

Online Appendix for Three World Wars: Fiscal-Monetary Consequences

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1 Budget Constraints

We derive the consolidated government budget constraint in levels, equation (1) in the paper, and as ratios of nominal GDP, equation (2) of the paper. We then describe the specific steps we took to decompose the sources of government revenues during each war reported in table 2 of the paper.

As we report in Table 1 of the paper, four different groups own Treasury debt: the Federal Reserve, Government Agencies and Trust Funds, Foreign Investors and Domestic Private Investors. Gross debt is the total debt held by all four groups of investors.

In the budget constraints presented below, there are three definitions of interest-bearing government debt:

$$\begin{aligned} B^n &= \text{Gross debt minus debt held by government agencies and trust funds} \\ B^f &= \text{Debt held by the Federal Reserve} \\ B &= \text{Debt held by private investors, both foreign and domestic} \end{aligned}$$

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The Treasury's budget constraint is

$$G_t + r_{t-1,t}^B B_{t-1}^n + (D_t - D_{t-1}) = T_t + (B_t^n - B_{t-1}^n) + OM_t \quad (1)$$

where

$$\begin{aligned} G_t &= \text{Government outlays (net of official interest payments)} \\ B_{t-1}^n &= \text{Nominal market value of interest bearing government debt held} \\ &\quad \text{outside the government at the end of } t-1 \\ r_{t-1,t}^B &= \text{Nominal value-weighted holding period return on government debt} \\ &\quad \text{between } t-1 \text{ and } t \\ T_t &= \text{Tax receipts} \\ D_t - D_{t-1} &= \text{Change in Treasury Cash} \\ OM_t &= \text{Funding by Other Means} \end{aligned}$$

The Federal Reserve's budget constraint is

$$(B_t^f - B_{t-1}^f) + (A_t - A_{t-1}) = r_{t-1,t}^B B_{t-1}^f + r_{t-1,t}^A A_{t-1} + (D_t - D_{t-1}) + (M_t - M_{t-1}) \quad (2)$$

where

$$\begin{aligned} B_{t-1}^f &= \text{Nominal market value of interest bearing government debt} \\ &\quad \text{held by the Federal Reserve at the end of } t-1 \\ A_{t-1} &= \text{Private Assets Held By the Fed at the end of } t-1 \\ r_{t-1,t}^A &= \text{Nominal holding period return on Fed-held private assets between } t-1 \text{ and } t \\ M_t - M_{t-1} &= \text{Change in the Monetary Base or Fed Credit} \end{aligned}$$

Define

$$B_t \equiv B_t^n - B_t^f = \text{Nominal market value of interest bearing government debt held by private investors at the end of } t$$

Adding (1) and (2) yields a consolidated budget constraint:

$$G_t + r_{t-1,t}^B B_{t-1} + (A_t - A_{t-1}) = T_t + (B_t - B_{t-1}) + r_{t-1,t}^A A_{t-1} + (M_t - M_{t-1}) + OM_t \quad (3)$$

This is equation [1] in the paper.

Following Hall and Sargent (2021), we decompose wartime increases in federal revenue. We start by dividing each term in equation (3) by nominal GDP, denoted Y_t , and rearranging terms

$$\begin{aligned} \frac{G_t}{Y_t} + \left(r_{t-1,t}^B \frac{B_{t-1}}{Y_{t-1}} - r_{t-1,t}^A \frac{A_{t-1}}{Y_{t-1}} \right) + \left(\frac{A_t}{Y_t} - \frac{A_{t-1}}{Y_{t-1}} \right) &= \frac{T_t}{Y_t} + \left(\frac{B_t}{Y_t} - \frac{B_{t-1}}{Y_{t-1}} \right) + \frac{M_t - M_{t-1}}{Y_t} \\ + \frac{OM_t}{Y_t} + g_{t-1,t} \frac{B_{t-1} - A_{t-1}}{Y_{t-1}} + \pi_{t-1,t} \frac{B_{t-1} - A_{t-1}}{Y_{t-1}} &+ (\pi_{t-1,t} + g_{t-1,t}) \left(r_{t-1,t}^B \frac{B_{t-1}}{Y_{t-1}} - r_{t-1,t}^A \frac{A_{t-1}}{Y_{t-1}} \right) \end{aligned} \quad (4)$$

