Ever Thought Possible*, Berrett-Koehler Publishers, Inc., 2003.

Veblen, Thorstein, *The Theory of the Leisure Class*, New American Library, 1953. First published in 1899.

A Appendix

A.1 Proof of Proposition 3

Each self prefers that future selves choose their thresholds in order to maximize the continuation value v rather than the current value w. Therefore, we look for the threshold $\overline{x}$ that maximizes the wait region of v^{SI}. The first order condition $\frac{\partial v^{SI}}{\partial \overline{x}} = 0$ gives us the first-best threshold $\overline{x} = (\frac{\gamma_1}{\gamma_1 - 1})I$, which is identical to the solution for a standard agent x^*. We can also see this feature by inspecting the wait region of v^{SI}, which is the same function of the threshold as v^*. Because utility incurred upon stopping is not directly distorted, the ex ante option value of waiting, given a stopping threshold, is not distorted by η directly.

However, the presence of $\eta > 0$ affects the equilibrium threshold by changing the marginal value of stopping. The current self's optimal threshold is $\overline{x}^{SI} = (\frac{\overline{\gamma}}{\overline{\gamma}-1-\eta})I$, so the first best can only be achieved for η^* such that $\frac{\overline{\gamma}}{\overline{\gamma}-1-\eta^*} = \frac{\gamma_1}{\gamma_1-1}$. We can verify that $\eta^* < \overline{\gamma} - 1$ by noting that $\frac{\overline{\gamma}-\gamma_1}{\gamma_1} = \frac{\overline{\gamma}}{\gamma_1} - 1$. Since $\gamma_1 > 1$, then this inequality is satisfied for any $\beta \in [0, 1]$.

A.2 Proof of Corollary 1

The first part follows from the above proof of Proposition 3. The second statement follows by comparing the value functions when $\eta > 0$ versus $\eta = 0$. When $\eta = 0$, the agent stops at threshold $(\frac{\overline{\gamma}}{\overline{\gamma}-1})I$. When $\eta > 0$, the agent stops at threshold $(\frac{\overline{\gamma}}{\overline{\gamma}-1-\eta})I$. Thus, goal-setting is detrimental when

$$
\begin{aligned}
[(\frac{\overline{\gamma}}{\overline{\gamma}-1-\eta})I - I](\frac{1}{(\frac{\overline{\gamma}}{\overline{\gamma}-1-\eta})I})^{\gamma_1} &< [(\frac{\overline{\gamma}}{\overline{\gamma}-1})I - I](\frac{1}{(\frac{\overline{\gamma}}{\overline{\gamma}-1})I})^{\gamma_1} \\
[(\frac{1+\eta}{\overline{\gamma}-1-\eta})I](\frac{1}{(\frac{\overline{\gamma}}{\overline{\gamma}-1-\eta})I})^{\gamma_1} &< [(\frac{1}{\overline{\gamma}-1})I](\frac{1}{(\frac{\overline{\gamma}}{\overline{\gamma}-1})I})^{\gamma_1} \\
0 &< (\frac{\overline{\gamma}-1}{\overline{\gamma}-1-\overline{\eta}})^{\gamma_1-1}(\frac{1}{1+\overline{\eta}}) - 1,
\end{aligned}
$$

and the lower bound $\overline{\eta}$ satisfies the above condition with equality. Define the function $H(\eta)$ such that

$$
H(\eta) = (\frac{\overline{\gamma}-1}{\overline{\gamma}-1-\overline{\eta}})^{\gamma_1-1}(\frac{1}{1+\overline{\eta}}) - 1.
$$

We can verify that $\overline{\eta}$ exists and is unique by noting that $H(0) = 0$, $H(\overline{\gamma}-1) \to \infty$, and $H(\eta)$ strictly decreases for $\eta < \eta^*$ and strictly increases for $\eta > \eta^*$. Thus, we have shown existence and uniqueness of $\overline{\eta}$ for any $\gamma \in (1, \infty)$, as well as the fact that $\eta^* < \overline{\eta} < \overline{\gamma} - 1$. Finally, the implicit function theorem yields

$$
\frac{\partial \overline{\eta}}{\partial \overline{\gamma}} = -\frac{-(\frac{\overline{\eta}}{1+\overline{\eta}})(\frac{\gamma_1-1}{(\overline{\gamma}-1-\overline{\eta})^2})(\frac{\overline{\gamma}-1}{(\overline{\gamma}-1-\overline{\eta})^{\gamma_1-2}})}{(\frac{1}{1+\overline{\eta}})(\frac{\overline{\gamma}-1}{(\overline{\gamma}-1-\overline{\eta})^{\gamma_1-1}})[-\frac{1}{1+\overline{\eta}} + (\frac{\gamma_1-1}{\overline{\gamma}-1-\overline{\eta}})]} > 0,
$$

which is positive since the numerator is always negative and the denominator is positive since $\overline{\eta} > \eta^*$.

A.3 Existence and Uniqueness of $\overline{x}^{NE}$

First, I prove the existence and uniqueness of $\overline{x}^{NE}$ for any given $r \geq 0$. Define the following function $G(x)$:

$$
G(x) = \frac{1}{(\gamma_2-1)(1+\eta)}[\beta(\gamma_2-\gamma_1)(\hat{x} - I + \eta(\hat{x} - I - r))(\frac{x}{\hat{x}})^{\gamma_1} + \gamma_2(1+\eta)I + \gamma_2\eta r] - x, \quad (32)
$$

where $\hat{x} = (\frac{\gamma_1}{\gamma_1-1})I + r(\frac{\eta}{1+\eta})(\frac{\gamma_1}{\gamma_1-1})$. Note that when $r = \hat{x} - I$, then $\hat{x} = \hat{x}^{NI}$.

Proof.

