A Stochastic Index of the Cost of Life; An Application to Recent and Historical Asset Price Fluctuations

Michael F. Bryan, Stephen G. Cecchetti, and Róisín O’Sullivan*

Introduction

This paper considers the role of asset prices in the construction of measures of inflation. Following work begun in Bryan, Cecchetti, and O’Sullivan (2001), we examine the potential bias in aggregate price statistics that can arise in measures that focus solely on the price of current consumption. If the role of the central bank is to eliminate changes in the overall price level, then the appropriate price index should capture the current cost of purchasing claims to future as well as current consumption. One interpretation of this is that policymakers should be concerned with the current cost of expected lifetime consumption rather than the more conventional measures that focus exclusively on the current cost of current consumption. The simplest way to think about this is that changes in the real interest rate affect the current price of future consumption. When the real interest rate falls, the cost of lifetime consumption rises, and this is inflationary. Ignoring future consumption in the computation of inflation is analogous to leaving out a good that belongs in the index and creates the potential for what we will call “intertemporal substitution bias”. Since asset prices measure the current cost of future consumption, including them in a measure of inflation potentially allows for this real interest rate impact and eliminates this bias. We call this a “cost-of-life index”.

Following the methodology in our earlier work, we are able to introduce a number of candidate asset prices into our cost-of-life index. This allows us to estimate the size of the intertemporal substitution bias, both on a year-by-year basis and on average over decades. We find that the bias from omitting asset prices in the measurement of inflation is roughly the same size as that associated with other well-known biases, including those that arise from commodity substitution and the like. We also show that this bias is time-varying and over periods of several years may significantly distort the magnitude and timing of how the central bank perceives inflation. Specifically, intertemporal substitution bias between current consumption and future consumption tended to understate the inflation being recorded by current-consumption price statistics such as the Consumer Price Index (CPI) during the latter half of the 1990s. This reversed the upward bias to the retail price measure that existed over much of the 1977 to 1995 period.

During some periods this bias can be extreme. Such appears to have been the case between the two world wars of the last century when the relatively low real interest rate in the mid-1920s suddenly rose. Our measure of inflation is between 1 and 2 percentage points higher than conventional (consumption-based) indicators suggested during the 1920s — and similarly more deflationary during the early years of the 1930s.

The organization of this paper is as follows. Section I describes how it is that ignoring asset prices introduces a bias into aggregate price statistics. In section II, we introduce the weighting methodology used to incorporate “excessively noisy” asset prices so as to produce a price index

* Federal Reserve Bank of Cleveland, The Ohio State University and NBER, and the Ohio State University and Smith College, respectively. The views expressed in this paper do not necessarily reflect the views of the Federal Reserve Bank of Cleveland, the Federal Reserve System, or the NBER.
for the current “cost of life” that eliminates the intertemporal substitution bias. Section III reports on the behavior of this index relative to the more common cost-of-living statistics used by economists and policymakers. In section IV we show how such a price measure would have changed our interpretation of price movements during the great deflation between the two World Wars. Section V concludes.

Asset Prices and the Cost of Life: A Case of Missing Goods Bias

Which goods should be used in the computation of a price statistic? This is one of the oldest questions in quantitative macroeconomics, and not surprisingly, the answer depends on the question the statistic is intended to answer. Whether or not to include asset prices in the computation of aggregate inflation depends on what relevant information they contain for the question at hand. Asset prices are essential in any calculation that incorporates intertemporal consumption decisions since they measure today’s cost of future consumption. Households allocate current income to both current and future consumption. In fact, theory suggests that current decisions by economic agents attempt to insure a particular level of lifetime consumption. This means that the appropriate price statistic—both for households’ decisions and policymakers—must take account of changes in the cost of current relative to future consumption. Ignoring changes in this relative price (changes in the real interest rate) will create a bias in price measurement that is exactly analogous to what is commonly called “omitted goods bias”.

This issue is especially important to a central bank that has among its objectives the management of inflation. The link connecting central bank policy to inflation is often presumed to be “long and variable,” as changes in target interest rates and central bank money affect prices slowly and unpredictably. Since these policies alter the real interest rate, causing intertemporal substitution in consumption, the central bank may be missing an important part of the inflationary process by focusing exclusively on the drift in current retail (consumption) prices.