where $g_{t-1,t}$ denotes the net growth rate of real GDP, and $\pi_{t-1,t}$ denotes the net inflation rate. The three terms on the left side are: 1) government purchases plus transfers, 2) interest payments on government debt net interest payments on private assets, and 3) the growth in Fed holdings of private assets, as shares of GDP. The first four terms on the right side are sources of government revenue as shares of GDP: 1) taxes, 2) new borrowing, 3) money creation and 4) other means. The next two terms record the diminution of the debt/GDP ratio due to real GDP growth and inflation. The final term is a cross product of the two growth rates. This is equation [2] in the paper.

Define a ‘‘peacetime baseline’’ version of equation (4):

$$\begin{aligned} \left(\frac{G}{Y} \right)^{base} + \left(r_{-1,0}^B \frac{B_{-1}}{Y_{-1}} - r_{-1,0}^A \frac{A_{-1}}{Y_{-1}} \right)^{base} + \left(\frac{A}{Y} - \frac{A_{-1}}{Y_{-1}} \right)^{base} &= \left(\frac{T}{Y} \right)^{base} \\ + \left(\frac{B}{Y} - \frac{B_{-1}}{Y_{-1}} \right)^{base} + \left(\frac{M - M_{-1}}{Y_{-1}} \right)^{base} + \left(\frac{OM}{Y} \right)^{base} & \\ + \left(g_{-1,0} \left(\frac{B_{-1} - A_{-1}}{Y_{-1}} \right) \right)^{base} + \left(\pi_{-1,0} \left(\frac{B_{-1} - A_{-1}}{Y_{-1}} \right) \right)^{base} & \\ + \left((\pi_{-1,0} + g_{-1,0}) \left(r_{-1,0}^B \frac{B_{-1}}{Y_{-1}} - r_{-1,0}^A \frac{A_{-1}}{Y_{-1}} \right) \right)^{base} &. \end{aligned} \quad (5)$$

We subtract equation (5) from equation (4). Then, for each war, we sum this difference from the

beginning to the end of the war:

$$\begin{aligned}
& \underbrace{\sum_{t=T_1}^{T_2} \left[\frac{G_t}{Y_t} - \left(\frac{G}{Y} \right)^{base} \right]}_{\text{government spending}} + \underbrace{\sum_{t=T_1}^{T_2} \left[\left(r_{t-1,t}^B \frac{B_{t-1}}{Y_{t-1}} - r_{t-1,t}^A \frac{A_{t-1}}{Y_{t-1}} \right) - \left(r_{-1,0}^B \frac{B_{-1}}{Y_{-1}} - r_{-1,0}^A \frac{A_{-1}}{Y_{-1}} \right)^{base} \right]}_{\text{nominal return on net debt}} \\
& + \underbrace{\sum_{t=T_1}^{T_2} \left[\left(\frac{A_t}{Y_t} - \frac{A_{t-1}}{Y_{t-1}} \right) - \left(\frac{A}{Y} - \frac{A_{-1}}{Y_{-1}} \right)^{base} \right]}_{\text{growth in Fed-held private assets}} \\
& = \underbrace{\sum_{t=T_1}^{T_2} \left[\frac{T_t}{Y_t} - \left(\frac{T}{Y} \right)^{base} \right]}_{\text{explicit tax revenue}} + \underbrace{\sum_{t=T_1}^{T_2} \left[\left(\frac{B_t}{Y_t} - \frac{B_{t-1}}{Y_{t-1}} \right) - \left(\frac{B}{Y} - \frac{B_{-1}}{Y_{-1}} \right)^{base} \right]}_{\text{interest-bearing debt growth}} \\
& + \underbrace{\sum_{t=T_1}^{T_2} \left[\frac{M_t - M_{t-1}}{Y_t} - \left(\frac{M - M_{-1}}{Y_{-1}} \right)^{base} \right]}_{\text{growth in Fed credit}} \\
& + \underbrace{\sum_{t=T_1}^{T_2} \left[g_{t-1,t} \frac{B_{t-1} - A_{t-1}}{Y_{t-1}} - \left(g_{-1,0} \frac{B_{-1} - A_{-1}}{Y_{-1}} \right)^{base} \right]}_{\text{debt dilution via real GDP growth}} \\
& + \underbrace{\sum_{t=T_1}^{T_2} \left[\pi_{t-1,t} \frac{B_{t-1} - A_{t-1}}{Y_{t-1}} - \left(\pi_{-1,0} \frac{B_{-1} - A_{-1}}{Y_{-1}} \right)^{base} \right]}_{\text{debt default via inflation}} \\
& + \underbrace{\sum_{t=T_1}^{T_2} \left[\frac{OM_t}{Y_t} - \left(\frac{OM}{Y} \right)^{base} \right]}_{\text{other means}} \\
& + \underbrace{\sum_{t=T_1}^{T_2} \left[(\pi_{t-1,t} + g_{t-1,t}) \left(r_{t-1,t}^B \frac{B_{t-1}}{Y_{t-1}} - r_{t-1,t}^A \frac{A_{t-1}}{Y_{t-1}} \right) - \left((\pi_{-1,0} + g_{-1,0}) \left(r_{-1,0}^B \frac{B_{-1}}{Y_{-1}} - r_{-1,0}^A \frac{A_{-1}}{Y_{-1}} \right) \right)^{base} \right]}_{\text{cross-product}} \tag{6}
\end{aligned}$$