Consider the function $G(x)$, given by Equation (32). Clearly, $\overline{x}^{NE}$ must satisfy $G(\overline{x}^{NE}) = 0$, where I have assumed that $\overline{x}^{NE} \leq \hat{x}$ by construction. Since $G(0) = \gamma_2[(1+\eta)I + \eta r] > 0$

and

$$G(\hat{x}) = -\frac{(1-\beta)(\gamma_2-\gamma_1)(r\eta+(1+\eta)I)}{(1+\eta)(\gamma_1-1)(\gamma_2-1)} < 0,$$

then $\overline{x}^{NE}$ exists for any $r \geq 0$. Next, note that

$$G'(x) = \frac{1}{(\gamma_2-1)(1+\eta)}[\beta(\gamma_2-\gamma_1)(\hat{x}-I+\eta(\hat{x}-I-r))(\frac{1}{\hat{x}})^{\gamma_1}(\gamma_1)(x)^{\gamma_1-1}] - 1$$
$$G''(x) = \frac{1}{(\gamma_2-1)(1+\eta)}[\beta(\gamma_2-\gamma_1)(\hat{x}-I+\eta(\hat{x}-I-r))(\frac{1}{\hat{x}})^{\gamma_1}(\gamma_1)(\gamma_1-1)(x)^{\gamma_1-2}],$$

so $G''(x) > 0$ for all $x > 0$. Then $G'(x) < 0$ for any x such that $G(x) = 0$. Thus, there exists an $\overline{x}^{NE} \leq \hat{x}$ such that $G(\overline{x}^{NE}) = 0$.

The expression for $\overline{x}^{NE}$, which is given by Equation 26, was constructed by assuming that $\overline{x}^{NE} \leq \hat{x}$. To ensure uniqueness of $\overline{x}^{NE}$, we must rule out the case where $\overline{x}^{NE} > \hat{x}$. Suppose that there exists another threshold $\tilde{x}$ such that it is optimal for the naif to stop when $x \geq \tilde{x}$ and wait otherwise, where $\tilde{x} > \hat{x}$. To construct the value function, note that the naif still believes that all future selves will employ the threshold $\hat{x} = (\frac{\gamma_1}{\gamma_1-1})I + r(\frac{\eta}{1+\eta})(\frac{\gamma_1}{\gamma_1-1})$, and the continuation value v is still given by Equation (11). When $x \in [0, \hat{x})$, then $v(x, \hat{r}) = [\hat{x} - I + \eta(\hat{x} - I - \hat{r})](\frac{x}{\hat{x}})^{\gamma_1} \equiv v_1(x, \hat{r})$. When $x \in [\hat{x}, \infty)$, then $v(x, \hat{r}) = x - I + \eta(x - I - \hat{r})$. When $x \in [0, \tilde{x})$, then w satisfies the differential equation given by Equation (7). When $x \in [\tilde{x}, \infty)$, then $w(x, r) = x - I + \eta(x - I - r)$. Since $0 \leq \hat{x} < \tilde{x}$, the continuation value function w now has three regions. Let w_1 describe w when $x \in [0, \hat{x})$, w_2 describe w when $x \in [\hat{x}, \tilde{x})$, and w_3 describe w when $x \in [\tilde{x}, \infty)$. Let $r = \hat{r}$.

When $x \in [0, \hat{x})$, then $v(x, r) = v_1(x, r) = A_1 x^{\gamma_1}$, where $A_1 \equiv [\hat{x} - I + \eta(\hat{x} - I - \hat{r})](\frac{x}{\hat{x}})^{\gamma_1}$, and w satisfies the differential equation given by Equation (7). Substituting $v(x, r) = A_1 x^{\gamma_1}$ into (7) yields the solution

$$w_1(x, r) = \beta A_1 x^{\gamma_1} + A_2 x^{\gamma_3}, \tag{33}$$

where γ_1 and γ_3 are the positive and negative roots of the quadratic equation $\frac{1}{2}\sigma^2\gamma + (\mu - \frac{1}{2}\sigma^2)\gamma - \rho = 0$, as before, where $\gamma_1 > 1$ and $\gamma_3 < 0$.

When $x \in [\hat{x}, \tilde{x})$, then $v(x, r) = x - I + \eta(x - I - r)$ and w again satisfies Equation (7). Substituting $v(x, r) = x - I + \eta(x - I - r)$ into (7) yields the solution

$$w_2(x, r) = \beta A_3 x^{\gamma_2} + A_2 x^{\gamma_4} + \beta(1+\eta)(\frac{\lambda}{\rho+\lambda-\mu})x - \beta[(1+\eta)I + \eta r], \tag{34}$$

where γ_2 and γ_4 are the positive and negative roots of the quadratic equation $\frac{1}{2}\sigma^2\gamma^2 + (\mu - \frac{1}{2}\sigma^2)\gamma - (\rho + \lambda) = 0$, as before, where $\gamma_2 \geq \gamma_1 > 1$ and $\gamma_4 < 0$.

The agent stops for any $x \in [\tilde{x}, \infty)$, so

$$w_3(x, r) = x - I + \eta(x - I - r). \tag{35}$$

By definition of the geometric Brownian motion x, $x = 0$ is an absorbing barrier of the project value process. Furthermore, the continuation value function v must be continuous, while the current value function w must be continuous and smooth everywhere. Since the naif *believes* that future selves will behave in the optimal manner, v is smooth as well. This gives the following boundary conditions:

$$\begin{aligned}
\text{Boundary: } & v_1(0, r) = 0, & (36)\\
& w_1(0, r) = 0, & (37)\\
\text{Value Matching: } & v_1(\hat{x}, r) = x - I + \eta(\hat{x} - I - r), & (38)\\
& w_1(\hat{x}, r) = w_2(\hat{x}, r), & (39)\\
& w_2(\tilde{x}, r) = w_3(\tilde{x}, r), & (40)\\
\text{Smooth Pasting: } & \frac{\partial v_1}{\partial x}(\hat{x}, r) = 1 + \eta, & (41)\\
& \frac{\partial w_1}{\partial x}(\hat{x}, r) = \frac{\partial w_2}{\partial x}(\hat{x}, r), & (42)\\
& \frac{\partial w_2}{\partial x}(\tilde{x}, r) = 1 + \eta. & (43)
\end{aligned}$$