Following the arguments first set out by Alchian and Klein (1973), this leads us to consider a price statistic that is designed to gauge the current (intertemporal) cost of life— that is—a measure that takes account of the current cost of both current and future consumption. The cost of life is distinct from the current cost of living that provides the conceptual basis for statistics such as the CPI or the deflator for personal consumption expenditures. Those measures are designed to capture the cost of current consumption and intentionally omit current prices of future consumption. Therefore, there exists a relative price change such that its exclusion from the set of prices distorts the aggregate so that it no longer gives an accurate answer to the question under consideration.

To illustrate this point, we can think of price changes for all goods, services, and assets today as having a common and idiosyncratic component. We write this as

\[ \pi_{it} = \pi_t + x_{it}, \]  

(1)

where \( i \) indicates the set of goods, services, or assets, and \( t \) is time, and so \( \pi_{it} \) is the inflation of individual good \( i \) at time \( t \), \( \pi_t \) is the common trend in inflation that we are trying to measure, and \( x_{it} \) is the deviation of good \( i \) inflation at time \( t \) from this trend. An inflation index is constructed by weighting together these individual inflation measures. That is,

\[ \hat{P}_t = \sum_i w_i \pi_{it} \]  

(2)
where the weights \( w_{it} \) can change over time, but have the property that they sum to one. That is
\[
\sum_t w_{it} = 1 \forall t
\]  (3)

Using this fact, we can now rewrite the aggregate price change as
\[
\hat{P}_t = \pi_t + \sum_i w_{it} x_{it}
\]  (4)

Our goal is the measurement of the common trend in all prices (inflation), \( \pi_t \). To do this we need to find a set of goods, services, and assets where the (weighted) relative changes, the \( x_{it} \)'s, cancel out. But the weighted sum of these relative inflation measures will only cancel out if we have a complete set of prices. If a good with a non-zero relative price disturbance \( x_{it} \) is left out, then the sum of the weighted price adjustments will not sum to zero, and so the index \( \hat{P}_t \) will not equal \( \pi_t \), the common inflation trend we are trying to measure. If asset prices are excluded from the cost-of-life statistic, any intertemporal substitution between current and future consumption originating from a change in the real interest rate will create a bias in the resulting measure of inflation. Specifically, during periods when the real interest rate is declining, prices of current consumption fall relative to current claims on future consumption, causing any aggregate price measure based only on current consumption prices to be too low. (The opposite would be true in situations when the real interest rate has risen.)

**Estimating Changes in the Cost of Life**

Using consumer theory, Pollack (1975) and Shibuya (1992) derive and implement a cost-of-life index that weights traditional consumer prices together with various asset price measures. We have argued elsewhere that such a methodology yields price measures that are implausibly volatile over short horizons.\(^1\) In order to create an index that is potentially useful in a high-frequency decision-making environment such as setting monetary policy, we follow the methodology first proposed in Bryan and Cecchetti (1993). In that work, we utilize the joint statistical properties of the individual price series in order to construct a dynamic factor index (DFI). The DFI exploits the fact that the information contained in individual \( \pi_{it} \)'s about the common trend \( \pi_t \) differs – that is the “signal-to-noise” ratio varies across different sets of goods, services, and assets. In particular, asset prices tend to be quite noisy, and so may not be very important in constructing the DFI. As in Bryan and Cecchetti (1993), we write the model as
\[
p_{it} = \pi_t + x_{it}
\]  (5)
\[
\psi(L) \pi_t = \delta + \xi_t
\]  (6)
\[
\theta_i(L) x_{it} = \eta_{it}
\]  (7)

where \( p_{it} \), \( \pi_t \), and \( x_{it} \) are the first differences of the logs of the observed variables, the common unobserved component representing inflation and the idiosyncratic relative price movement in the \( i \)th series, respectively. \( \psi(L) \) and \( \theta_i(L) \) are vectors of lag polynomials and \( \xi_t \) and \( \eta_{it} \) are i.i.d.

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\(^1\) See Bryan, Cecchetti and O’Sullivan (2001).
random variables. Throughout, it is assumed that both the common element, $\pi_t$, and the idiosyncratic components, $x_{it}$, can be modeled as AR(2) processes.