where T_1 is the first year of the war or the first year of US involvement, and T_2 is the final year of the war. We construct each term's peacetime baseline as the mean of the appropriate term for the five years immediately prior to the start of the war.

1.1 Decomposing the monetary base

In the paper, we measure M_t as Federal Reserve Credit. We discuss reasons for and ramifications of this choice in more detail in section 2 below. In this subsection, we measure M_t by the monetary base and decompose it into currency, C_t , and interest bearing reserves, R_t . The Federal Reserve's budget constraint, (2) becomes

$$(B_t^f - B_{t-1}^f) + (A_t - A_{t-1}) + r_{t-1,t}^R R_{t-1} = r_{t-1,t}^B B_{t-1}^f + r_{t-1,t}^A A_{t-1} + (D_t - D_{t-1}) + (C_t + R_t - C_{t-1} - R_{t-1}) \quad (7)$$

Adding (1) and (7) yields a consolidated budget constraint:

$$G_t + r_{t-1,t}^B B_{t-1} + r_{t-1,t}^R R_{t-1} + (A_t - A_{t-1}) = T_t + ((B_t + R_t) - (B_{t-1} + R_{t-1})) + r_{t-1,t}^A A_{t-1} + (C_t - C_{t-1}) + OM_t \quad (8)$$

As fractions of GDP, the consolidated budget constraint becomes

$$\begin{aligned} \frac{G_t}{Y_t} + \left(r_{t-1,t}^B \frac{B_{t-1}}{Y_{t-1}} + r_{t-1,t}^R \frac{R_{t-1}}{Y_{t-1}} - r_{t-1,t}^A \frac{A_{t-1}}{Y_{t-1}} \right) + \left(\frac{A_t}{Y_t} - \frac{A_{t-1}}{Y_{t-1}} \right) &= \frac{T_t}{Y_t} + \left(\frac{B_t}{Y_t} + \frac{R_t}{Y_t} - \frac{B_{t-1}}{Y_{t-1}} - \frac{R_{t-1}}{Y_{t-1}} \right) \\ &+ \frac{C_t - C_{t-1}}{Y_t} + g_{t-1,t} \frac{B_{t-1} + R_{t-1} - A_{t-1}}{Y_{t-1}} + \pi_{t-1,t} \frac{B_{t-1} + R_{t-1} - A_{t-1}}{Y_{t-1}} \\ &+ \frac{OM_t}{Y_t} + (\pi_{t-1,t} + g_{t-1,t}) \left(r_{t-1,t}^B \frac{B_{t-1}}{Y_{t-1}} + r_{t-1,t}^R \frac{R_{t-1}}{Y_{t-1}} - r_{t-1,t}^A \frac{A_{t-1}}{Y_{t-1}} \right) \end{aligned} \quad (9)$$

To compute cumulative deviations from the pre-war baseline, we repeat the steps described in equations (5) and (6) stated above.

In figure 4 of the paper, we plot the par value of the gross or total debt as a percent of GDP.