Using the above boundary conditions in conjunction with $v_1(x, r) = A_1 x^{\gamma_1}$ and Equations (33), (34), and (35), we obtain the following non-linear equation for $\tilde{x}$:

$$\tilde{x} = \frac{\gamma_2(1+\beta)[(1+\eta)I + \eta r]}{(\gamma_2 - 1)(1+\eta)(1 - \frac{\lambda\beta}{\rho+\lambda-\mu})} + (\frac{\tilde{x}}{\hat{x}})^{\gamma_4} \left(\frac{\beta(\rho - \mu)\hat{x}}{\rho + (1-\beta)\lambda - \mu} \right), \tag{44}$$

where $\hat{x} = (\frac{\gamma_1}{\gamma_1})I + r(\frac{\gamma_1}{\gamma_1 - 1})(\frac{\eta}{1+\eta})$. Now, define the function $F(x)$ as follows:

$$F(x) = \frac{\gamma_2(1+\beta)[(1+\eta)I + \eta r]}{(\gamma_2 - 1)(1+\eta)(1 - \frac{\lambda\beta}{\rho+\lambda-\mu})} + (\frac{x}{\hat{x}})^{\gamma_4}(\hat{x}) \left(\frac{\beta - \frac{\lambda\beta}{\rho+\lambda-\mu}}{1 - \frac{\lambda\beta}{\rho+\lambda-\mu}} \right) - x, \tag{45}$$

where the first term and second terms are positive since $\gamma_2 > 1$, $\beta < 1$, $\mu < \rho$, and $r \geq 0$. Clearly, $\tilde{x}$ satisfies $F(\tilde{x}) = 0$. Since $\gamma_4 < 0$, then $\lim_{x \to 0} F(x) \to \infty$ and $\lim_{x \to \infty} F(x) \to -\infty$. Thus, $\tilde{x}$ exists. Moreover, $\gamma_4 < 0$ implies that $F'(x) < 0$ and $F''(x) < 0$, so $\tilde{x}$ is unique.

Evaluating $F(\cdot)$ at $\hat{x}$, we obtain

$$\begin{aligned} F(\hat{x}) &= \frac{\gamma_2(1+\beta)[(1+\eta)I+\eta r]}{(\gamma_2-1)(1+\eta)(1-\frac{\lambda\beta}{\rho+\lambda-\mu})} + \left(\frac{\beta-\frac{\lambda\beta}{\rho+\lambda-\mu}}{1-\frac{\lambda\beta}{\rho+\lambda-\mu}}\right)(\hat{x}) - \hat{x} \\ &= \frac{-(1-\beta)(\gamma_2-\gamma_1)[(1+\eta)I+\eta r]}{(\gamma_1-1)(\gamma_2-1)(1+\eta)(1-\frac{\lambda\beta}{\rho+\lambda-\mu})}, \end{aligned}$$

which is negative since $\gamma_2 > \gamma_1$. Since $G(\hat{x}) < 0$, then $\tilde{x} < \hat{x}$, which violates the assumption used to construct $\tilde{x}$, that $\tilde{x} > \hat{x}$. Thus, there does not exist an optimal threshold $\tilde{x}$ such that $\tilde{x} > \hat{x}$ and the naif stops when $x \geq \tilde{x}$ and waits otherwise. ■

A.4 Comparative Statics for $\overline{x}^{NE}$

Having shown existence and uniqueness of $\overline{x}^{NE}$, we can use the implicit function theorem to derive the relevant comparative statics.

Proof. By the implicit function theorem,

$$\frac{\partial x}{\partial \beta} = -\frac{\frac{\partial G}{\partial \beta}}{\frac{\partial G}{\partial x}}.$$

From the preceding proof of uniqueness, it is clear that $G'(x) < 0$ for all $x \in [0, \hat{x}]$. Turning to the numerator,

$$\frac{\partial G}{\partial \beta} = \frac{1}{(\gamma_2-1)(1+\eta)}(\gamma_2-\gamma_1)(\hat{x}-I+\eta(\hat{x}-I-r))(\frac{x}{\hat{x}})^{\gamma_1} > 0.$$

Therefore, $\frac{\partial \overline{x}^{NE}}{\partial \beta} > 0$. The implicit function theorem also gives us

$$\frac{\partial x}{\partial r} = -\frac{\frac{\partial G}{\partial r}}{\frac{\partial G}{\partial x}},$$

Turning to the numerator,

$$\begin{aligned} \frac{\partial G}{\partial r} &= \frac{1}{(\gamma_2-1)(1+\eta)}[\beta(\gamma_2-\gamma_1)(\frac{x}{\hat{x}})^{\gamma_1}(\frac{1}{\hat{x}})[\hat{x}(1+\eta)(\frac{\partial \hat{x}}{\partial r}) - \eta\hat{x} \\ &\quad - \gamma_1(\hat{x}-I+\eta(x-I-r))(\frac{\partial \hat{x}}{\partial r})] + \gamma_2\eta] \\ &= \frac{1}{(\gamma_2-1)(1+\eta)}\left(\beta(\gamma_2-\gamma_1)(\frac{x}{\hat{x}})^{\gamma_1}(-\eta) + \gamma_2\eta\right). \end{aligned}$$

Since $x \leq \hat{x}$, then $\frac{\partial G}{\partial r} > 0$, since $\beta(\gamma_2-\gamma_1)(\frac{x}{\hat{x}})^{\gamma_1} < \gamma_2$. Thus, $\frac{\partial \overline{x}^{NE}}{\partial r} > 0$.

Consider the effect of increasing β on $\frac{\partial \overline{x}^{NE}}{\partial r}$. It is clear that $\frac{\partial G}{\partial r}$ is decreasing in β, while $\frac{\partial G}{\partial x}$ is increasing in β. Therefore, $\frac{\partial^2 \overline{x}^{NE}}{\partial r \partial \beta} > 0$.

Now, consider the effect of increasing η in equilibrium. Let $\hat{r} = \hat{x} - I$. Since it is shown in the following proof that $\frac{\partial \overline{x}^{NE}}{\partial \hat{x}} > 0$ and clearly $\frac{\partial \hat{x}}{\partial \eta} > 0$, it is sufficient to show that $\frac{\partial G}{\partial \eta} > 0$ holding $\hat{x}$ fixed. By the implicit function theorem,

$$\frac{\partial x}{\partial \eta} = -\frac{\frac{\partial G}{\partial \eta}}{\frac{\partial G}{\partial x}},$$

where $\frac{\partial G}{\partial x} < 0$. Turning to the numerator,

$$\begin{aligned}\frac{\partial G}{\partial \eta} &= \frac{1}{(\gamma_2 - 1)}\left(-(\frac{1}{1+\eta})^2[\beta(\gamma_2 - \gamma_1)(\hat{x} - I)(\frac{x}{\hat{x}})^{\gamma_1} + \gamma_2 I + \gamma_2 \eta \hat{x}] + (\frac{1}{1+\eta})(\gamma_2 \hat{x})\right) \\ &= (\frac{1}{(\gamma_2 - 1)})(\frac{1}{1+\eta})^2(\hat{x} - I)[-\beta(\gamma_2 - \gamma_1)(\frac{x}{\hat{x}})^{\gamma_1} + \gamma_2] > 0,\end{aligned}$$

since $\overline{x}^{NE} < \hat{x}$. Thus, $\frac{\partial \overline{x}^{NE}}{\partial \eta} > 0$. ■

A.5 Proof of Proposition 4

Since the result that $\overline{x}^{SE} < \overline{x}^{NE} < \hat{x}^{NE}$ is slightly more general and implies that $\overline{x}^{SI} < \overline{x}^{NI} < \hat{x}^{NI}$, I will show the former. Note that in "rational expectations" equilibrium ($r = \hat{x} - I$), the naif's goal is the same as that of the time-consistent agent due to his mistaken beliefs, and is higher than that of the sophisticate.

