Things to Know for the Physics GRE

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Some ideas borrowed from review sheets by Marty Ligare and Dave Shoepf of Bucknell University, as well as from Wikipedia.

1 Math

1.1 Coordinate systems

Cartesian to Spherical:
$r = \sqrt{x^2 + y^2 + z^2}$ $\phi = \arctan(y/x)$ $\theta = \arccos(z/\sqrt{x^2 + y^2 + z^2})$
Spherical to Cartesian:
$x = r \cos\phi \sin\theta$ $y = r \sin\phi \sin\theta$ $z = r \cos\theta$

Cylindrical to Cartesian:
$x = \rho \cos\phi$ $y = \rho \sin\phi$ $z = z$
Cartesian to Cylindrical:
$\rho = \sqrt{x^2 + y^2}$ $\phi = \pm \arcsin(y/\rho)$ $z = z$

Cylindrical to Spherical:
$r = \sqrt{\rho^2 + z^2}$ $\theta = \arctan(\rho/z)$ $\phi = \phi$
Spherical to Cylindrical:
$\rho = r \sin\theta$ $\phi = \phi$ $z = r \cos\theta$

Spherical:
$d\vec{l} = dr\hat{r} + r d\theta\hat{\theta} + r \sin\theta d\phi\hat{\phi}$
$d\tau = r^2 \sin\theta dr d\theta d\phi$

Cylindrical:
$d\vec{l} = d\rho\hat{\rho} + \rho d\phi\hat{\phi} + dz\hat{z}$
$d\tau = \rho d\rho d\phi dz$

1.2 Vector Calculus

Laplacian:

$$\nabla^2 T = \vec{\nabla} \cdot \left(\vec{\nabla} T\right) = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}$$

$$\nabla^2\vec{v} = \nabla^2 v_x\hat{x} + \nabla^2 v_y\hat{y} + \nabla^2 v_z\hat{z}$$

$$\vec{\nabla}\times\left(\vec{\nabla}T\right) = 0 \qquad\qquad \vec{\nabla}\cdot\left(\vec{\nabla}\times\vec{v}\right) = 0$$

$$\vec{\nabla}\times\left(\vec{\nabla}\times\vec{v}\right) = \vec{\nabla}\left(\vec{\nabla}\cdot\vec{v}\right) - \nabla^2\vec{v}$$

$$\int_a^b \left(\vec{\nabla}T\right)\cdot d\vec{l} = T(b) - T(a) \text{ Independent of Path. } \oint \left(\vec{\nabla}T\right)\cdot d\vec{l} = 0$$

Stoke's Theorem:

$$\int_{surface} \left(\vec{\nabla}\times\vec{v}\right)\cdot d\vec{a} = \oint_{path} \vec{v}\cdot d\vec{l} \text{ (=0 for closed surface)}$$

Green's (Divergence) Theorem:

$$\int_{volume} \left(\vec{\nabla}\cdot\vec{v}\right) d\tau = \oint_{surface} \vec{v}\cdot d\vec{a}$$

1.3 Miscellaneous

$$(1+x)^n \approx 1 + nx \text{ for } x \ll 1$$

2 Classical mechanics

2.1 Kinematics

$$\text{Force: } \vec{F} = m\vec{a} = \frac{d\vec{p}}{dt}$$

$$\text{Work: } W = \int \vec{F}\cdot d\vec{r}$$

$$\text{Kinetic energy: } T = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

$$\text{Work-Energy Theorem: } W = \Delta T = \frac{1}{2}m\left(v_2^2 - v_1^2\right)$$

$$\text{Power: } \frac{dW}{dt} = \vec{F}\cdot\vec{v}$$

Potential Energy:

$$\text{Gravitational: } PE = mgh = -GmM_{Earth}/r$$

$$\text{Spring: } PE = \frac{1}{2}kx^2$$

$$\text{Conservative force: } \vec{F} = -\nabla U(\vec{r}) \text{ where } U = \text{potential energy}$$

$$\frac{d}{dt}(T+U) = \frac{d}{dt}E = 0$$

$$\text{Impulse: } \vec{I} = \int_{t_1}^{t_2} \vec{F}dt = \Delta\vec{p}$$

2.2 Rotational kinematics

Angular position:

$$\theta = s/r$$

Angular velocity:

$$\omega = \frac{d\theta}{dt} = \frac{v}{r}$$

Angular acceleration:

$$\alpha = \frac{d^2\theta}{dt^2} = \frac{a}{r}$$

Angular momentum: $\vec{L} = \vec{r} \times \vec{p}$ $\qquad L_{body} = I\omega$

Moment of inertia:

$$I = \int r^2 dm$$

$$I = kMR^2$$

where k is given for various objects on the formulas page at the front of the test.