- B^n is the gross debt minus the light tan area representing debt held by government agencies and trust funds.
- B^f , debt held by the Fed, is represented by the purple area
- B , debt held by foreign and domestic private investors, is represented by the sum of the blue and dark tan areas

In figure 7 of the paper, we plot as percents of GDP

- par and market values of privately-held debt: B_t (the dark blue and red lines).
- privately-held debt plus interest-bearing reserves and reverse repos: $B_t + R_t$ (the green line).
- privately-held debt plus interest-bearing reserves and reverse repos minus private assets: $B_t + R_t - A_t$ (the light blue line).

2 Federal Reserve Balance Sheet

In this section, we describe the Federal Reserve’s balance sheet and describe our decisions on measure interest-bearing debt, B_t , and non-interest bearing money, M_t .

Assets		Liabilities	
Gold + SDR Cert. Account	\$16,237	Currency	\$2,186,877
Loans to Depository Institutions	1,212	Deposits Held by Depository Institutions	4,039,860
Bills Discounted	34,952	Reverse Repurchase Agreements	1,920,789
US Treasury Securities	5,987,192	Government Deposits	283,995
Federal Agency Debt	2,347	Other Liabilities	325,939
Mortgage Backed Securities	2,615,492		
Other Private Assets	39,897		
Other	60,131		
Total	\$8,757,460	Total	\$8,757,460
Narrow Fed Credit	\$6,022,144	Narrow Monetary Base	\$6,226,737
Broad Fed Credit	\$8,637,636	Broad Monetary Base	\$8,147,526

Table 1: Federal Reserve Balance Sheet, December 29, 2021, in millions of dollars

We measure Federal Reserve Credit two ways. In our narrow measure, it is the sum of Bills Discounted and US Treasury Securities. In our broad measure we add Mortgage Backed Securities and Other Private Assets to the narrow measure. In Federal Reserve’s data release, *Factors Affecting Reserve Balances - H.4.1*, the Fed reports unamortized premiums and discounts on securities held outright. This adjustment of the securities’ par values to reflect their market values does not decompose the premiums and discounts across Treasury securities and MBSs. Therefore, we measure the US Treasury Securities at their market value using our calculations and measure the par value of the Fed’s MBS and private holdings at their par value.

The narrow measure of the monetary base is the sum of currency and deposits held by depository institutions, the traditional measure of the monetary base. In our broad measure of the monetary base, we add the Reserve Repurchase Agreements to the narrow measure.

As in Hall and Sargent (2019, 2021), we have computed M_t in the budget constraint as Federal Reserve credit rather than the monetary base. Thus, we are measuring M_t from the asset side of the Fed balance sheet. In the paper, we use Broad Federal Reserve Credit as our measure of M_t . In table 2 of the paper:

- In the case $\text{reserves} \subset M$, we set $M_t = \text{fedcredit}$, and $\text{debt} = B_t^n - B_t^f$.
- In the case $\text{reserves} \subset B$, we set $M_t = \text{fedcredit} - R_t$, and $\text{debt} = B_t^n - B_t^f + R_t$.

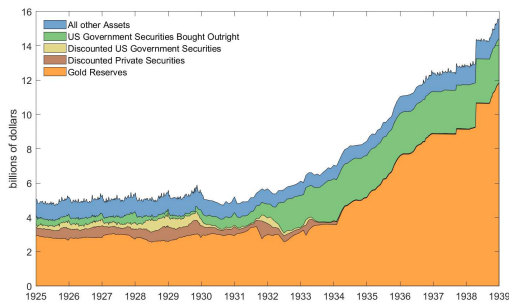
where R_t is the cum of Deposits Held by Depository Institutions and Reserve Repurchase Agreements.

War Start - End	(1) gov't spending	(2) payouts on net debt	(3) priv asset growth	(4) (1)+(2) +(3)	(5) tax revenue	(6) debt growth	(7) money growth	(8) GDP growth	(9) inflation	(10) Other
COVID-19										
2020:1 - 2021:12	21.37	0.22	5.85	27.45	0.95	-0.59	21.16	1.02	3.03	1.88
reserves $\subset M$					3.5	-2.2	77.1	3.7	11.0	6.9
2020:1 - 2021:12	21.37	0.17	5.85	27.40	0.95	18.36	1.07	1.48	3.99	1.55
reserves $\subset B$					3.5	67.0	3.9	5.4	14.6	5.7