Proof.

In the preceding proof of uniqueness of $\overline{x}^{NE}$, I already showed that $\overline{x}^{NE} < \hat{x}^{NE}$ when $\beta < 1$ for any $r \geq 0$. Since $r = \hat{x} - I > 0$, then it follows that $\overline{x}^{NI} < \hat{x}^{NI}$ as well. Thus, the naif falls short of his reference point.

Now, consider the function $G(x)$ as defined by Equation (32) and reproduced here:

$$G(x) = \frac{1}{(\gamma_2 - 1)(1+\eta)}[\beta(\gamma_2 - \gamma_1)(\hat{x} - I + \eta(x - I - r))(\frac{x}{\hat{x}})^{\gamma_1} + \gamma_2(1+\eta)I + \gamma_2 \eta r] - x,$$

The sophisticate's threshold solves $G(x) = 0$ when $\hat{x} = (\frac{\overline{\gamma}}{\overline{\gamma}-1})I + r(\frac{\eta}{1+\eta})(\frac{\overline{\gamma}}{\overline{\gamma}-1}) \equiv \hat{x}^{SE}$, while the naif's threshold solves $G(x) = 0$ when $\hat{x} = (\frac{\gamma_1}{\gamma_1 - 1})I + r(\frac{\eta}{1+\eta})(\frac{\gamma_1}{\gamma_1 - 1}) \equiv \hat{x}^{NE}$. Thus, $\hat{x}^{SE} < \hat{x}^{NE}$ and it is sufficient to show that $\frac{\partial x}{\partial \hat{x}}|_{\hat{x}=\hat{x}^{SE}} > 0$. By the implicit function theorem,

$$\frac{\partial x}{\partial \hat{x}} = -\frac{\frac{\partial G}{\partial \hat{x}}}{\frac{\partial G}{\partial x}}.$$

From the preceding proof, we have that $\frac{\partial G}{\partial x} < 0$. Turning to the numerator,

$$\frac{\partial G}{\partial \hat{x}} = \frac{1}{(\gamma_2 - 1)(1+\eta)} \left(\beta(\gamma_2 - \gamma_1)(\frac{x}{\hat{x}})^{\gamma_1+1}(\frac{1}{\hat{x}})[(1+\eta)\hat{x} - \gamma_1(\hat{x} - I + \eta(\hat{x} - I - r))] \right)$$

which is strictly positive for any $\hat{x} < \hat{x}^{NE}$. Thus, $\frac{\partial x}{\partial \hat{x}} > 0$, so $\overline{x}^{NE} > \overline{x}^{SE}$ for any fixed r. Moreover, when the goal is endogenous, the naif's goal is greater than the sophisticate's ($r^{NI} > r^{SI}$). Since we have already shown that the threshold is increasing in r, then $\overline{x}^{NI} > \overline{x}^{SI}$ when $r = \hat{x} - I$ for each type of agent. ■

A.6 Proof of Proposition 5

In Proposition 3, I showed that the sophisticate's first-best is achieved by stopping at the threshold $x^* = (\frac{\gamma_1}{\gamma_1 - 1})I$, so that the first-best ex ante value of the option (v^{S^*}) equals v^*. Given that the naif sets the goal $\hat{x}^{NI} - I$ where $\hat{x}^{NI} = (\frac{\gamma_1}{\gamma_1 - 1 - \eta})I$, his true ex-ante option value of waiting is given by Equation (29). To find the threshold $\overline{x}^N$ that would maximize $\tilde{v}^N$, we have the first order condition:

$$\begin{aligned}\frac{\partial \tilde{v}^N}{\partial \overline{x}^N} = 0 &= (x)^{\gamma_1} \left((1+\eta)(\frac{1}{\overline{x}^N})^{\gamma_1} - \gamma_1(\frac{1}{\overline{x}^N})^{\gamma_1+1}[\overline{x}^N - I + \eta(\overline{x}^N - \hat{x}^{NI})] \right) \\ &= (x)^{\gamma_1}(\frac{1}{\overline{x}^N})^{\gamma_1+1}(1+\eta)(\gamma_1 - 1)(\hat{x}^{NI} - \overline{x}^N),\end{aligned}$$

which holds when $\overline{x}^{NI} = \hat{x}^N$. Thus, the upper bound on the naif's option value of waiting is given by $\tilde{v}^N(x) = (\hat{x}^{NI} - I)(\frac{x}{\hat{x}^{NI}})^{\gamma_1}$. But we have shown that the option value of waiting when there is zero comparative utility is maximized at x^*. Since $\hat{x}^{NI} > x^*$, then $\tilde{v}^{N^*}(x) < v^{S^*}(x)$.

A.7 Verification

To verify that the constructed current value function w is optimal for any given $r \geq 0$, note that Equation (7) implies that any solution must satisfy the following two conditions for all $x \in (0, \infty)$, whether the agent is sophisticated or naive:

$$w(x, r) \geq x - I + \eta(x - I - r) \tag{46}$$

$$0 \geq -\rho w(x, r) + \lambda(\beta v(x, \hat{r}) - w(x, r)) + \mu x(\frac{\partial w}{\partial x}) + \frac{1}{2}\sigma^2 x^2(\frac{\partial^2 w}{\partial x^2}). \tag{47}$$

Let $\overline{x}$ denote current self's stopping threshold and $\hat{x}$ denote the (perceived) future self's stopping threshold. By construction, $w(x, r) = x - I + \eta(x - I - r)$ when $x \geq \overline{x}$ so equation

(46) holds with equality. When $x < \overline{x}$, $w(x,r)$ is of the form $w(x,r) = A_1 x^{\gamma_1} + A_2 x^{\gamma_2}$, where $\gamma_2 \geq \gamma_1 > 1$, $A_1 > 0$, and $A_2 > 0$. Since $w(x,r)$ is convex and increasing, it must lie above the line $x - I + \eta(x - I - r)$ for all $x < \overline{x}$.