Parallel axis theorem:

$$I_{anywhere} = I_{center} + MR^2$$

at a point a distance R away from the center.

Radius of gyration (about an axis):

$$R_g = \sqrt{I/M}$$

Torque:

$$\vec{\tau} = \frac{d\vec{L}}{dt} = \vec{r} \times \vec{F} = I\alpha$$

Kinetic energy:

$$K = \frac{1}{2} I\omega^2$$

Centripetal acceleration:

$$|\vec{a}_C| = v^2/r$$

Tangential acceleration:

$$|\vec{a}_T| = \frac{d\,|\vec{v}|}{dt}$$

Circular motion:
Period $T = 2\pi r/v$
Centripetal acceleration:

$$|\vec{a}| = \left(\frac{2\pi r}{T}\right)^2 /r = 4\pi^2 r/T^2$$

2.3 Waves and Oscillations

$f(x, t) = A\sin{(kx \pm \omega t)}$
$k = 2\pi/\lambda \qquad \omega = 2\pi/T = 2\pi f$
Phase velocity: $\omega_{ph} = \omega/k$
Group velocity: $\omega_g = \frac{\partial\omega}{\partial k}$
$\omega = \sqrt{k/m} \longleftrightarrow \sqrt{\frac{\kappa}{I}}$

2.4 Simple Harmonic Oscillation

If

$$m\frac{d^2x}{dt^2} = F = -kx$$

then

$$x(t) = A\cos{(\omega t + \phi)}$$

or alternately

$$x(t) = c_1\cos{(\omega t)} + c_2\sin{(\omega t)}$$

where

$$\omega = \sqrt{\frac{k}{m}}$$

or, in other words,

$$T = 2\pi\sqrt{\frac{m}{k}}.$$

Consequently,

$$a(x) = -\omega^2 x.$$

Kinetic Energy:

$$T = \frac{1}{2}m\left(\frac{dx}{dt}\right)^2 = \frac{1}{2}kA^2\sin^2(\omega t + \phi)$$

Potential Energy:

$$U = \frac{1}{2}kx^2 = \frac{1}{2}kA^2\cos^2(\omega t + \phi)$$

so the total energy is constant:

$$E = \frac{1}{2}kA^2$$

Pendulum of length l (with gravitational acceleration g) for small angle oscillations:

$$T = 2\pi\sqrt{\frac{l}{g}}$$

2.5 Damped Oscillator

For viscous damping $F = -c\frac{dx}{dt}$, equation of motion is

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = 0$$

where $\omega_0 = \sqrt{\frac{k}{m}}$ is the angular frequency of the undamped oscillator.
Damping ratio $\zeta = c/2m\omega_0$ determines behavior:
• Overdamped ($\zeta > 1$): Exponential decay (without oscillation) to equilibrium, slower for higher ζ.

• Critically damped ($\zeta = 1$): Fastest possible return to equilibrium; no oscillation. $C_{critical} = 2m\sqrt{k/m}$
• Underdamped ($\zeta < 1$): System oscillates with amplitude slowly decreasing to zero, with frequency $\omega_1 = \omega_0\sqrt{1-\zeta^2}$.

2.6 Miscellaneous

Doppler (sound):

$$f' = f\left(\frac{v \pm v_{observer}}{v \mp v_{source}}\right)$$

3 Electricity and Magnetism

3.1 The Basics

Maxwell's Equations:

$$\vec{\nabla} \cdot \vec{E} = \rho/\epsilon_0 \text{ (Gauss)}$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \text{ (Faraday)}$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \epsilon_o \mu_o \frac{\partial \vec{E}}{\partial t} \text{ (Ampere)}$$

Coulomb's Law:

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int_{volume} \frac{1}{\mathscr{r}^2}\rho(\vec{r}')d\tau'\hat{\mathscr{r}}$$

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int_{surface} \frac{1}{\mathscr{r}^2}\sigma(\vec{r}')da'\hat{\mathscr{r}}$$

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int_{line} \frac{1}{\mathscr{r}^2}\lambda(\vec{r}')dl'\hat{\mathscr{r}}$$

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{1}{\mathscr{r}}\rho(\vec{r}')d\tau'$$