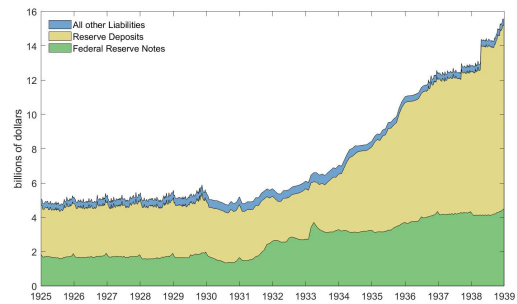
Table 2: Decomposition of Wartime Revenues from Equation (6) with M_t measured by the Broad Monetary Base

For each war, the elements in first row are in percent of GDP. Columns 5-10 sum to column 4. The numbers in the second row are percentages of the sum of war-related spending and returns to bondholders (column 4) accounted for by each term on the right side of equation (6). Column 10 is the sum of other means, the cross product, and a residual.

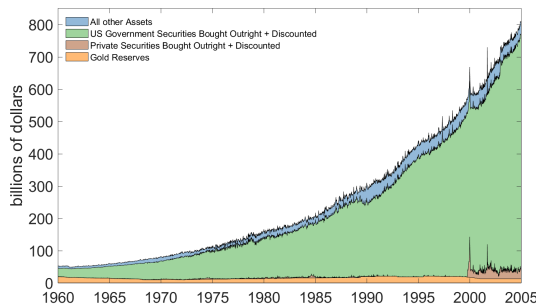
In table 2 of this appendix, we report the decomposition of wartime revenues for the COVID-19 War using our Broad Monetary Base measure.



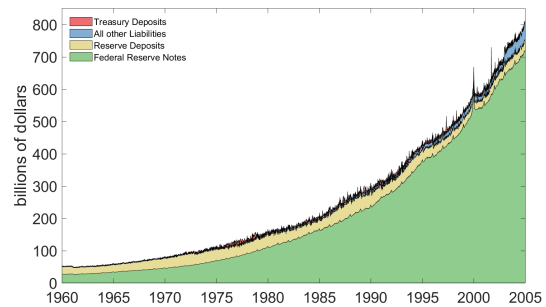
(a) Assets: 1925-1938



(b) Liabilities: 1925-1938



(c) Assets: 1960-2004



(d) Liabilities: 1960-2004

Figure 1: Federal Reserve Balance Sheets During Two “Peacetimes”

2.1 Federal Reserve balance sheets during “peacetime”

As promised in the text of the paper, in figure 1 of this appendix, we plot the Federal Reserve’s assets and liabilities from its balance sheet for the “boring peacetime” periods not reported in the paper. Without big wars to contend with, the balances sheets evolve in more tranquil ways than they do during war times when the Fed is enlisted to help finance a surge in government expenditures.

3 Data Sources

In this section, we report the sources of our data.

- Nominal and Real GDP are from www.measuringworth.com from 1900 to 1947 and the BEA: National Income and Product Accounts thereafter.
- Figure 1:
 - Activity Duty Military Personnel from 1900 to 2005 are from Table 2-11 of *Selected Manpower Statistics: Fiscal Year 2005* Statistical Information Analysis Division, Defense Manpower Data Center, Department of Defense. Data from 2005 to the present are from <https://dwp.dmdc.osd.mil/dwp/app/dod-data-reports/workforce-reports>
 - Insured Unemployment is from FRED¹
 - Population is total population including Armed Forces overseas from the Census Bureau.
- Figures 2 and 3
 - Federal expenditures and revenues from 1900 to 1940 are from the *Annual Report of the Secretary of the Treasury on the State of the Finances*.² The numbers we use are summarized on pages 642 - 650 of the 1940 Annual Report.³ Federal expenditures and revenues from 1940 to 2011 are from the Office of Management and Budget (OMB).⁴
 - From 2011 to 2021 Monthly Receipts, Outlays, Reduction of Operating Cash and Funding by Other Means are from the *Monthly Treasury Statement*.⁵ Interest on Treasury Debt Securities (Gross) and Interest Paid to Government Accounts are from the US Treasury Report: *Interest Expense on the Debt Outstanding*.⁶
 - The forecasts of revenues and outlays for 2022-2031 are computed using Congressional Budget Office forecasts.
 1. We start with the baseline forecast from Table 1-1 of the CBO's July 2021 report *Additional Information About the Updated Budget and Economic Outlook: 2021 to 2031*.⁷

¹See <https://fred.stlouisfed.org/series/CCSA>.