Whether the agent is sophisticated or naive, Equation (47) holds with equality when $x < \overline{x}$ by construction. Define the function $J(x)$ as follows:

$$J(x) = -\rho w(x,r) + \lambda(\beta v(x,\hat{r}) - w(x,r)) + \mu x(\frac{\partial w}{\partial x}) + \frac{1}{2}\sigma^2 x^2 (\frac{\partial^2 w}{\partial x^2})$$

When $x \geq \overline{x}$, we have $w(x,r) = x - I + \eta(x - I - r)$. Since we have shown that $\overline{x} \leq \hat{x}$ (with equality only if the individual is sophisticated), then $v(x,r) = x - I + \eta(x - I - r)$ if $x \geq \overline{x}$. Then we have

$$\begin{aligned} J(x) &= -\rho[x - I + \eta(x - I - r)] + \lambda\left(\beta[x - I + \eta(x - I - r)] - [x - I + \eta(x - I - r)]\right) + \mu x(1+\eta) \\ &= (1+\eta)[\mu - \rho - \lambda(1-\beta)]x + [\rho + \lambda(1-\beta)][(1+\eta)I + \eta r], \end{aligned}$$

which is strictly decreasing in x since $\mu < \rho$. We have previously shown that $\frac{\partial \overline{x}}{\partial \hat{x}} > 0$, and recall that $\hat{x}^{SE} < \hat{x}^{NE}$ and $\overline{x} = \hat{x}^{SE}$ when $\hat{x} = \hat{x}^{SE}$. So it is sufficient to show that $J(\hat{x}^{SE}) < 0$ to satisfy Equation (47).

$$\begin{aligned} J(x) &\leq J(\hat{x}^S) \\ &= [(1+\eta)I + r\eta](\frac{1}{\overline{\gamma} - 1})[\overline{\gamma}\mu - \rho - \lambda(1-\beta)] \\ &= [(1+\eta)I + r\eta](\frac{1}{\overline{\gamma} - 1})[\beta(\mu\gamma_1 - \rho) + (1-\beta)(\mu\gamma_2 - \rho - \lambda)]. \end{aligned}$$

Recall that $\gamma_1 > 1$ satisfies $0 = -\rho + \mu\gamma_1 + \frac{1}{2}\sigma^2\gamma_1(\gamma_1 - 1)$. Then $\mu\gamma_1 - \rho = -\frac{1}{2}\sigma^2\gamma_1(\gamma_1 - 1) < 0$. Likewise, $\gamma_2 > 1$ satisfies $0 = -(\rho+\lambda) + \mu\gamma_2 + \frac{1}{2}\sigma^2\gamma_2(\gamma_2 - 1)$, so $\mu\gamma_2 - \rho - \lambda = -\frac{1}{2}\sigma^2\gamma_2(\gamma_2 - 1) < 0$. Thus, $J(x) \leq J(\hat{x}^S) < 0$. Since $\overline{x}^{SE} < \overline{x}^{NE}$, then Equation (47) is also satisfied when the agent is naive. Therefore, the constructed value function w is at least as good as the value function generated by any alternative Markov strategy.

A.8 Proof of Proposition 7

Since the linear comparison utility function is a special case of the general comparison utility function $\psi(\cdot)$, Proposition 6 is a special case of Proposition 7. Choosing a goal to maximize total utility from an ex-ante perspective is equivalent to self 0's problem of choosing $\hat{r}$ that maximizes $v(x,\hat{r})$ in its wait region described by Equation (11), where he anticipates future threshold $\hat{x}(\hat{r})$. When the comparison utility function is linear, then $\hat{x} = \overline{x}^{SE}(\gamma_1)$.

Consider a comparison utility function given by $\psi(x - I - r)$, where $\psi(\cdot)$ exhibits the following properties:

A1. $\psi(y)$ is continuous and thrice differentiable for all y.

A2. $\psi(0) = 0$.

A3. $\psi(y)$ is increasing everywhere: $\psi'(y) \geq 0, \ \forall y \in \mathbb{R}$.

A4. $\psi(y)$ is concave: $\psi''(y) \leq 0, \ \forall y \in \mathbb{R}$.

A5. $\psi(y)''' \geq 0, \ \forall y \in \mathbb{R}$.

The case studied thus far, in which the comparison utility function is linear, corresponds to the case in which $\psi'(y) = \eta$ and $\psi''(y) = 0$ for all $y \in \mathbb{R}$. A1-A3 are standard and intuitive features of comparison utility. A4 implies that when $\psi''(y) < 0$ everywhere, the agent is diminishingly sensitive to gains and increasingly sensitive to losses. While the former is a feature of Kahneman and Tversky's (1979) value function, the latter property is less commonly assumed. Given that the effects of time discounting in the optimal stopping problem bias the agent toward setting goals that are *lower* than anticipated (and realized) outcomes, the key region of interest is that of *gains*, and its feature is consistent with previous work.[46] When $\psi''(y) < 0$ everywhere, comparison utility is asymmetric across gains and losses, and the agent exhibits loss aversion. We also can obtain an exact mapping to $\psi(\cdot)$ through an alternative model. A formally equivalent specification is that the comparison utility function is linear, but the agent faces convex costs, incurred upon stopping, of setting a goal that deviates from his actual net terminal payoff, whether that goal is below or above his realized outcome.[47] A5 is a regularity condition.

When $\psi(\cdot)$ is non-linear, we must derive the optimal stopping threshold, which, as in the linear case, is determined by joining the waiting and stopping regions of the value function. It is only the stopping region that now changes, since the comparison of an outcome against the goal upon stopping differs. Then the optimal stopping threshold $\overline{x}^{NL}$ given a goal r is given by the following implicit function:

$$0 = (\overline{\gamma} - 1)\overline{x}^{NL} - \overline{\gamma} I + \overline{\gamma}\psi(\overline{x}^{NL} - I - r) - \overline{x}^{NL}\psi'(\overline{x}^{NL} - I - r), \quad (48)$$

[46]Moreover, the assumption of increasing sensitivity to losses can be interpreted as a regularity condition, to ensure that the continuation value function can be single-peaked, but is not strictly necessary to obtain a positive optimal goal.

