NAME:

COLLEGE ALGEBRA -ANALYTIC GEOMETRY MIDTERM 2 SOLUTIONS

INSTRUCTOR: HAROLD SULTAN

1. INSTRUCTIONS

(1) Timing: You have exactly 1 hour 55 minutes for this exam.
(2) There are 6 questions in total, with the following scoring breakdown:

Question	Total Points	Your points
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
Total	60	

(3) Please show your work and JUSTIFY all answers unless otherwise specified. Partial credit will be awarded
(4) Many of questions have multiple parts. Each question is on its own page. Please feel free to work on the back of the page or scrap paper if necessary.
(5) Challenging Questions are marked (**)
(6) Good Luck!

Date: 11/06/10.

2. Questions

Question 2.1.

(a) (5 points) Simplify the following complex expression:

$$\frac{(1+2i)(3-i)}{(2+i)}$$

[Your answer should be a complex number in the form $a+bi$]
ANSWER:

$$\frac{(1+2i)(3-i)}{(2+i)} = \frac{(1+2i)(3-i)}{(2+i)}\frac{(2-i)}{(2-i)} = \frac{15+5i}{5} = 3+i$$

(b) (5 points) Evaluate the following expression:

$$\log_8(6) - \log_8(3) + \log_8(2)$$

[Your answer should be a fraction]
ANSWER:

$$\begin{aligned}
\log_8(6) - \log_8(3) + \log_8(2) &= \log_8(\frac{6}{3}) + \log_8(2) \\
&= \log_8(\frac{6 \times 2}{3}) \\
&= \log_8(4) \\
&= \frac{\log_2(4)}{\log_2(8)} \\
&= \frac{2}{3}
\end{aligned}$$

Question 2.2.
Consider the polynomial function $P(x) = 12x^3 - 28x^2 + 17x - 3$

(a) (3 points) Explicitly list all the different possible rational zeros (roots) of $P(x)$. How many are there?
ANSWER:

$$\pm 1/4,\ \pm 1/2,\ \pm 3/4,\ \pm 1,\ \pm 3/2,\ \pm 3,\ \pm 1/12,\ \pm 1/6,\ \pm 1/3$$

In total there are 18 possible rational roots.

(b) (4 points) Find all rational zeros (roots) of the polynomial $P(x)$
[Hint: **All the rational zeros are positive**, so you don't have to try everything from your list in (a).]
ANSWER: After trying possible positive roots from the list above we can see that the roots are

$$\frac{3}{2}, \frac{1}{2}, \frac{1}{3}$$

(c) (3 points) Graph the polynomial $P(x)$.

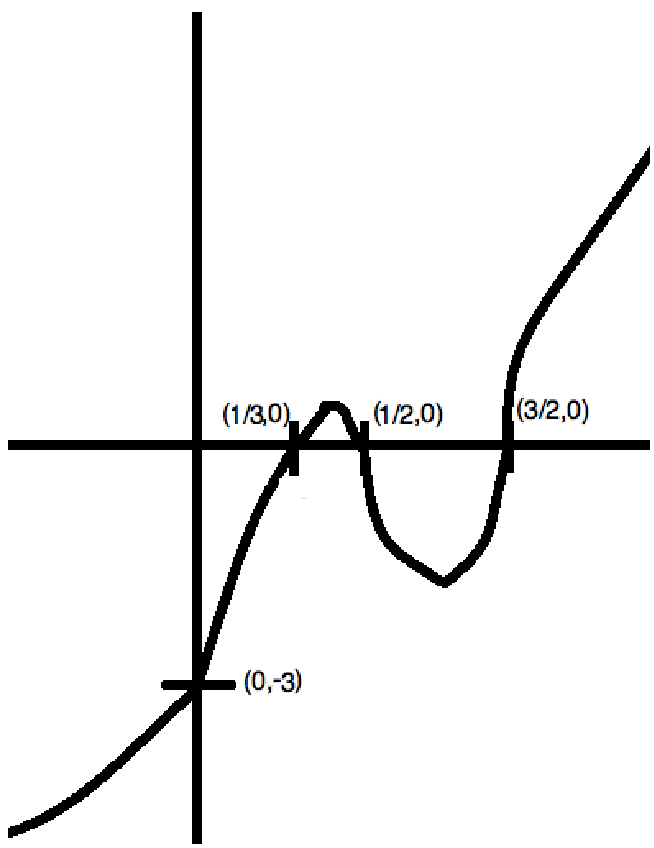

FIGURE 1.