²See <http://fraser.stlouisfed.org/publication/?pid=194>.

³See http://fraser.stlouisfed.org/docs/publications/treasar/AR_TREASURY_1940.pdf.

⁴See <https://www.whitehouse.gov/omb/historical-tables/>.

⁵See <https://www.fiscal.treasury.gov/reports-statements/mts/>.

⁶See <https://fiscaldata.treasury.gov/datasets/interest-expense-debt-outstanding>.

⁷See <https://www.cbo.gov/publication/57263>.

2. We then adjust this forecast for the additional spending authorized by the Infrastructure Investment and Jobs Act. We add estimated outlays and revenues from Table 1 of the CBO report *Senate Amendment 2137 to H.R. 3684, the Infrastructure Investment and Jobs Act, as Proposed on August 1, 2021*.⁸ To take into account spending from the Highway Trust Fund, we add to outlays the contract authority spending reported in Table 2.
- Figures 4, 6, 8, 9, 10, and Tables 1, 2, and 3.
 - For 1900-1960 the price and quantity data of outstanding Treasury securities are from Hall et al. (2018).
 - For 1960-2021 the price and quantity data are from the CRSP US Treasury Database and the US Treasury *Monthly Statement of Public Debt*.⁹
 - Federal Reserve Holdings of Treasury securities are from the Federal Reserve System Open Market Account (SOMA).¹⁰
 - Foreign Holdings of US Treasury Securities
 - * for 1939 to 1999 are from the Department of the Treasury’s *Treasury Bulletin*.¹¹
 - * for 2000 to 2021 are from the Department of the Treasury’s *Treasury International Capital (TIC) System*.¹²
 - During the World War I and World War II periods, Federal Reserve credit is the sum of bills discounted, bills bought, United States Government securities bought outright and discounted, deposits in foreign banks, industrial and commercial loans, municipal warrants, and Federal Reserve bank float. For the COVID period, see the discussion in the previous section of this appendix. The sources of these data are the tables of assets and liabilities of the twelve Federal Reserve Banks reported in each issue of the *Federal Reserve Bulletin*.
 - We measure reserves balances and reserve repo agreements using the balances reported in the Federal Reserve’s H.4.1 statistical release, “Factors Affecting Reserve Balances of Depository Institutions and Condition Statement of Federal Reserve Banks.” We used the policy rates reported in <https://www.federalreserve.gov/monetarypolicy/reserve-balances.htm>.
 - For federal expenditures and receipts, see the sources cited above.

⁸See <https://www.cbo.gov/publication/57406>.

⁹See <http://www.crsp.org/products/research-products/crsp-us-treasury-database> and <https://www.treasurydirect.gov/govt/reports/pd/mspd/mspd.htm>.

¹⁰See <https://www.newyorkfed.org/markets/soma-holdings>.

¹¹See <https://fraser.stlouisfed.org/title/407>.

¹²See <https://home.treasury.gov/data/treasury-international-capital-tic-system>.

- Figure 5
 - The balance sheet data are from the tables of assets and liabilities of the twelve Federal Reserve Banks reported in each issue of the *Federal Reserve Bulletin* and the Federal Reserve’s H.4.1 statistical release, “Factors Affecting Reserve Balances of Depository Institutions and Condition Statement of Federal Reserve Banks.”
- Figure 7
 - Consumer Price Index for All Urban Consumers, Not Seasonally Adjusted, from FRED.¹³

¹³See <https://fred.stlouisfed.org/series/CPIAUCNS>.

References

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- . 2021. Debt and Taxes in Eight U.S. Wars and Two Insurrections. In *The Handbook of Historical Economics*, edited by Alberto Bisin and Giovanni Federico, Chapter 27, 825–880. Academic Press.
- Hall, George J., Jonathan Payne, Thomas J. Sargent, and Bálint Szőke. 2018. US Federal Debt 1776-1940: Prices and Quantities. <https://github.com/jepayne/US-Federal-Debt-Public>.