Forces:

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

Gauss's Law:

$$\text{Flux } \Phi \equiv \oint \vec{E} \cdot d\vec{a} = Q_{enc}/\epsilon_0 \Leftrightarrow \vec{\nabla} \cdot \vec{E} = \rho(\vec{r})/\epsilon_0$$

Electric field inside a sphere of radius R with uniformly distributed charge Q:

$$E_{inside} = \frac{Q}{4\pi\epsilon_0}\frac{r}{R^3}$$

$$E_{outside} = \frac{Q}{4\pi\epsilon_0}\frac{1}{r^2}$$

$$\vec{\nabla} \times \vec{E} = \vec{0} \text{ always.}$$

Electrical Potential:

$$V(\vec{a}) = -\int_{\mathcal{O}}^{\vec{a}} \vec{E}(\vec{r}) \cdot d\vec{l}$$

$$V(a) - V(b) = \int_a^b \vec{E}(\vec{r}) \cdot d\vec{l}$$

$$\vec{E} = -\vec{\nabla} V$$

Poisson's Equation:

$$\nabla^2 V = -\rho/\epsilon_0$$

$$\Rightarrow \text{ Laplace's Equation: If } \rho = 0,\ \nabla^2 V = 0$$

Energy of a point charge:

$$W = QV$$

Energy of a collection of n point charges:

$$W_{tot} = \frac{1}{8\pi\epsilon_0}\sum_{j=0}^{n}\sum_{i=0, i\neq j}^{n}\frac{q_i q_j}{r_{ij}}$$

Energy of a continuous distribution of charge:

$$W = \frac{1}{2}\int_V \rho V d\tau = \frac{\epsilon_0}{2}\int_{allspace} E^2 d\tau$$

Conductor:
$\vec{E}_{inside} = \vec{0}$ $\rho_{inside} = 0$ V constant $\vec{E}_{surface} \perp$ surface

3.2 Dielectric Media

Dipole moment:

$$\vec{p} = \alpha \vec{E}$$

where α=atomic polarizability.
Polarization:

$$\vec{P} = \vec{p}/\text{unit volume}$$

Bound charge:

$$\sigma_b = \vec{P} \cdot \hat{n}$$

$$\rho_b = -\vec{\nabla} \cdot \vec{P}$$

Electric displacement field:

$$\vec{D} \equiv \epsilon_0 \vec{E} + \vec{P}$$

$$\vec{\nabla} \cdot \vec{D} = \rho_{free}$$

$$\oint \vec{D} \cdot d\vec{a} = Q_{free_enclosed}$$

Linear Dielectrics:

$$\vec{P} = \epsilon_0 \chi_e \vec{E}$$

where $\chi_e \equiv$ electric susceptibility.

$$\vec{D} = \epsilon_0 (1 + \chi_e) \vec{E} = \epsilon \vec{E}$$

where permittivity $\epsilon \equiv \epsilon_0 (1 + \chi_e)$
Dielectric constant (relative permittivity):

$$\epsilon_r \equiv 1 + \chi_e \equiv \epsilon / \epsilon_0$$

If linear dielectric fills all relevant space,

$$\vec{E} = \frac{1}{\epsilon_r} \vec{E}_{vac}$$

i.e. replace ϵ_0 with ϵ.
Boundary conditions:

$$\epsilon_a E_a^{\perp} - \epsilon_b E_b^{\perp} = \sigma_f$$

$$V(a) = V(b)$$

3.3 Magnetostatics

$$\vec{F}_{mag} = \vec{I}l \times \vec{B} \text{ or } Q(\vec{v} \times \vec{B})$$

Circular motion of a charge in a $\vec{B}$ field:

$$p = mv = qBR$$

Currents:

$$\vec{I} = \lambda \vec{v} \qquad \vec{K} = \sigma \vec{v} \qquad \vec{J} = \rho \vec{v}$$

Biot-Savart Law:

$$\vec{B} = \frac{\mu_0}{4\pi} \int \frac{\vec{I} \times \hat{\mathscr{r}}}{\mathscr{r}^2} dl' = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{l}' \times \hat{\mathscr{r}}}{\mathscr{r}^2}$$