Question 2.3.
Consider the rational function

$$R(x) = \frac{4x^3 - 4x}{2x^2 + 3x + 1}$$

(a) (2 points) Factor the numerator and denominator of $R(x)$
ANSWER:

$$R(x) = \frac{4x^3 - 4x}{2x^2 + 3x + 1} = \frac{4x(x-1)(x+1)}{(x+1)(2x+1)} = \frac{4x(x-1)}{2x+1}$$

(b) (2 points) Find the x and y-intercepts of $R(x)$
ANSWER:
The x-intercepts are the roots of the numerator which after factoring and simplifying as in the previous part are 0 and 1.
To find the y-intercept we plug in $x = 0$ to $R(x)$ and solve, specifically,

$$R(0) = \frac{4(0^3) - 4(0)}{2(0^2) + 3(0) + 1} = \frac{0}{1} = 0$$

so the y-intercept is 0.

(c) (2 points) Find the vertical asymptotes of $R(x)$, and determine whether $R(x) \to +\infty$ or $R(x) \to -\infty$ as x approaches each asymptote from the left and right.
ANSWER:
The vertical asymptotes are the roots of the denominator. After factoring and simplifying as in part (a) we see that there is a single vertical asymptote of $\frac{-1}{2}$.
Moreover, as $x \to (\frac{-1}{2})^-$, plugging in we see that $R(x) \to -\infty$, while as $x \to (\frac{-1}{2})^+$, plugging in we see that $R(x) \to +\infty$

(d) (2 points) Find the slant horizontal asymptote of $R(x)$.
[Hint: long division]
ANSWER:
Since the degree of the numerator (3) is larger than the degree of the denominator (2) in $R(x)$ strictly speaking there is no horizontal asymptote. However, there is a slant horizontal asymptote. Specifically, after long division on $R(x)$ we have:

$$R(x) = \frac{4x^3 - 4x}{2x^2 + 3x + 1} = (2x - 3) + \frac{3x + 3}{2x^2 + 3x + 1}$$

It follows that the slant horizontal asymptote of $R(x)$ is the line $y = 2x - 3$.

(e) (2 points) Graph $R(x)$.