[47]Costs that rise more quickly with the gap between the goal and the expected payoff thus correspond to a $\psi(\cdot)$ that is more concave.

where we can verify that, unsurprisingly, $\overline{x}^{NL}$ is increasing in r and decreasing in $\overline{\gamma}$ given assumptions A1-A4. As in the linear case, the optimal aspirational goal r^* is chosen to maximize the continuation value function $v(x, \hat{r})$ in its wait region, where the comparison utility is now evaluated non-linearly according to $\psi(\cdot)$ and $\overline{x}^{NL}(\hat{r})$ is given by Equation (48):

$$r^* = \underset{\hat{r}}{\operatorname{argmax}}\, v(x, \hat{r}) = \underset{\hat{r}}{\operatorname{argmax}}\ [\overline{x}^{NL} - I + \psi(\overline{x}^{NL} - I - \hat{r})](\frac{x}{\overline{x}^{NL}})^{\gamma_1}. \tag{49}$$

An optimal, non-degenerate aspirational goal (i.e. $r^* > -I$) must satisfy the first-order condition, which is equivalent to finding the r^* such that

$$\begin{aligned} &\overline{x}^{NL}\psi''(\overline{x}^{NL} - I - r^*)[-\overline{\gamma} + \gamma_1(1 + \psi'(\overline{x}^{NL} - I - r^*)] \\ &- \overline{\gamma}(\gamma_1 - 1)\psi'(\overline{x}^{NL} - I - r^*)(1 + \psi'(\overline{x}^{NL} - I - r^*)) = 0, \end{aligned} \tag{50}$$

where $\overline{x}^{NL}(r^*)$ is given by (48). The second term of (50) is non-positive for all values of r^* by A3. Since $\psi(\cdot)$ is concave, the first term is only positive if $\overline{\gamma} > \gamma_1(1 + \psi'(\overline{x}^{NL} - I - r^*))$, where the right-hand side is greater than γ_1. Unsurprisingly, setting a non-generate goal is only desirable if it is needed to correct impulsiveness, which must be sufficiently high to counteract the marginal benefit of setting a lower goal.[48] Moreover, the first term of (50) must be sufficiently positive for the equality to hold, which implies that $\psi''(\cdot)$ must be sufficiently negative. The incentive to set a very low goal diminishes when the marginal benefit of exceeding it decreases sufficiently quickly.

To verify that r^* is a maximum, we need to check the second-order condition. The first derivative can be written as

$$\frac{\partial v}{\partial r} = (\frac{x}{\overline{x}^{NL}})^{\gamma_1}\left(\overline{x}^{NL}[\frac{\partial \overline{x}^{NL}}{\partial r}(1 + \psi') - \psi'] - \gamma_1(\frac{\partial \overline{x}^{NL}}{\partial r})(\overline{x}^{NL} - I + \psi)\right). \tag{51}$$

Evaluated at r^*, the sign of $\frac{\partial v^2}{\partial^2 r}$ is given by the sign of the second term of (51). Given that Equation (50) must hold at r^*, the following must hold at r^* if it exists:

$$\frac{\partial \overline{x}^{NL}}{\partial r}|_{r=r^*} = (\frac{\partial \psi'(\overline{x}^{NL} - I - r^*)}{1 + \psi'(\overline{x}^{NL} - I - r^*)})(\frac{\overline{\gamma}}{\overline{\gamma} - \gamma_1}) \tag{52}$$

$$\overline{x}^{NL}(1 + \psi'(\overline{x}^{NL} - I - r^*)) = \overline{\gamma}[\overline{x}^{NL} - I + \psi(\overline{x}^{NL} - I - r^*)]. \tag{53}$$

Note that $\overline{\gamma} > \gamma_1$ at r^*. Using these two facts and Equation (50), we need $\frac{\partial v^2}{\partial^2 r}(r^*) < 0$, where

[48] Note that this condition has a clear analog to the case in which goals are expectations over outcomes. As described in Proposition 3, a given degree of reference dependence is only welfare improving if impulsiveness is sufficiently severe relative to marginal comparison utility.

I suppress the argument for $\psi(\cdot)$ for brevity:

$$\frac{\partial v^2}{\partial^2 r}|_{r=r^*} = -\frac{\overline{\gamma}(\gamma_1 - 1)\psi'}{(\overline{\gamma} - \gamma_1)(1 + \psi')} + \frac{\partial^2 \overline{x}^{NL}}{\partial r^2}|_{r=r^*}[(\overline{\gamma} - \gamma_1)(\overline{x}^{NL} - I + \psi)] < 0,$$

where the first term is negative and the second is positive. Evaluating $\frac{\partial^2 \overline{x}^{NL}}{\partial r^2}|_{r=r^*}$, the second-order condition is satisfied when the following upper bound on $\psi'''(\overline{x}^{NL} - I - r^*)$ holds:

$$\psi'''(\overline{x}^{NL} - I - r^*)K(r^*) < L(r^*)M(r^*), \tag{54}$$

where

$$\begin{aligned}
K(r^*) &= \overline{x}^{NL}\left(\frac{\overline{x}^{NL} - I - \psi}{\overline{\gamma} - 1 - \psi'}\right)\left(\frac{[\overline{\gamma} - \gamma_1(1 + \psi')]^3}{(\overline{\gamma} - \gamma_1)^2(1 + \psi')^3}\right), \\
L(r^*) &= \frac{\overline{\gamma}(\gamma_1 - 1)\psi'}{(\overline{\gamma} - \gamma_1)^2(1 + \psi')(\overline{\gamma} - 1 - \psi')}, \\
M(r^*) &= (\overline{\gamma} - \gamma_1)(\overline{\gamma} - 1 - \psi') - [\overline{\gamma} - \gamma_1(1 + \psi')[2\psi' + \overline{\gamma} - \gamma_1(1 + \psi')].
\end{aligned}$$

We can verify that $K(r^*), L(r^*)$, and $M(r^*)$ are positive, so the upper bound on $\psi'''(\overline{x}^{NL} - I - r^*)$ is positive. Given properties A3, A4, and A5, a $\psi(\cdot)$ satisfying the second-order condition exists. Thus, an optimal $r^* > -I$ exists when the Equations (50) and (54) are satisfied.

Thus, comparison utility must diminish sufficiently quickly in gains for the agent to prefer setting a non-degenerate aspirational goal. If either of these conditions is not satisfied at a given goal r, then the left-hand side of (50) is negative and the optimal aspirational goal is the lowest possible. Note that when $\psi(\cdot)$ is linear, Equation (50) cannot hold and the left-hand side is negative, implying that the optimal aspirational goal is the lowest possible, as stated in Proposition (6).