Ampere's Law:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$$

Solenoid with n turns and current I:

$$\vec{B} = \mu_0 n I \hat{z}$$

Magnetic vector potential:

$$\vec{B} = \vec{\nabla} \times \vec{A}$$

$$\vec{A} \parallel \vec{J}$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}')}{\mathscr{r}} d\tau'$$

$$\oint \vec{A} \cdot d\vec{l} = \Phi_B$$

Magnetic dipole moment:

$$\vec{m} = I\vec{a}$$

where $\vec{a}$ is area normal vector.
Magnetization:

$$\vec{M} \equiv \vec{m}/\text{unit volume}$$

H field:

$$\vec{H} \equiv \frac{1}{\mu_0}\vec{B} - \vec{M}$$

$$\vec{\nabla} \times \vec{H} = \vec{J}_f$$

$$\vec{\nabla} \cdot \vec{H} = -\vec{\nabla} \cdot \vec{M}$$

Generally, $\vec{H} \parallel \vec{B} \parallel \vec{M}$.

3.4 Linear magnetic materials

$$\vec{M} = \chi_m \vec{H}$$

where χ_m= magnetic susceptibility.

$$\vec{B} = \mu_0(1 + \chi_m)\vec{H} = \mu\vec{H}$$

where $\mu \equiv \mu_0(1 + \chi_m)$=magnetic permeability.

$$B_{material} = \frac{\mu}{\mu_0} B_{vacuum} = \mu_r B_{vacuum}$$

where $mu_r \equiv \mu/\mu_0$=relative permeability.

3.5 Index of Refraction

$$n \equiv \frac{c}{v_m edium} = \sqrt{\epsilon_r \mu_r}$$

Speed of light in medium is $v = c/n$.

3.6 Electrodynamics

Electromotive force $\mathscr{E}$=work done per unit charge

$$\mathscr{E} = \oint \vec{f}_{mag} \cdot d\vec{l} = vBl$$

$$\mathscr{E} = -\frac{d\Phi_B}{dt} = \oint \vec{E}_{induced} \cdot d\vec{l}$$

$$\vec{\nabla} \times \vec{E}_{induced} = -\frac{\partial \vec{B}}{\partial t}$$

$$\oint \vec{E}_{induced} \cdot d\vec{l} = -\frac{\partial \Phi_B}{\partial t}$$

$$\vec{\nabla} \cdot \vec{E}_{induced} = 0.$$

$$\vec{J} = \sigma \vec{E} = \sigma f$$

where σ=conductivity.
Mutual inductance:

$$M = \Phi_{21}/I_1 = \Phi_{12}/I_2$$

Self-inductance:

$$L = \Phi/I$$

$$\mathscr{E} = -\frac{d\Phi}{dt} = -L\frac{dI}{dt}$$

For solenoid with n turns, radius R, height l:

$$L = \mu_0 n^2 \pi R^2 l$$

3.7 Miscellaneous

Doppler (light):

$$f = \left(1 - \frac{v}{c}\right) f_0$$

4 Electronics

Voltage: $V = \int \vec{E} \bullet d\vec{r}$

Power= $Vi = Ri^2 = V^2/R$

Resistors: $V_{across} = iR = R\frac{dQ}{dt}$ (Ohm's Law)
$R_{series} = R_1 + R_2 + ...$
$R_{parallel} = \left(R_1^{-1} + R_2^{-2} + ...\right)^{-1} = \frac{R_1 R_2}{R_1+R_2}$ (for 2 resistors)

Capacitors: $V_{across} = Q/C$
$C_{series} = \left(C_1^{-1} + C_2^{-2} + ...\right)^{-1} = \frac{C_1 C_2}{C_1+C_2}$ (for 2 capacitors)
$C_{parallel} = C_1 + C_2 + ...$
$E_{cap} = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{1}{2}Q^2/C$

Discharging capacitor: $V_C = V_0 e^{-t/RC}$ $\qquad Q = Q_0 e^{-t/RC}$
Charging capacitor: $V_C = V_0\left(1 - e^{-t/RC}\right)$
Inside capacitor: $E = \sigma_{free}/\epsilon_0$

Inductor: $V_{across} = L\frac{dI}{dt} = L\frac{d^2Q}{dt^2}$

To measure R_x, need a voltmeter in parallel with resistance $R_v >> R_x$ so $R_{eq} \approx R_x$.

Superposition Theorem: The voltage across any two points is
(what you get if you open all current sources) +
(what you get if you instead short all voltage sources).