The main identifying assumption of the model is that the common component and the idiosyncratic components are mutually uncorrelated at all leads and lags. This is achieved by assuming that $\theta(L)$ is diagonal and that all the error terms in the model are mutually uncorrelated. This is consistent with the notion that the common component captures all the comovement in the observed series, leaving $x_{it}$ to reflect only idiosyncratic movements. To set the scale of $\pi_t$, the variance of $\xi_t$ is normalized to one. The parameters of the model are then estimated via maximum likelihood using the Kalman filter. As a by-product, the Kalman filter recursively constructs MMSE estimates of the unobserved components $\pi_t$ and $x_{it}$ given observations of $p_{it}$. The common index can be written as a linear component of current and past values of the observed series

$$\hat{\pi}_t = \sum_i \hat{w}_i(L) p_{it}$$  (8)

These are the (implicit) weights used to construct the common inflation component.

In an alternative approach to this “signal-extraction” problem, Wynne (2000) describes the implementation of a simple variance-weighted price index where the weights,

$$w_j = \frac{1}{\sigma_i^2}$$

(9)

are based on the variance of the rate of change in the price of each good $i$, $\sigma_i^2$.

A simple variance-weighting scheme of this type is a good indicator of the likely importance of a particular series in the construction of more complex (and difficult to compute) dynamic factor indices. To see why, note that the variance of the “common” element in any scheme, similar to that describe in equation (9) above, will have the property that the estimated inflation index will have variance equal to or less than the variance of the least volatile component used.\(^2\) In the analysis that follows in sections III and IV, we also report the variance weights for comparison to the weights used in our dynamic factor index.

\(^2\) This is strictly true in the DFI model we use given the identification assumptions employed. In the variance weighting case, certain restrictions are necessary on the covariances between the constituent series.
Comparing the Cost-of-Living to the Cost-of-Life

Using nine sub-indices of the current consumer price index, together with prices of six major assets, we estimate a cost of life index for 1977 to 2001. The assets include housing, stocks, bonds, commodities, money, and gold.

Table 1 displays the implied weights for the data set using the alternative methodologies. In the first three columns of the table we report results for the cost-of-living approach—that is, retail prices excluding asset prices. The remaining columns incorporate asset prices and so are the weights for similarly constructed cost-of-life indices. The first observation we make is that even on a conceptually similar basis, dynamic factor weights are significantly different from the expenditure weights used in the official CPI. This is because the Kalman filter technology tends to reduce the weight assigned to price series with a relative high time-series variance, giving more weight to those series that provide a better signal of the common element (the DFI). These weights are oblivious to the relative importance of the good in the typical consumer market basket. The result is an index similar in spirit to a “core” inflation statistic like the CPI excluding food and energy, where the market basket is altered to adjust for transitory fluctuations in the data that are not believed to be part of a more generalized inflation process. Note that the DFI cost-of-living weights (those weights based on CPI component series only) are more closely aligned with the conceptually similar cost-of-living variance weights. That is, food, transportation, and housing are given a substantially smaller share of the overall price index compared with the ordinary expenditure-weighted CPI, while medical care, personal care, and education get larger weights in these reduced-noise price statistics.

Using the DFI procedure, we constructed a cost-of-life index that includes six asset price series along with nine retail price series. We find that stocks (0.8 percent), bonds (0.4 percent), and gold (0.7 percent) are assigned very small weights in the computation of the cost-of-life index. This reflects the extreme volatility of these series at a monthly frequency (also evidenced by the near zero weight these assets are given on the basis of the variance weighting criteria.) Nevertheless, some assets are assigned a relatively large share of the “market basket,” like new homes (2.3 percent), commodities (5.6 percent) and the money supply (5.3 percent).

Turning to the estimated price indices themselves, Figure 1 shows the 12-month growth rates of the three key inflation statistics for the recent era, the CPI, the cost-of-living dynamic factor index (DFI cost-of-living) and the cost-of-life dynamic factor index (DFI cost-of-life). First, as we expect, the two DFI series are considerably smoother than the expenditure-weighted CPI. More importantly for the purposes of this paper, there are persistent differences between the DFI cost-of-living measure and the DFI cost-of-life measure. This is the “bias” between these two conceptually different approaches. Specifically, while the price series including asset prices tended to track below the consumption-only price measure during the 1980-94 period, it tended to be higher between 1995 and 2000, and only very recently have the two measures converged.