Given these findings, it is evident that setting an optimal aspirational goal that is realistic ($r^* = \overline{x}^{NL} - I$) is simply a special case in which the agent sets an aspiration goal that coincides with his outcome. But it is only ex-ante optimal when the conditions described in Proposition (7) are met when $\overline{x}^{NL}(r^*) - I - r^* = 0$. In particular, the marginal benefit of setting a goal slightly lower than is achievable must be diminishing sufficiently quickly at the origin, implying that he must be sufficiently loss averse over small stakes in order to set a realistic goal.

A.8.1 Comparative Statics for $\overline{x}^{NL}$

Using the implicit function theorem and suppressing the argument for ψ for brevity, we have that $\frac{\partial \overline{x}}{\partial r}$ is:

$$\frac{\partial \overline{x}}{\partial r} = \frac{\overline{\gamma}\psi'(\overline{x}-I-r) - \overline{x}\psi''(\overline{x}-I-r)}{(\overline{\gamma}-1)[1+\psi'(\overline{x}-I-r)] - \overline{x}\psi''(\overline{x}-I-r)}, \tag{55}$$

which is positive whenever $\psi''(y) \leq 0$, so $\frac{\partial \overline{x}}{\partial r} > 0$ for all $r \in [-I, \infty)$. We can also verify that $\frac{\partial \overline{x}}{\partial r} < 1$. The implicit function theorem also yields $\frac{\partial \overline{x}}{\partial \overline{\gamma}}$:

$$\frac{\partial \overline{x}}{\partial \overline{\gamma}} = -\frac{\overline{x} - I + \psi(\overline{x}-I-r)}{(\overline{\gamma}-1)[1+\psi'(\overline{x}-I-r)] - \overline{x}\psi''(\overline{x}-I-r)}, \tag{56}$$

which is clearly negative since $\overline{x} - I + \psi(\overline{x}-I-r) \geq 0$ (otherwise, the agent would be better off never stopping).

The optimal threshold $\overline{x}^{NL}$ is given by the implicit function described by Equation (48). For $\overline{x}$ to be defined and positive, I assume that $\psi'(\overline{x}-I-r) < \overline{\gamma}-1$ for all $\overline{x}-I-r \in [-I, \infty)$. To see that $I - \psi(\overline{x}^{NL} - I - r) > 0$, suppose that it is negative, implying that $\overline{x}^{NL} < 0$. But if $\overline{x}^{NL} < 0$, then $\psi(\overline{x}^{NL} - I - r) < 0$ because $\psi(0) = 0$ and $\psi'(y) \geq 0$ for all y. This implies that $I - \psi(\overline{x}^{NL} - I - r) > 0$, since $I > 0$, which is a contradiction. Note that $\overline{x}^{NL}$ is equivalent to Equation (20) when $\psi(\cdot)$ is linear.

Differentiating Equation (11) and suppressing the argument for $\psi(\cdot)$ for brevity we obtain:

$$\frac{\partial v}{\partial r} = [\frac{1}{(\overline{\gamma}-1)(1+\psi') - \overline{x}\psi''}][\frac{I-\psi}{\overline{\gamma}-1-\psi'}](\frac{x}{\overline{x}})^{\gamma_1}\{\overline{x}\psi''[-\overline{\gamma}+\gamma_1(1+\psi')] - \overline{\gamma}(\gamma_1-1)\psi'(1+\psi')\}.$$

The first three terms are positive given our regularity conditions, so the sign of $\frac{\partial v}{\partial r}$ is determined by the last term, which is Equation (50). If $\psi(y)$ is linear, i.e. $\psi''(y) = 0$ for all $y \in \mathbb{R}$, then $\frac{\partial v}{\partial r} < 0$ so the optimal r^* is the lowest value of r possible.

A.9 Proof of Proposition 8

To show the first part, we can apply the implicit function theorem to Equation (50). Since the second-order condition must be satisfied at the optimal r^*, it is sufficient to sign the partial derivative of (50) with respect to $\overline{\gamma}$ at r^*, which is positive only if $\frac{\partial^2 \overline{x}^{NL}}{\partial r \partial \overline{\gamma}}$ is sufficiently high.[49] This implies that $\psi'''(\overline{x}^{NL} - I - r^*)$ must be sufficiently high:

$$\psi'''(\overline{x}^{NL} - I - r^*)P(r^*) > Q(r^*) + S(r^*) + U(r^*), \tag{57}$$

[49] Algebraic details are omitted for brevity.

where

$$
\begin{aligned}
P(r^*) &= \frac{[\overline{x}^{NL}(\overline{\gamma}-\gamma_1(1+\psi'))]^3}{\overline{\gamma}^2(\overline{\gamma}-\gamma_1)^2(1+\psi')(\overline{\gamma}-1-\psi')^2} > 0,\\
Q(r^*) &= \frac{\overline{x}^{NL}(\gamma_1-1)[-\overline{\gamma}\psi'-\gamma_1(1+f')]\psi'}{\overline{\gamma}(\overline{\gamma}-\gamma_1)(\overline{\gamma}-1-\psi')} < 0,\\
S(r^*) &= -\frac{\overline{x}^{NL}(\overline{\gamma}-1)(\gamma_1-1)[\overline{\gamma}-\gamma_1(1+\psi')]^2\psi'}{\overline{\gamma}(\overline{\gamma}-\gamma_1)^2(\overline{\gamma}-1-\psi')^2} < 0,\\
U(r^*) &= -\frac{\overline{\gamma}^2(\gamma_1-1)(\psi')^2}{(\overline{\gamma}-\gamma_1)[\overline{\gamma}-\gamma_1(1+\psi')]} < 0.
\end{aligned}
$$

Thus, given that r^* exists, $\frac{\partial\overline{x}^{NL}}{\partial\overline{\gamma}} > 0$ if $\psi'''(\overline{x}^{NL}-I-r^*)$ exceeds a negative lower bound. Given that $\psi(\cdot)$ must satisfy A5, such a $\psi(\cdot)$ exists satisfying both Equations (54) and (57).