Voltage Divider: $V_{AB} = V_{in}\left(\frac{R_2}{R_1+R_2}\right)$

Current Divider: $i_2 = I\left(\frac{R_1}{R_1+R_2}\right)$

Kirchoff's Laws: $\sum_{\text{Loop}} V = 0 \qquad \sum_{\text{Node}} i = 0$

Thevenin's Theorem: The voltage across any two points in a circuit is the same as that across some resistor R_{TH} in series with some voltage source V_{TH}. To find R_{TH}, short V_{in}, find R_{eq} for V_{AB}. Then $V_{TH} = V_{AB}$ and

$R_{TH} = V_{TH}/I$

Load: Use a large resistor R_L across A and B; then V_{AB} is as through the resistor isn't there.

Max power dissipated when $R_L = R_{TH}$. Circuit unchanged for $R_L << R_{TH}$

Impedance: $Z = v(t)/i(t)$
$Z_R = R$
$Z_L = j\omega L$ Large impedance at high freq (coil opposes change in B field). Acts like short at low freq (acts like a wire).
$Z_C = \frac{1}{j\omega C}$ Large impedance at low freq (charge just builds on plates). Acts like short at high freq.
Reactance: $X =$Re$[Z]$ $X_L = \omega L$ $X_C = \frac{1}{\omega C}$

Transfer function $H = V_{out}/V_{in}$
Phase $\phi = \arctan\left(\text{Im}[H]/\text{Re}[H]\right)$
$\Rightarrow V_{out}$ lags by ϕ behind V_{in}.

Voltage divider: $V_{out} = \frac{Z_2}{Z_1+Z_2}V_{in}$

RC circuit: time constant $\tau = RC$

RL circuit: time constant $\tau = L/R$

LC Circuit (L and C in parallel): $L\frac{d^2Q}{dt^2} + \frac{Q}{C} = 0 \Rightarrow \omega = \frac{1}{\sqrt{LC}}$

RLC Circuit (Series resonant circuit):
$|H| = R/\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = 1/\sqrt{1 + Q^2\left(1 - \frac{\omega_{res}^2}{\omega^2}\right)^2}$
$|H|_{max} = 1$ for $\omega_{res} = \frac{1}{\sqrt{LC}}$
Quality factor $Q \equiv$ (Power in L) / (Power in R) $= \omega L/R$
This is a band pass filter: lets through ω_{res}. High $Q \rightarrow$ narrow peak.
Half power points: $H = \frac{1}{\sqrt{2}}|H|_{max}$ at $\omega = \omega_1, \omega_2$. $Q = \frac{\omega_{res}}{\omega_2 - \omega_1}$

Low pass filter: $H = \frac{1}{j\omega RC+1}$. $|H|_{\omega\to 0} \longrightarrow 1$, $|H|_{\omega\to\infty} \longrightarrow \dfrac{1}{\omega RC}$
Corner frequency $\omega_C = \frac{1}{RC}$, $|H| = \frac{1}{\sqrt{2}}$

High pass filter: $H = \frac{j\omega RC}{1+j\omega RC}$. $|H|_{\omega\to 0} \longrightarrow \omega RC$, $|H|_{\omega\to\infty} \longrightarrow 1$

Tank Circuit: $Z_{eq} = \frac{Z_C Z_L}{Z_C+Z_L} = \frac{j\omega L}{1-\omega^2 LC}$
used in band pass filter: $|H| = 1$ for $\omega = \omega_R = \frac{1}{\sqrt{LC}}$

used in band stop filter:

Diode: For forward current, voltage across diode drops by about 0.7 V with 1 Ω resistance.

If you are faced with two wires of non-negligible resistance, the thing that's the same about them is $\rho = R * a/l$ where R is the resistance, a the area, and l the length.

5 Thermal Physics

1st Law of Thermodynamics:

$$dE = \delta Q - \delta W$$

that is, the change in internal energy is the heat energy absorbed by the system minus the work done BY the system.

2nd Law of Thermodynamics:

$$dS = \frac{\delta Q}{T}$$

that is, the change in entropy is the heat absorbed by the system divided by the temperature.
Interpretations:
• The entropy of an isolated system that is not in equilibrium increases over time, obtaining its maximum at equilibrium.
• Heat generally does not flow spontaneously from a material of lower temperature to one of higher temperature.
• A cyclic process cannot convert heat into work.
Also, a reversible process must have zero change in entropy.