The significance of this observation is that a real interest rate drop in the mid-1990s appears to have been responsible for an intertemporal substitution between current consumption and future consumption.3 This suggests that the CPI (or other similarly conceptualized retail price statistics).

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3 In fact, the difference between the DFI cost-of-living and the DFI cost-of-life is an estimate of the change in the ex ante real interest rate. This is supported by the fact that this difference and the change in the ex post real interest rate, computed from the three month U.S. Treasury bill rate and headline CPI inflation, are significantly positively correlated with a correlation coefficient of 0.13 and a robust t-statistic of (2.1).
measures) were biased downward by \( \frac{1}{4} \) to \( \frac{1}{2} \) percentage point per year as measures of inflation compared with the rise in prices recorded by current claims to current and future consumption. Since 1995, the bias we have identified has worked in the opposite direction to the more commonly discussed biases in retail price measures that come from other (but related) sources. But in the first half of our sample, this bias worked to reinforce the belief that measurement biases lead to the overestimation of inflation by conventional indices. Furthermore, the tendency of the intertemporal substitution bias to fluctuate suggests that measures of real interest rates and other nominal, intertemporal phenomena can, at times, be significantly over- or under-stated if the price deflator does not include the price of current purchases of future consumption—assets.

**A Retrospective of the Inter-War Deflation**

We have demonstrated that the failure to include asset prices when calculating a “cost of life” index can create an intertemporal substitution bias in the cost-of-life statistic and that this exclusion may influence one’s interpretation of inflation. In this section, we ask if correcting this bias might lead to a reinterpretation of the events during the 1920’s and 1930’s – the period between the two world wars – and one of the great deflations in U.S. history.

The extreme movements in stock prices and the historical fall in the price level during this period are well documented, and figure 2 illustrates the movements in consumer and stock prices between 1920 and 1937.\(^4\) We computed our DFI measures, with and without asset prices using data series that closely match those used in the previous section.\(^5\) Consumer prices were obtained from Sayre (1948), where series on the prices of food, housing, clothing, fuel and house furnishings and sundries were available along with the overall CPI index. Stock and bond prices and money stock data were taken from the NBER macrohistory database while the commodity price index was available from the BLS.

Two series used in the modern analysis could not be included directly. The first of these – house prices – are simply not available on a monthly basis. As a compromise, we used an index of building materials costs instead. The second series that presented a problem was gold prices. While price data are available, the behavior of gold prices was dictated by the central bank’s gold standard and therefore is not suitable for our purpose. From 1920 to 1934 we were able to substitute the price of silver for the price of gold. After 1934, these data become unusable because of the Silver Purchase Act of 1934, which resulted in significant government intervention in the market for silver.

Table 2 summarizes the weights obtained when the DFI technique is applied to various combinations of the historical series. Variance-based weights and expenditure weights for the components of the CPI are included for comparison purposes.\(^6\) Looking first at the constituent series of the CPI, we see that the category with the largest expenditure weight - food - attracted a significantly smaller weight when both the DFI and variance-based signal extraction techniques were used. In contrast, the relatively stable house furnishings and sundries category, which

\(^4\) See, for example, Friedman and Schwartz (1963).
\(^5\) Reasonable data on bond prices were not available for the period between 1937 and 1940 and so only data up to January 1937 were used. Results based on series excluding bonds showed little difference when the additional four years of data were included.
\(^6\) When silver prices were added for the period up to 1934, the series attracted a small weight (3.37) – the second smallest after stocks.
includes costs associated with such items as medical care, personal care and transportation, was attributed DFI and variance-based weights more than double its expenditure share.

Looking more closely at the DFI cost-of-life index we see that the building costs index – included here as a proxy for house prices – attracted the largest weight among the non-CPI series. This may overstate the strength of the common price signal in house prices, as they tend to be more volatile than building costs in the short run. It is also worth noting that the unprecedented developments in the stock market during this time period resulted in a higher weight being given to bond prices than stock prices. This is in contrast to the outcome obtained using the modern data. Indeed, according to the variance-weighted approach, the stock price series contains virtually no information at all about the common trend in prices.

Still, despite the volatility experienced in asset markets during this period, the DFI approach attributes almost one third of the weight to the non-CPI component series. The thirty-three percent share of DFI weights attributed to asset prices is about twice the share assigned by a variance-weighting approach. In other words, these asset data, though volatile, appear to have an important common signal embedded in their movement, suggesting that their exclusion would have biased the cost-of-life measure compared with the conventional cost-of-living statistic.