To show the second part, we can apply the implicit function theorem to Equation (53):

$$
\frac{\partial\overline{x}^{NL}}{\partial\overline{\gamma}} = -\frac{-[\overline{x}^{NL}-I+\psi(\overline{x}^{NL}-I-r^*)]}{-(\overline{\gamma}-1)(1+\psi'(\overline{x}^{NL}-I-r^*))+\overline{x}^{NL}\psi''(\overline{x}^{NL}-I-r^*)}.
$$

The numerator is strictly negative by the optimality of $\overline{x}^{NL}$. The denominator is strictly negative since $\overline{\gamma} > 1$ and $\psi'(y) \geq 0$ and $\psi''(y) \leq 0$ for all $y \in \mathbb{R}$. Thus, $\frac{\partial\overline{x}^{NL}}{\partial\overline{\gamma}} < 0$.

A.10 Proof of Proposition 9

The equilibrium defined by agents i and j's optimal threshold functions, given by Equations (30) and (31), is

$$
\overline{x}_i = \frac{\overline{\gamma}_i I[\eta\overline{\gamma}_j+(1+\eta)(\overline{\gamma}_j-1)]}{(1+\eta)^2(\overline{\gamma}_i-1)(\overline{\gamma}_j-1)-\eta^2\overline{\gamma}_i\overline{\gamma}_j} \tag{58}
$$

$$
\overline{x}_j = \frac{\overline{\gamma}_j I[\eta\overline{\gamma}_i+(1+\eta)(\overline{\gamma}_i-1)]}{(1+\eta)^2(\overline{\gamma}_i-1)(\overline{\gamma}_j-1)-\eta^2\overline{\gamma}_i\overline{\gamma}_j}, \tag{59}
$$

where $\overline{\gamma}_k = \beta_k\gamma_1+(1-\beta_k)\gamma_2$ for $k = i, j$. The first two parts of Proposition 9 are obtained by differentiating the equilibrium thresholds (58) and (59) directly. In particular, note that

$$
\begin{aligned}
\lim_{\overline{\gamma}_j\to\infty}\overline{x}_i &= \left(\frac{(1+2\eta)\overline{\gamma}_i}{(1+2\eta)\overline{\gamma}_i-(1+\eta)^2}\right)I \geq \left(\frac{\overline{\gamma}_i}{\overline{\gamma}_i-1}\right)I\\
\lim_{\overline{\gamma}_j\to\infty}\overline{x}_j &= \left(\frac{(1+2\eta)\overline{\gamma}_i-(1+\eta)^2}{(1+2\eta)\overline{\gamma}_i-(1+\eta)}\right)I \geq I,
\end{aligned}
$$

with inequality only if $\overline{\gamma}_i \to \infty$ as well. This implies that as long as $\overline{\gamma}_i$ is finite, both agents behave more patiently than they would in the absence of a goal.

The third part of the proposition follows by noting that

$$\overline{x}_i - \overline{x}_j = \frac{(\overline{\gamma}_j - \overline{\gamma}_i)(1+\eta)I}{(1+\eta)^2(\overline{\gamma}_i - 1)(\overline{\gamma}_j - 1) - \eta^2\overline{\gamma}_i\overline{\gamma}_j},$$

so $\overline{x}_i - \overline{x}_j > 0$ whenever $\overline{\gamma}_j - \overline{\gamma}_i > 0$.

A.11 Proof of Proposition 10

In an interpersonal equilibrium, agent i's continuation value function is given by

$$v_i(x, r_i = \overline{x}_j - I) = \begin{cases} [\overline{x}_i - I + \eta(\overline{x}_i - \overline{x}_j)](\frac{x}{\overline{x}_i})^{\gamma_1} & \text{if} \quad x < \overline{x}_i \\ x - I + \eta(x - \overline{x}_j) & \text{if} \quad x \geq \overline{x}_i, \end{cases}$$

where $\overline{x}_i$ and $\overline{x}_j$ are given by (58) and (59), respectively. To find the peer j^* who maximizes ex ante welfare, we can find the γ_j^* such that the value of v_i in its wait region is maximized given equilibrium behavior. The first order condition is

$$\frac{\partial v_i}{\partial \overline{\gamma}_j} = (\frac{x}{\overline{x}_i})^{\gamma_1}\Big(\frac{\partial \overline{x}_i}{\partial \overline{\gamma}_j} + \eta(\frac{\partial \overline{x}_i}{\partial \overline{\gamma}_j} - \frac{\partial \overline{x}_j}{\partial \overline{\gamma}_j}) - (\frac{\gamma_1}{\overline{x}_i})(\frac{\partial \overline{x}_i}{\partial \overline{\gamma}_j})[\overline{x}_i - I + \eta(\overline{x}_i - \overline{x}_j)]\Big).$$

After making the appropriate substitutions and simplifying, the first order condition is of the following form:

$$\frac{\partial v_i}{\partial \overline{\gamma}_j} = (\frac{x}{\overline{x}_i})^{\gamma_1}\frac{A}{B(\overline{\gamma}_j)}[C + \overline{\gamma}_j D],$$

where

$$\begin{aligned} A &= \eta(1+\eta)^2(\gamma_1 - 1)[(\overline{\gamma}_i - 1)(1+\eta) + \eta\overline{\gamma}_i]I \\ B(\overline{\gamma}_j) &= \Big(\frac{1}{(\overline{\gamma}_j - 1)(1+\eta) + \eta\overline{\gamma}_j}\Big)\Big(\frac{1}{(1+\eta)^2(\overline{\gamma}_i - 1)(\overline{\gamma}_j - 1) - \eta^2\overline{\gamma}_i\overline{\gamma}_j}\Big)^2 \\ C &= -(1+\eta) \\ D &= (1+2\eta). \end{aligned}$$

Since $A > 0$, $B(\delta) > 0$, $C < 0$, and $D > 0$, it is clear that v_i is an asymmetric function of $\overline{\gamma}_j$, with a unique minimum at $\hat{\gamma}_j < 0$ such that $\frac{\partial v_i}{\partial \overline{\gamma}_j}(\hat{\gamma}_j) = 0$. Since $\hat{\gamma}_j = \frac{1+\eta}{1+2\eta} < 1$, then v_i is monotonically increasing for all $\overline{\gamma}_j \in (1, \infty)$. Hence, the value function is maximized as $\overline{\gamma}_j \to \infty$. Since $\overline{\gamma}_j = \beta_j\gamma_1 + (1-\beta_j)\gamma_2$, this is equivalent to desiring a peer such that $\beta_j^* < 1$ and $\lambda_j^* \to \infty$.