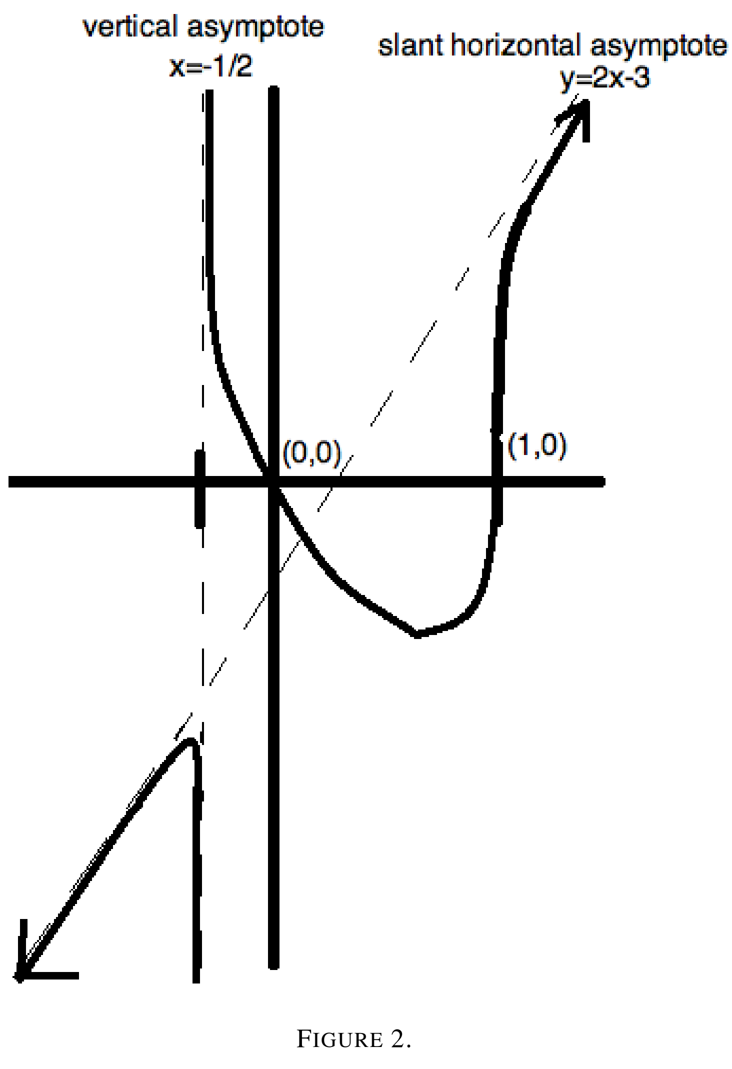

FIGURE 2.

Question 2.4.

(a) (10 points) Find a real, degree four polynomial with zeros i and $1+i$, whose graph has y-intercept 12

Answer:
Since we are looking for a real polynomial with imaginary zeros of i and $1 + i$, we know that the polynomial must have imaginary zeros of $-i$ and $1 - i$ as well because they are conjugates of imaginary zeros.

Hence our polynomial must be of the form:

$$\begin{aligned} P(x) &= a(x-i)(x+i)(x-(1+i))(x-(1-i)) \\ &= a(x^2+1)(x^2-2x+2) \end{aligned}$$

where a above is any real constant. Since we are given that the y-intercept is 12 we know that $P(0) = 12$. Plugging we have

$$\begin{aligned} 12 = P(0) &= a((0)^2+1)((0)^2-2(0)+2) = 2a \\ &\implies 12 = 2a \implies a = 6 \end{aligned}$$

Putting things together our final answer is

$$P(x) = 6(x^2+1)(x^2-2x+2)$$

Question 2.5.

(a) (7 points) Solve for x in the following equation:

$$2^{(-2/\log_5(x))} = \frac{1}{16}$$

Answer:

$$\begin{aligned}
2^{(-2/\log_5(x))} &= \frac{1}{16} \\
&\implies \log_2\left(2^{(-2/\log_5(x))}\right) = \log_2(\frac{1}{16}) \\
&\implies \frac{-2}{\log_5(x)} = -4 \\
&\implies \frac{-2}{-4} = \log_5(x) \\
&\implies \frac{1}{2} = \log_5(x) \\
&\implies x = 5^{1/2} = \sqrt{5}
\end{aligned}$$

(b) (**) (3 points) For what values of x is the following expression true:

$$\log_{10}(x+4) = \log_{10}(x) + \log_{10}(4)$$

Answer:
Using a law of logs to combine the right hand side of the equation, we can rephrase the question as:
For what values of x is the following expression true:

$$\log_{10}(x+4) = \log_{10}(4x)$$

However, it is now clear that the above expression is true only if $x + 4 = 4x$ and $x + 4, 4x > 0$ (this last condition is necessary as the function $\log_{10}$ can only be applied to positive numbers). Solving the linear equation $x + 4 = 4x$ we have

$$x = \frac{4}{3}$$

This is our only answer, and moreover it is not a pseudo answer as $4x$ and $x+4$ are greater than zero.

Question 2.6.