Helmholtz free energy:

$$F = E - TS$$

where E is the total energy.
Gibbs Free Energy:

$$G = E - TS + PV$$

Specific heats (the heat energy required for a unit temperature increase):

$$\text{At const pressure: } C_P \equiv \left(\frac{\partial Q}{\partial T}\right)_P$$

$$\text{At const pressure: } C_V \equiv \left(\frac{\partial Q}{\partial T}\right)_V$$

Heat energy:

$$Q = mC\Delta T$$

Work:

$$dW = -PdV$$

$$W = -\int_{V_i}^{V_f} PdV$$

6 Quantum Mechanics

Schrodinger Equation:

$$i\hbar\frac{\partial}{\partial t}\Psi(x,t) = H\Psi(x,t) = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi(x,t) + V(x)\Psi(x,t)$$

Time Independent Schrodinger Equation (Eigenvalue equation for H, with standing waves $\Psi(x,t) = \psi_E(x)e^{-iEt/\hbar}$):

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi_E(x)}{\partial x^2} + V(x)\psi_E(x) = H\psi_E = E \cdot \psi_E$$

Norm of wavefunction:

$$<\psi|\psi> = \int_{allspace} \psi^*\psi dx$$

Normalized:

$$<\psi|\psi> = 1$$

Energy eigenfunctions corresponding to different energy eigenvalues are orthogonal:

$$\int u_E^*(x)u_{E'}(x)dx = 0$$

Dimensions of $\hbar$: Energy $\cdot$ time. Momentum operator:

$$p\psi = -i\hbar\frac{\partial}{\partial x}\psi$$

Energy of photon:

$$E = h\nu = \hbar\omega = pc$$

where the momentum of a photon is

$$p = h/\lambda = \hbar k$$

Compton Scattering: When a photon of wavelength λ is scattered by a particle of mass m through an angle θ, the change in wavelength is

$$\lambda' - \lambda = \frac{h}{mc}(1 - \cos\theta)$$

Uncertainty Principle:

$$\Delta x \Delta p = \frac{\hbar}{2}$$

Expectation Value:

$$<F> = \int_{allspace} \psi^* F \psi dx$$

6.1 Classic Early Quantum Experiments

Photoelectric effect: • Demonstrated that light is quantized, with quanta of energy $E = hf$
• When electromagnetic radiation above a certain threshold frequency, the radiation is absorbed and electrons are emitted with kinetic energy

$$\frac{1}{2}mv_{max}^2 = E_{kmax} = hf - \phi$$

where h is Planck's constant, f is the light's frequency, and ϕ (or sometimes W) is the Work Function, the electron's binding energy that the photon must possess just to free it from the atom.

6.2 Miscellaneous

Hydrogen atom: Energy levels: $E_n = -13.6$ eV $/n^2$

7 Nuclear Decays

Notation:

$${}_Z^AX$$

A=mass number (number of protons + number of neutrons)
Z=atomic number (number of protons)
Alpha decay: Emission of a $\alpha =_2^4 He^{2+}$, so $A \to A-4$, $Z \to Z-2$.
Beta decay: Emission of β^+=electron or β^-=positron, along with a neutrino or antineutrino. $A \to A$, $Z \to Z \pm 1$
(Think of it as a neutron becoming a proton + electron, or a proton becoming a neutron + positron.)
Gamma decay: Emission of γ=photon. A and Z stay the same.

8 Special Relativity

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Lorentz Transformations for motion along x direction:

$$t' = \gamma \left(t - vx/c^2\right)$$

$$x' = \gamma \left(x - vt\right)$$

$$y' = y$$

$$z' = z$$

Time dilation: for clock at rest in unprimed system,

$$\Delta t' = \gamma \Delta t$$

Length contraction (length along x direction):

$$\Delta x' = \frac{\Delta x}{\gamma}$$

Energy:

$$E = \gamma mc^2$$

Spacetime interval (Lorentz invariant)

$$\Delta s^2 = c^2 \Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2$$

and all observers agree on this value!
Velocity:

$$u'_x = \frac{u_x - v}{1 - u_x v/c^2}$$

9 Laboratory formulas

• If $y = y(x_1, x_2, ...)$ and each x_i is measured with uncertainty σ_i then the uncertainty in y is

$$\sigma_y = \sqrt{\left(\frac{\partial y}{\partial x_1}\sigma_1\right)^2 + \left(\frac{\partial y}{\partial x_2}\sigma_2\right)^2 + ...}$$

• When n events (uncorrelated, random) are measured to give a mean with standard deviation σ, the standard error of the mean is $\sigma/\sqrt{n}$.

• Reading graphs:
If a log-log graph is linear with slope m, then $y = ax^m$.
If a log-lin graph is linear with slope m, then $y = ae^{mx}$.