Turning to the growth rates of the various inflation series, figure 3 plots the year-to-year growth rates for the headline CPI, the DFI series based on CPI components only (DFI cost-of-living), and the DFI including all ten series (DFI cost-of-life). The growth in the cost-of-living based DFI series is obviously smoother than the headline CPI rate over the period, with this series recording deflation rates in the early 1930’s of about half those obtained using the headline CPI rate. When asset prices are included, however, the deflation was higher than that calculated from the headline rate, while the rebound in the mid-thirties was also more pronounced. That is, the DFI cost-of-life index shows more extreme movements than the conventional cost-of-living index.

As noted for the modern period, the gap between the DFI cost-of-living and the DFI cost-of-life (all assets) series can be interpreted as reflecting movements in the real interest rate. The correlation between the change in the ex ante real interest rate taken from Cecchetti (1992) and the difference between the two DFI series is significantly positive– the correlation coefficient is 0.16 with a robust t-ratio of 2.05. In particular, the greater fall in the cost-of-life DFI series compared with the DFI series without asset prices during the early 1930’s reflected the relatively steep drop in stock prices at that time. Our interpretation is that this fall in asset prices reflected a higher discount rate or real interest rate at a time when nominal interest rates were falling quite dramatically. But more fundamentally, it suggests a strong intertemporal substitution bias in the conventional retail price measure of a rather extreme level (peaking at around 6 percentage points in mid-1932) that substantially alters the inflation pattern we record for the period compared with what is shown by the CPI.

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7 To examine the robustness of the weighting methodology during this turbulent time, DFI and variance-based weights were also calculated for rolling ten-year windows, beginning with 1920-30 and ending with 1927-37. The weights assigned to the various series were relatively stable across the time period windows, with a fall in the weight attached to the money stock reflected mainly in an increase in the house furnishings and sundries weight. That is, the DFI weights appear to be robust throughout this sample.
Conclusion

This paper considers a particular problem associated with the failure to include asset prices in an aggregate price statistic. If the statistic is intended to gauge inflation, meaning a persistent change in the cost of life as distinct from the current cost of living, then the failure to include asset prices in the aggregate price measure can bias your inflation estimate. Using a modified Kalman filter—a dynamic factor index—we compute an aggregate cost-of-life index and compare it with both the CPI (an expenditure-weighted cost-of-living measure) and a methodologically similar dynamic factor index that also measures only the cost-of-living. Over time, we find the differences between these statistics to be somewhat small, often less than \( \frac{1}{4} \) percentage point per year. However, over some periods, namely, periods when the real interest rate fluctuates, we find that the exclusion of asset prices can significantly distort one’s reading of monetary inflation common to all goods. Such appears to have been the case during the great deflation in the early 1930s, and also in the latter half of the last decade.

References


Table 1: Alternative Weights for the Recent Era (1977-2001)

<table>
<thead>
<tr>
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<th>Cost-of-Living Weights (CPI Components only)</th>
<th>Cost-of-Life Weights (CPI components with asset prices)</th>
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<td>DFI  Variance</td>
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<td>Food/Beverages</td>
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<td>4.8  8.0  5.0</td>
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<td>Transportation</td>
<td>18.1  3.8  1.7</td>
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<td>Medical Care</td>
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<td>11.5  5.8</td>
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<td>Gold</td>
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Figure 1: The CPI, DFI Cost-of-Living and DFI Cost-of-Life

12-month percent change

Figure 1a: The Intertemporal Substitution Bias

12-month growth in DFI cost-of-living less DFI cost-of-life
Table 2: Alternative Weights for the Historical Series (1920-1937)

<table>
<thead>
<tr>
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<th>Cost-of-Living Weights (CPI Components only)</th>
<th>Cost-of-Life Weights (CPI components with asset prices)</th>
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Figure 2: Consumer Prices and Stock Prices in the Interwar Period
Figure 3: The CPI, DFI Cost-of-Living, and DFI Cost-of-Life in the Interwar Period

Figure 3a: The Intertemporal Substitution Bias

12-month percent change

12-month growth in DFI cost-of-living less DFI cost-of-life