(a) (4 points) Graph of the function

$$f(x) = -\log_2(x+1)$$

Answer:
The graph of the function $f(x)$ can be obtained from the graph of the usual graph of the $\log_2(x)$ by shifting the graph of $\log_2(x)$ horizontally to the left by one unit and then reflecting about the x-axis. The graph of $f(x) = -\log_2(x+1)$ is drawn below

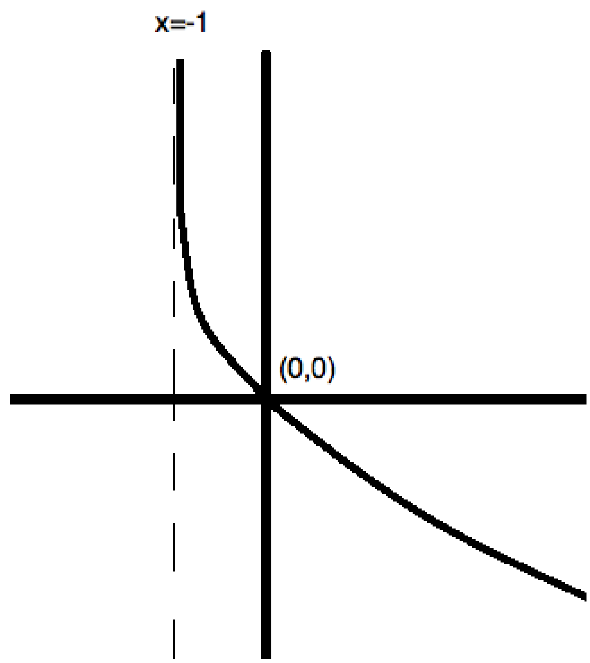

FIGURE 3.

(b) (4 points) Graph of the function

$$g(x) = \log_{\frac{1}{2}}(x+1) - 1$$

Answer:
The graph of the function $g(x)$ can be obtained from the graph of the usual graph of the $\log_{1/2}(x)$ by shifting the graph of $\log_{1/2}(x)$ horizontally to the left by one unit and shifting vertically down by one unit. The graph of $g(x) = \log_{1/2}(x+1) - 1$ is drawn below

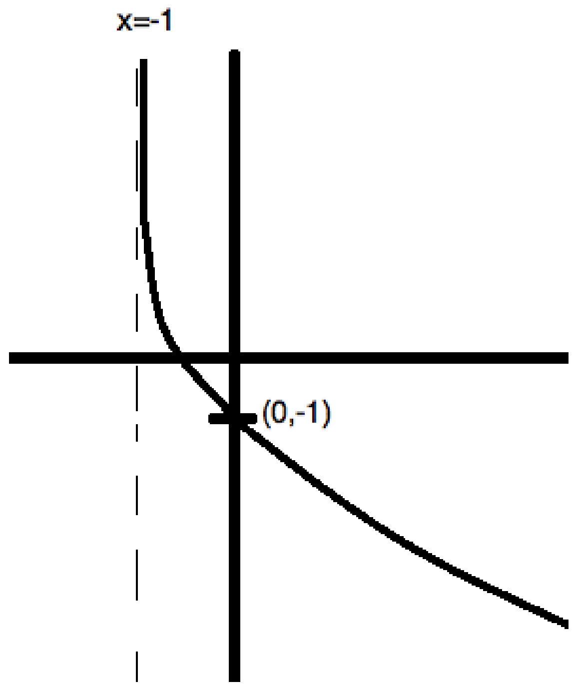

FIGURE 4.

(c) (**) (2 points) Explain the relationship between the functions $f(x), g(x)$ of parts (a) and (b). Justify your answer

The graph of $g(x)$ is obtained by shifting the graph of $f(x)$ vertically down one unit, or equivalently, the graph of $f(x)$ is obtained by shifting the graph of $g(x)$ vertically up one unit.

The reason for this is that from consideration of the definition of the log function and the laws of logs we have the identity

$$-\log_2(x) = \log_2(\frac{1}{x}) = \log_{1/2}(x)$$

Hence, in particular it follows that

$$f(x) = -\log_2(x+1) = \log_{1/2}(x+1) = g(x) + 1$$